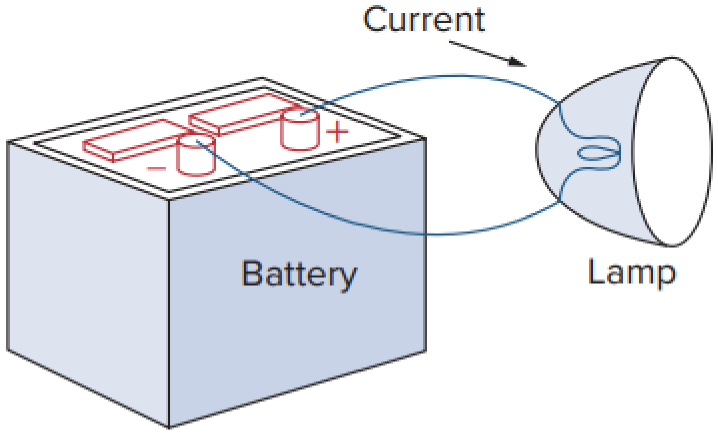
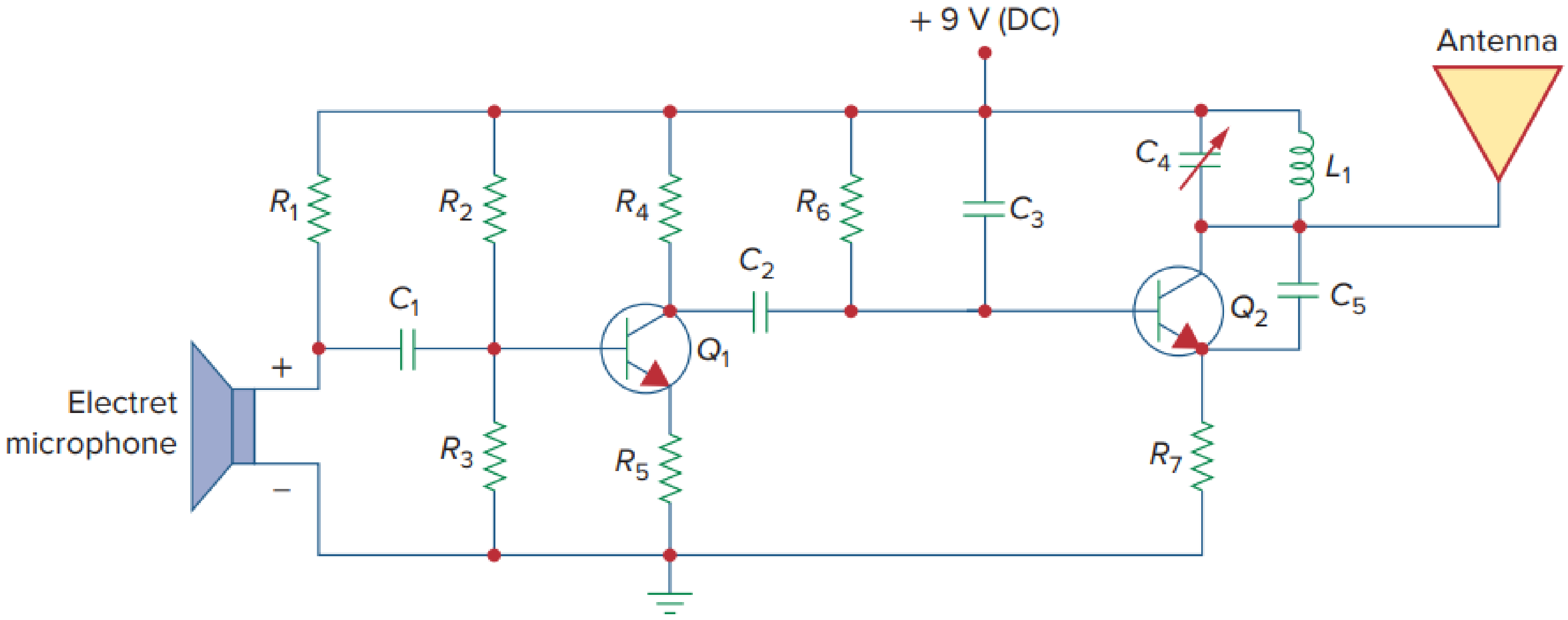
Electrical Circuits I

EEE 101

BUBT

Introduction





System of Units

Six basic SI units and one derived unit relevant to this text.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Charge	coulomb	C

The SI prefixes.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

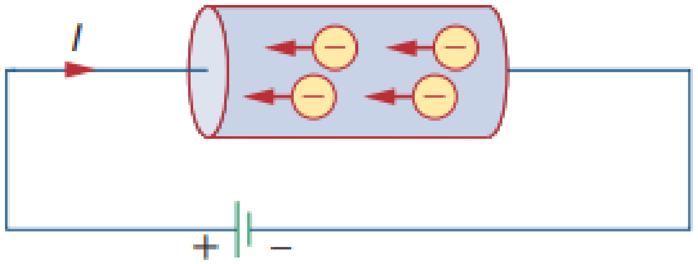
Charge and Current

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

$$Q \triangleq \int_{t_0}^{t} i \, dt$$

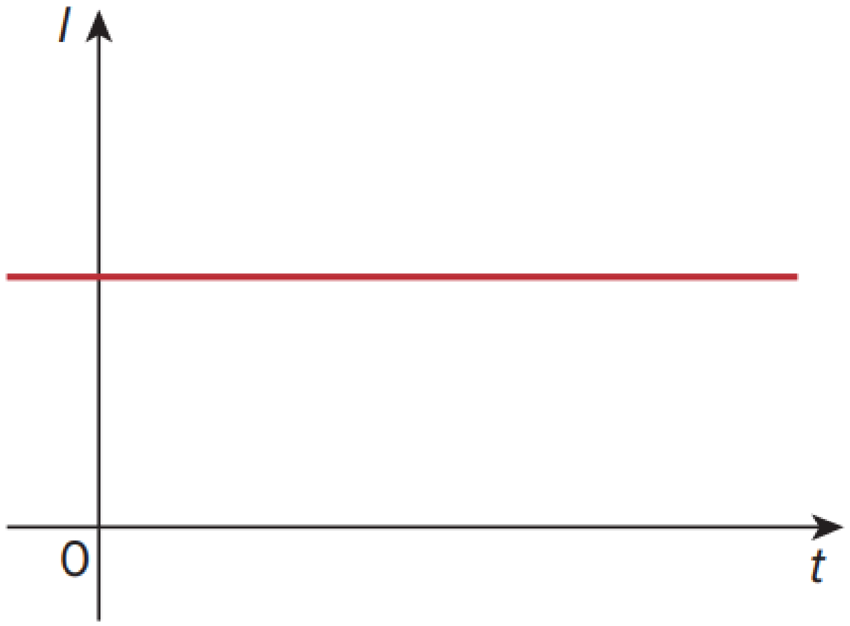
Electric current is the time rate of change of charge, measured in amperes (A).

$$i \triangleq \frac{dq}{dt}$$

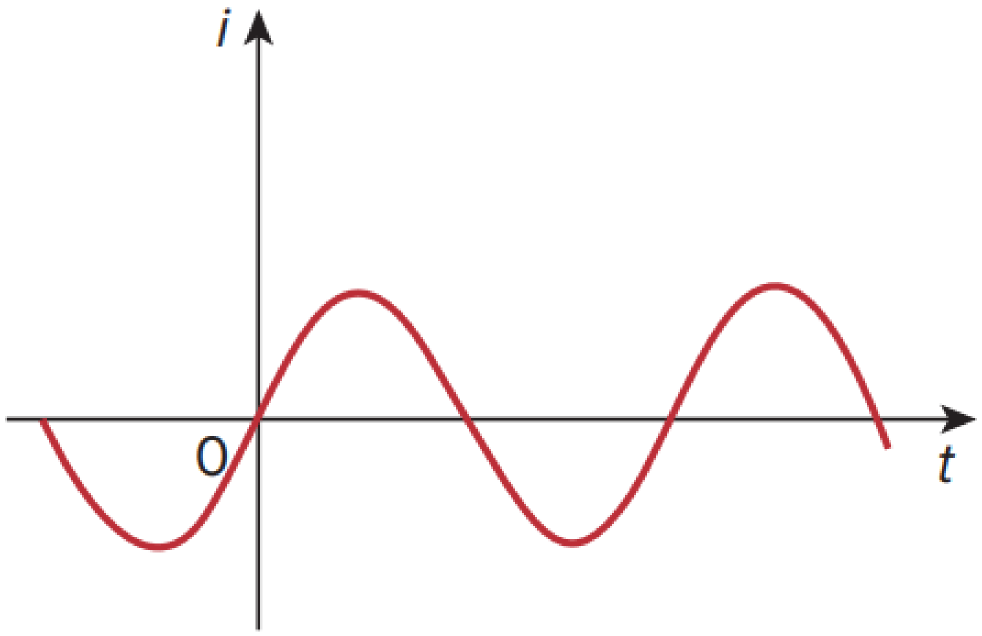


DC and AC

A direct current (dc) flows only in one direction and can be constant or time varying.

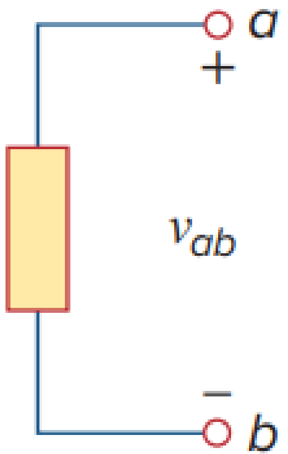


An alternating current (ac) is a current that changes direction with respect to time.



Voltage

Voltage (or potential difference) is the energy required to move a unit charge from a reference point (−) to another point (+), measured in volts (V).



$$v_{ab} \triangleq \frac{dw}{dq}$$

$$v_{ab} = -v_{ba}$$

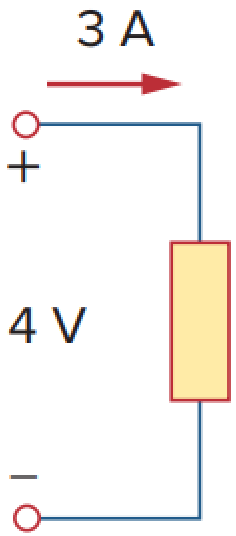
Power and Energy

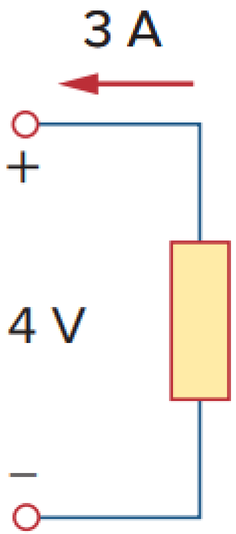
Power is the time rate of expending or absorbing energy, measured in watts (W).

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi$$

p = +vi or vi > 0 implies that the element is absorbing power.

p = −vi or vi < 0 implies that the element is supplying power.





continued…

Total power supplied to the circuit must balance the total power absorbed.

$$\sum p = 0$$

Energy is the capacity to do work, measured in joules (J).

$$w = \int_{t_0}^{t} p\, dt = \int_{t_0}^{t} vi\, dt$$

Circuit Element

Active Element

Capable of generating energy

Example:

- *generators*
- *batteries*
- *operational amplifiers*

The most important active elements are voltage sources or current sources that generally deliver power to the circuit connected to them.

Passive Element

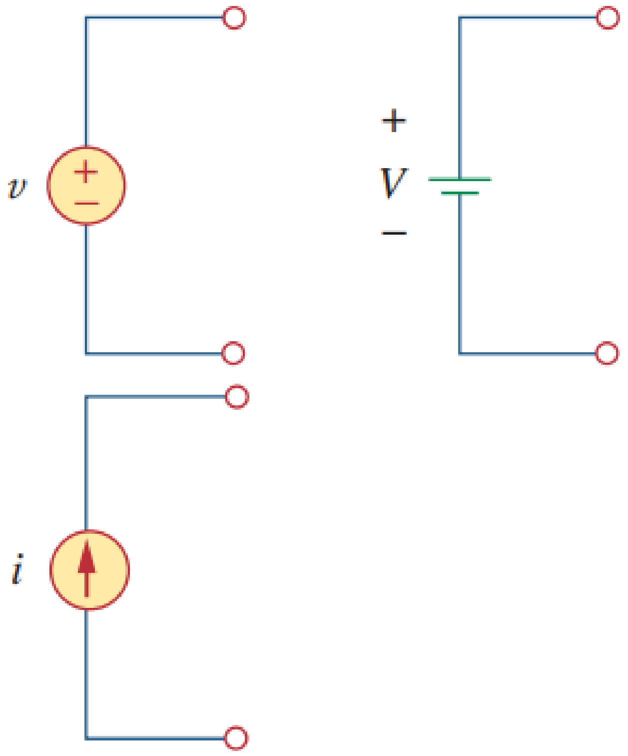
Not capable of generating energy

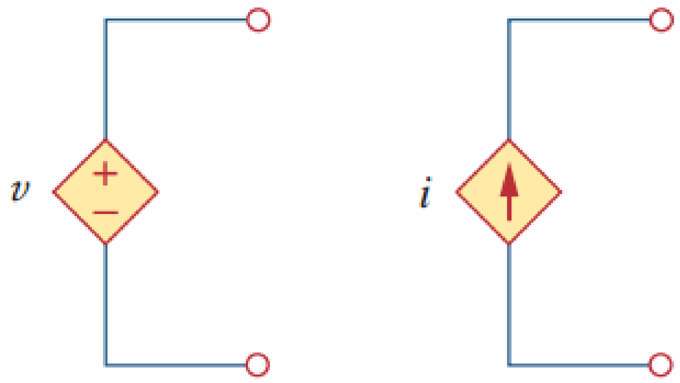
Example:

- *resistors*
- *capacitors*
- *inductors*

Voltage Source and Current Source

- Ideal Independent Source: an active element that provides a specified voltage or current that is completely independent of other circuit elements
- Ideal Dependent (or controlled) Source: an active element in which the source quantity is controlled by another voltage or current





- voltage-controlled voltage source (VCVS)
- current-controlled voltage source (CCVS)
- voltage-controlled current source (VCCS)
- current-controlled current source (CCCS)

Example

How much charge is represented by 4,600 electrons?

-1.602×10^{-19} C/electron $\times$ 4,600 electrons $= -7.369 \times 10^{-16}$ C

The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

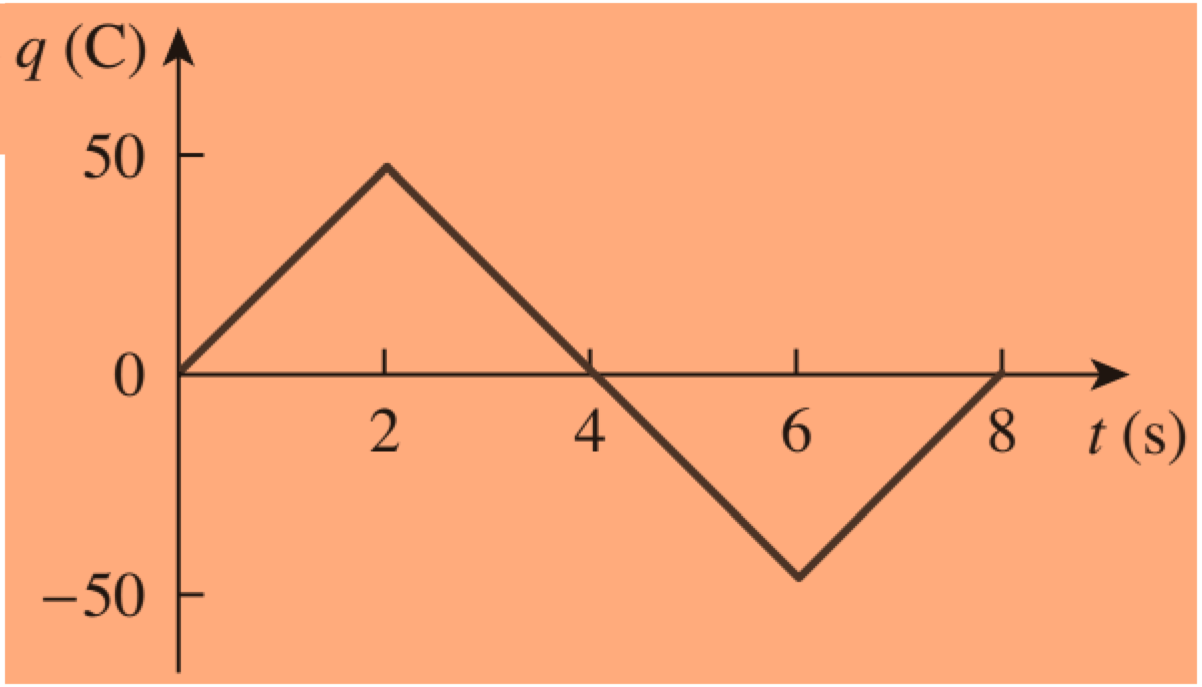
At $t = 0.5$, $i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42$ mA

Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

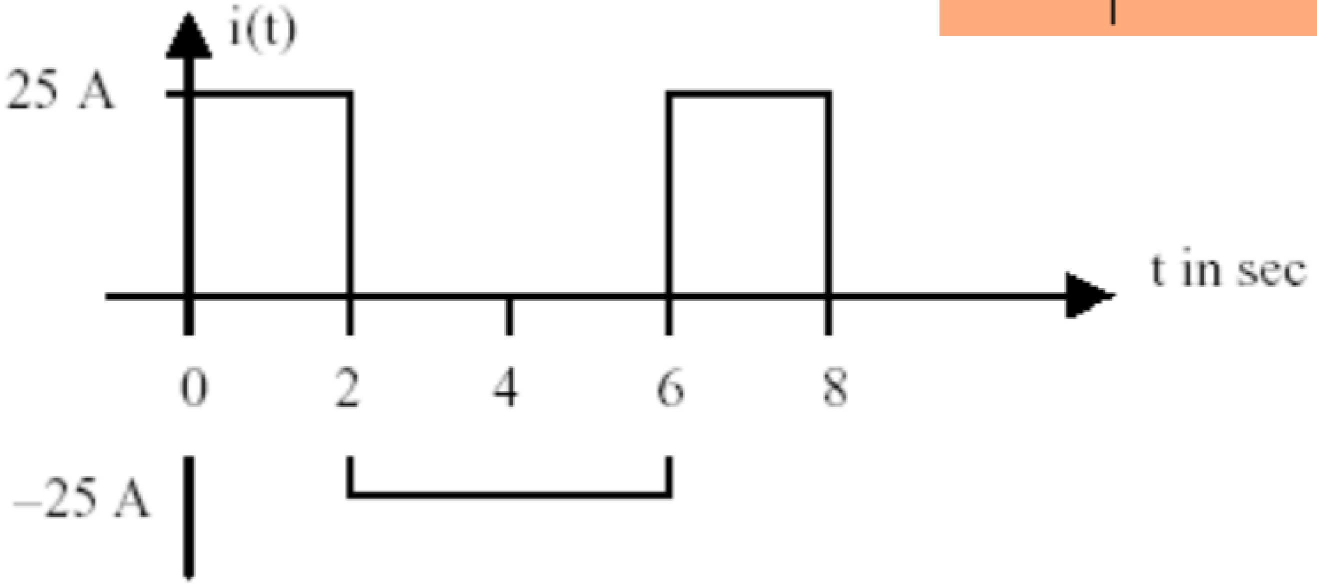
$$Q = \int_{t=1}^{2} i\, dt = \int_{1}^{2} (3t^2 - t)\, dt = \left(t^3 - \frac{t^2}{2}\right)\Bigg|_1^2 = (8 - 2) - \left(1 - \frac{1}{2}\right) = 5.5 \text{ C}$$

continued...

The charge flowing in a wire is plotted. Sketch the corresponding current.

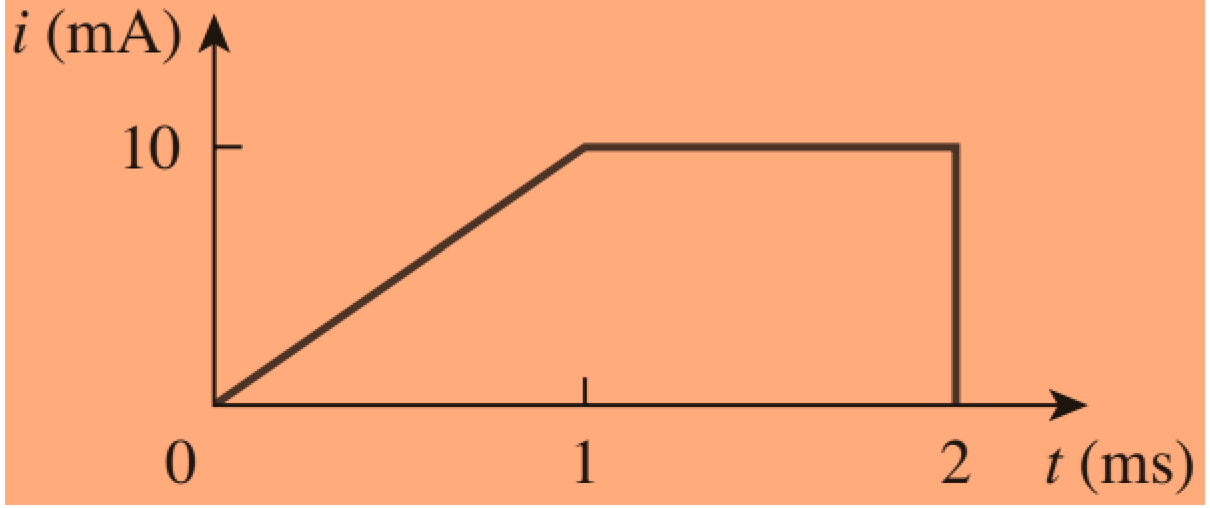


$$i = \frac{dq}{dt} = \begin{cases} 25A, & 0 < t < 2 \\ -25A, & 2 < t < 6 \\ 25A, & 6 < t < 8 \end{cases}$$



continued...

The current flowing past a point in a device is shown Calculate the total charge through the point.



$$q = \int i\,dt$$

$$= \frac{10 \times 1}{2} + 10 \times 1$$

$$= 15\ \mu C$$

Find the charge $q(t)$ flowing through a device if the current is:

$i(t) = 20\cos(10t + \pi/6)\,\mu A,\ q(0) = 1\ \mu C$

$$q(t) = \int 20\cos\left(10t + \pi/6\right) + q(0) \;= (2\sin(10t + \pi/6) + 1)\,\mu C$$

continued…

An energy source forces a constant current of 2 A for 10 s to flow through a light bulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

$$\Delta q = i\,\Delta t = 2 \times 10 = 20\text{ C}$$

$$v = \frac{\Delta w}{\Delta q} = \frac{2.3 \times 10^3}{20} = 115\text{ V}$$

Find the power delivered to an element at $t = 3$ ms if the current entering its positive terminal is

$$i = 5 \cos 60\pi t \text{ A}$$

and the voltage is: (a) $v = 3i$, (b) $v = 3\,di/dt$.

$$v = 3i = 15 \cos 60\pi t$$

$$p = vi = 75 \cos^2 60\pi t \text{ W}$$

At $t = 3$ ms, $p = 75 \cos^2 (60\pi \times 3 \times 10^{-3}) = 75 \cos^2 0.18\pi = 53.48$ W

(a)

continued…

$$v = 3\frac{di}{dt} = 3(-60\pi)5 \sin 60\pi t = -900\pi \sin 60\pi t \text{ V}$$

$$p = vi = -4500\pi \sin 60\pi t \cos 60\pi t \text{ W}$$

At $t = 3$ ms, $p = -4500\pi \sin 0.18\pi \cos 0.18\pi$ W

$$= -14137.167 \sin 32.4^\circ \cos 32.4^\circ = -6.396 \text{ kW}$$

(b)

How much energy does a 100-W electric bulb consume in two hours?

$$w = pt = 100 \text{ (W)} \times 2 \text{ (h)} \times 60 \text{ (min/h)} \times 60 \text{ (s/min)}$$

$$= 720{,}000 \text{ J} = 720 \text{ kJ}$$

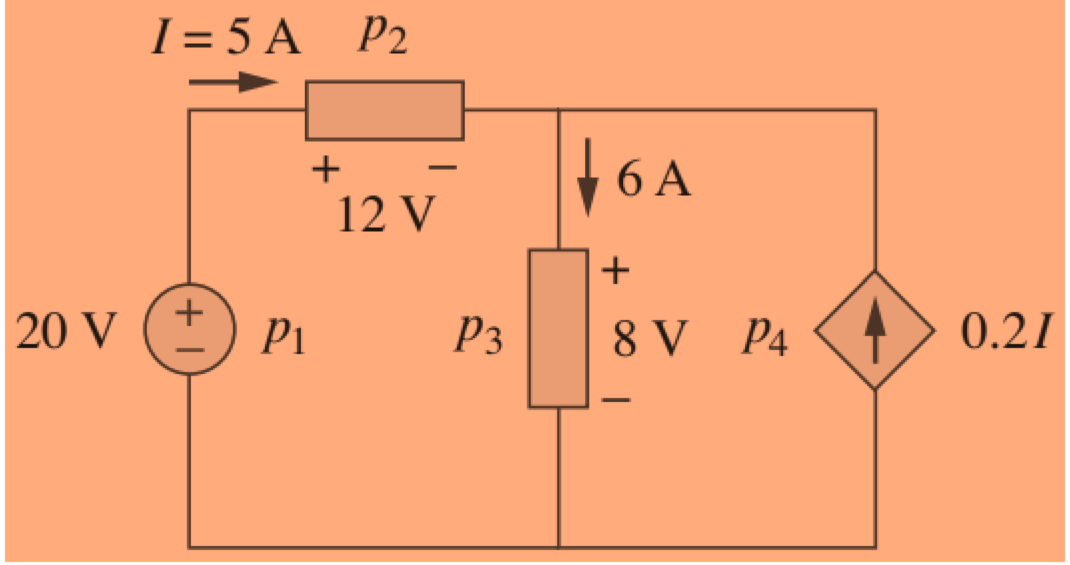
or,

$$w = pt = 100 \text{ W} \times 2 \text{ h} = 200 \text{ Wh}$$

$$= 720 \text{ kJ}$$

continued...

Calculate the power supplied or absorbed by each element



$$p_1 = 20(-5) = -100\text{ W} \quad \text{Supplied power}$$

$$p_2 = 12(5) = 60\text{ W} \quad \text{Absorbed power}$$

$$p_3 = 8(6) = 48\text{ W} \quad \text{Absorbed power}$$

$$\begin{aligned} p_4 &= 8(-0.2I) \\ &= 8(-0.2 \times 5) \\ &= -8\text{ W} \quad \text{Supplied power} \end{aligned}$$

$$p_1 + p_2 + p_3 + p_4 = -100 + 60 + 48 - 8 = 0$$

continued…

The voltage v across a device and the current i through it are

$$v(t) = 10 \cos 2t \text{ V}, \qquad i(t) = 20(1 - e^{-0.5t}) \text{ mA}$$

Calculate:

(a) the total charge in the device at $t = 1$ s

(b) the power consumed by the device at $t = 1$ s.

(a) $q = \int i dt = \int_0^1 0.02(1 - e^{-0.5t})dt = 0.02(t + 2e^{-0.5t})\Big|_0^1$

$= 0.02(1 + 2e^{-0.5} - 2) = \mathbf{4.261\ mC}$

(b) $p(t) = v(t)i(t)$

$p(1) = 10\cos(2)x0.02(1 - e^{-0.5}) = (-4.161)(0.007869)$

$= \mathbf{-32.74\ mW}$

continued…

The current entering the positive terminal of a device is $i(t) = 6e^{-2t}$ mA and the voltage across the device is $v(t) = 10di/dt$ V.

(a) Find the charge delivered to the device between $t = 0$ and $t = 2$ s.

(b) Calculate the power absorbed.

(c) Determine the energy absorbed in 3 s.

(a) $q = \int i\,dt$

$$= \int_0^2 0.006e^{-2t}\,dt$$

$$= \frac{-0.006}{2} e^{-2t} \Big|_0^2$$

$$= -0.003\left(e^{-4} - 1\right)$$

$$= \mathbf{2.945\ mC}$$

(b) $$v = \frac{10di}{dt} = -0.012e^{-2t}(10) = -0.12e^{-2t}\ \text{V}$$

$$p(t) = v(t)i(t) = (-0.12e^{-2t})(0.006e^{-2t}) = \mathbf{-720e^{-4t}\ \mu W}$$

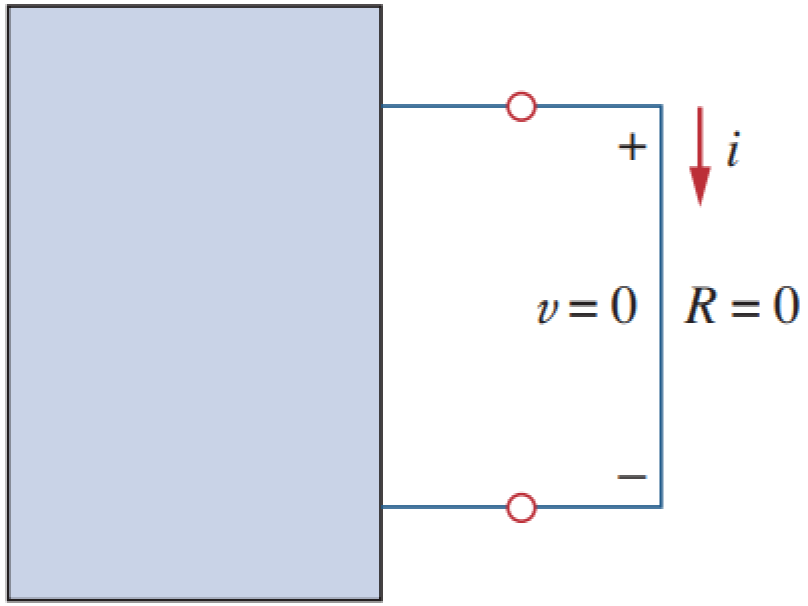
(c) $$w = \int p\,dt = \frac{-720}{-4} e^{-4t} 10^{-6} \Big|_0^3 = \mathbf{-180\ \mu J}$$

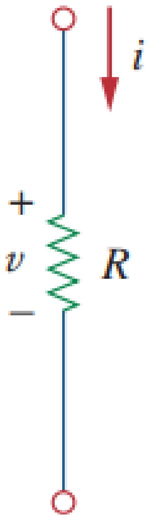
Ohm's Law

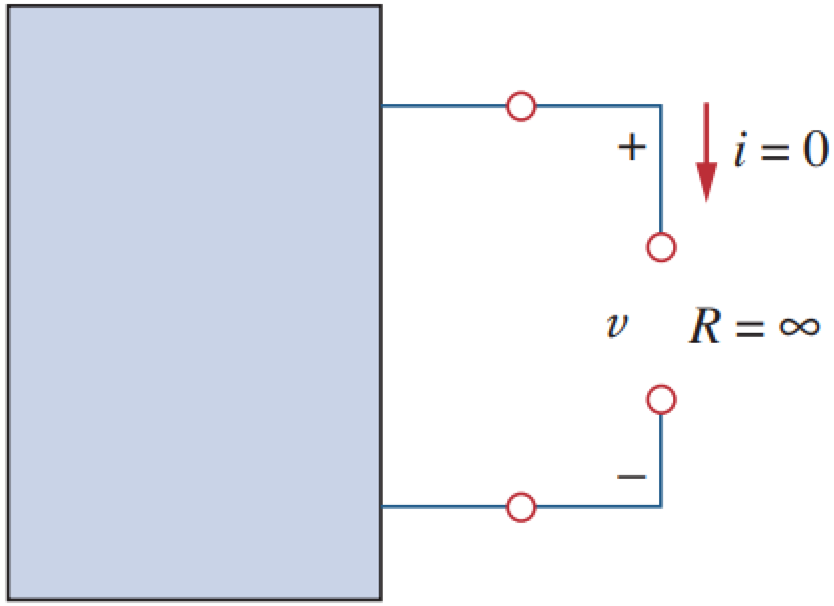
The voltage (v) across a resistor is directly proportional to the current (i) flowing through the resistor.

$$v \propto i$$

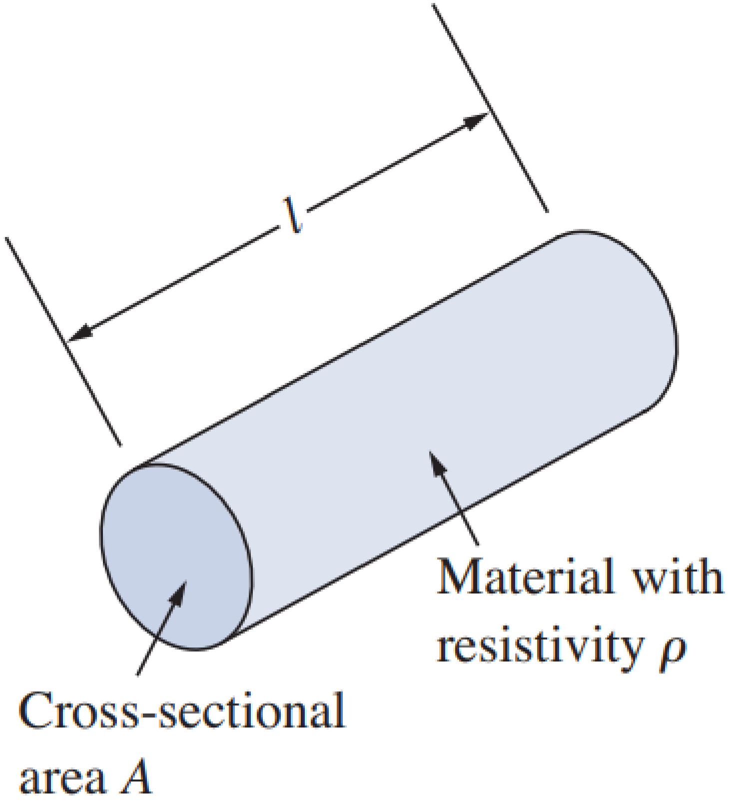
$$v = iR$$







Resistivity



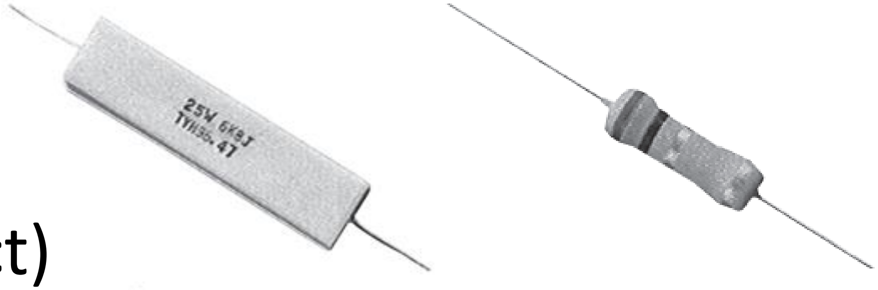
$$R = \rho\frac{\ell}{A}$$

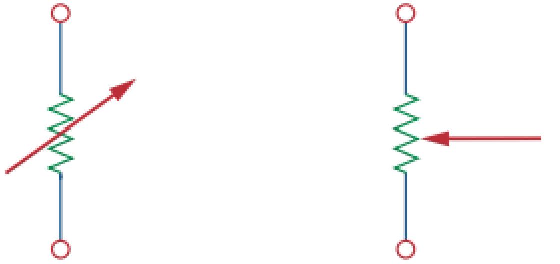
Resistivities of common materials.

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

Some Terms

- Resistance: Physical property of object
- Resistivity: Physical property of material
- Resistor or Conductor: Circuit Element (Object)
- Conductance: 1 / Resistance
- Conductivity: 1 / Resistivity





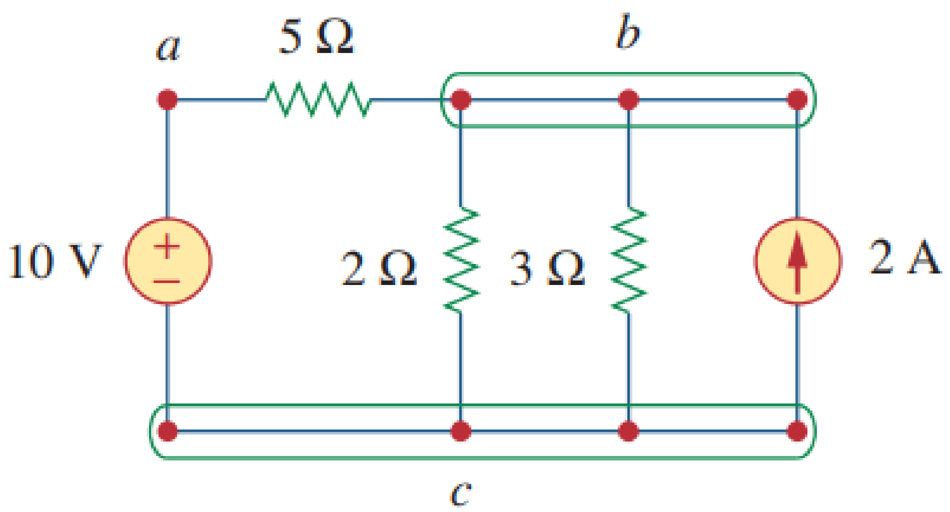
$$G = \frac{1}{R} = \frac{i}{v}$$

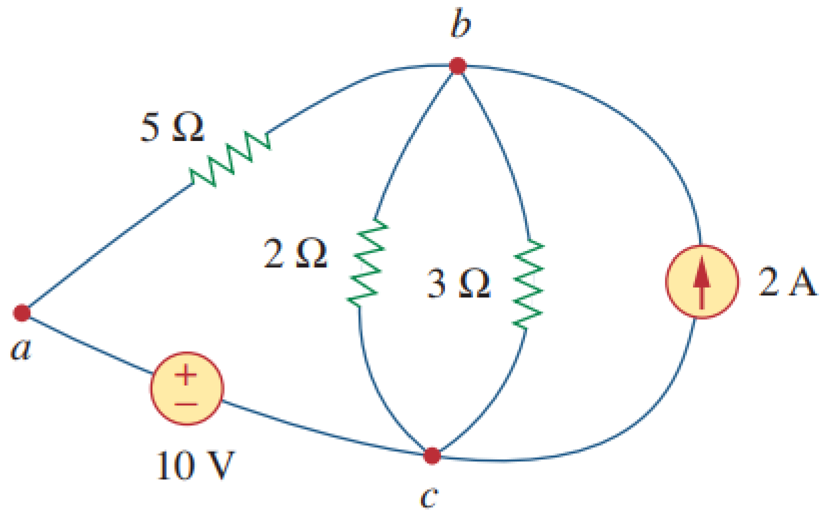
$$1\ \Omega = 1\ \mathrm{V/A}$$

$$1\ \mathrm{S} = 1\ \mho = 1\ \mathrm{A/V}$$

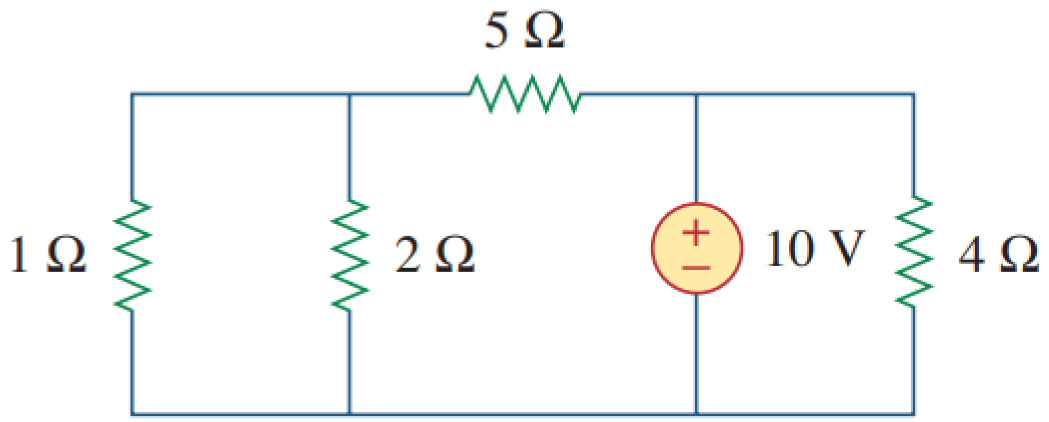
$$p = vi = i^2R = \frac{v^2}{R} = v^2G = \frac{i^2}{G}$$

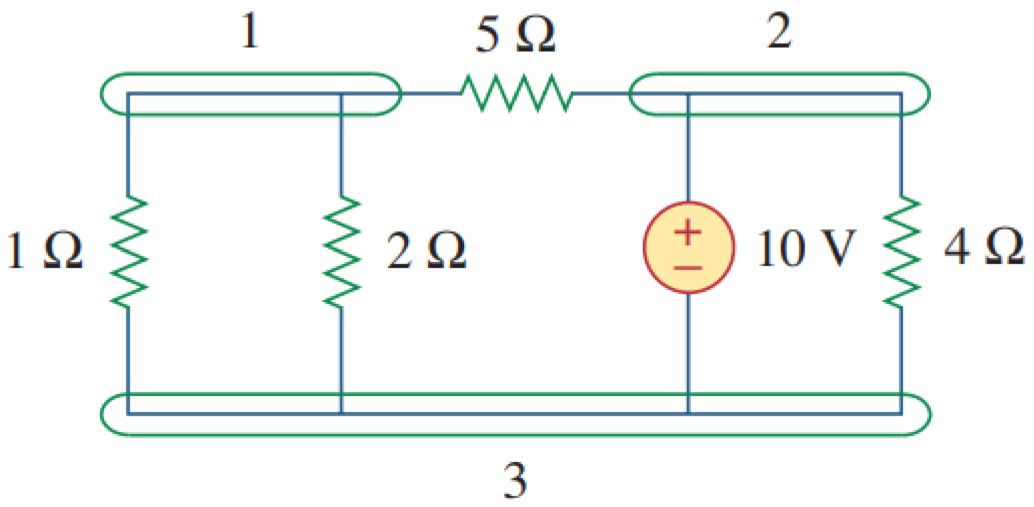
Nodes, Branches, and Loops





$$b = l + n - 1$$

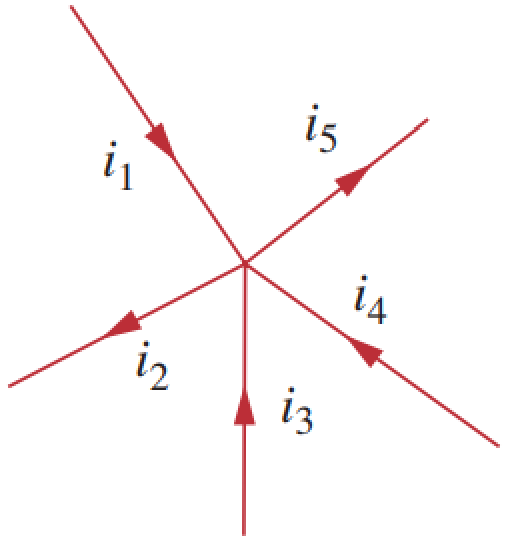




Kirchhoff's Laws

KCL

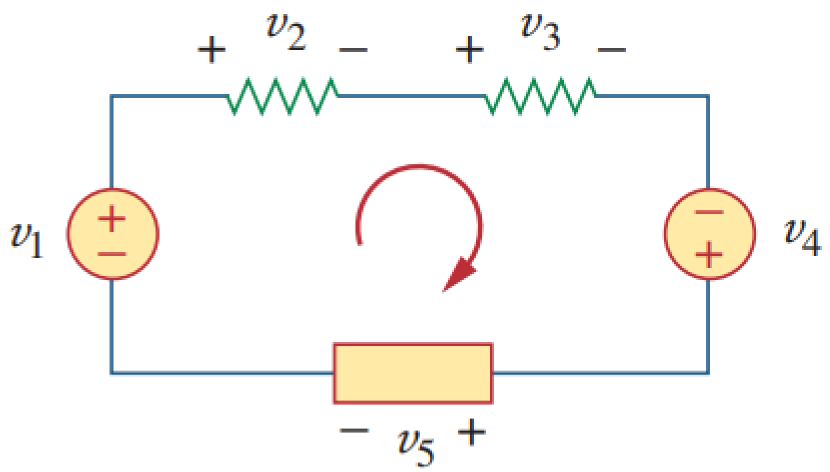
Algebraic sum of all currents entering a node is zero.



$$i_1 + i_3 + i_4 = i_2 + i_5$$

KVL

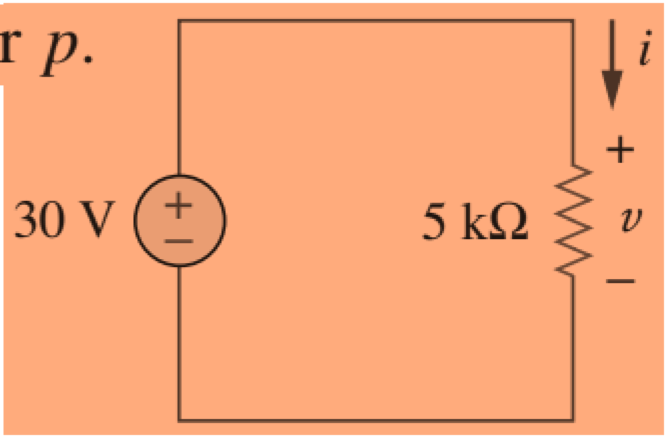
Algebraic sum of all voltages around a closed path is zero.



$$v_2 + v_3 + v_5 = v_1 + v_4$$

Example

calculate the current i, the conductance G, and the power p.



$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6\text{ mA}$$

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2\text{ mS}$$

$$p = vi = 30(6 \times 10^{-3}) = 180\text{ mW}$$

or $\quad p = i^2R = (6 \times 10^{-3})^2 5 \times 10^3 = 180\text{ mW}$

or $\quad p = v^2G = (30)^2 0.2 \times 10^{-3} = 180\text{ mW}$

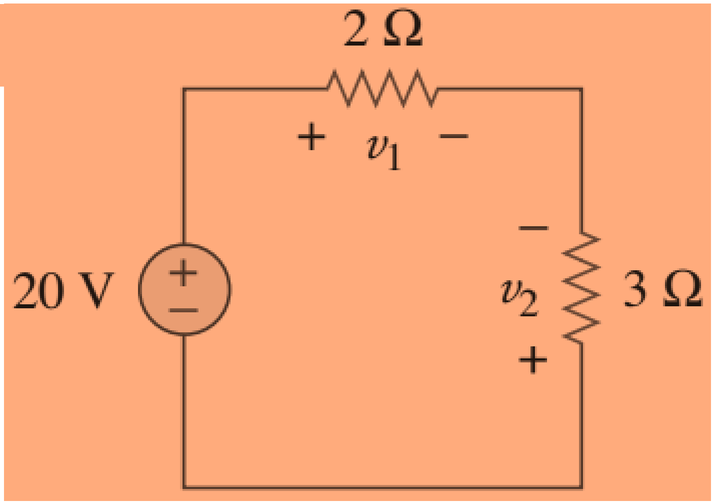
A voltage source of $20 \sin \pi t$ V is connected across a 5-kΩ resistor. Find the current through the resistor and the power dissipated.

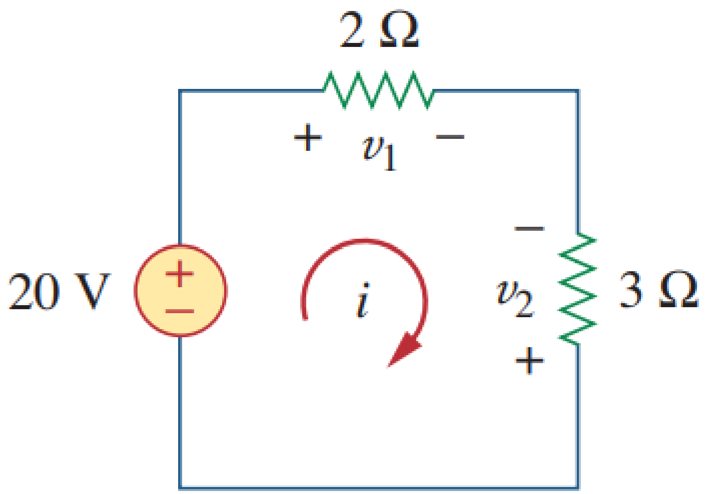
$$i = \frac{v}{R} = \frac{20 \sin \pi t}{5 \times 10^3} = 4 \sin \pi t\text{ mA}$$

$$p = vi = 80 \sin^2 \pi t\text{ mW}$$

continued...

find voltages v_1 and v_2.





$$-20 + v_1 - v_2 = 0$$

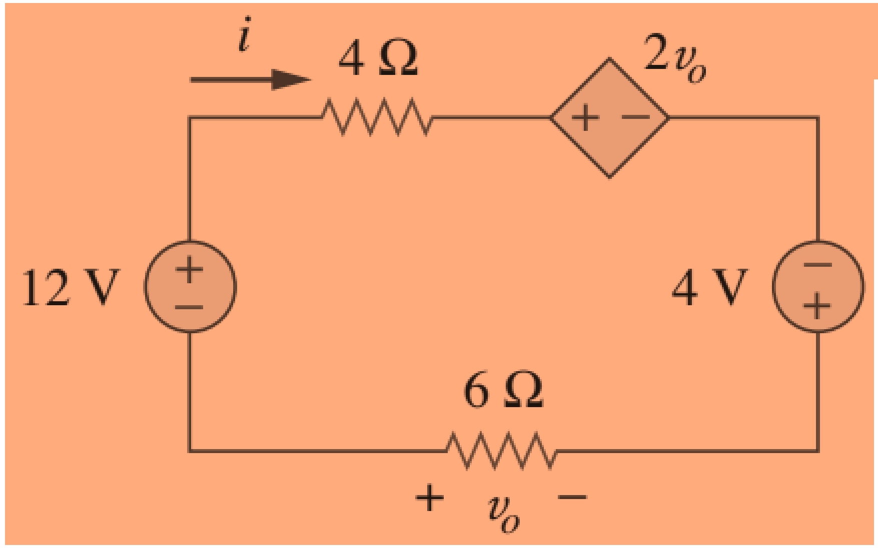
$$v_1 = 2i$$

$$v_2 = -3i$$

$$-20 + 2i + 3i = 0$$

$$\Rightarrow \qquad i = 4 \text{ A}$$

$$v_1 = 8 \text{ V}, \qquad v_2 = -12 \text{ V}$$



Determine v_o and i in the circuit

$$-12 + 4i + 2v_o - 4 + 6i = 0$$

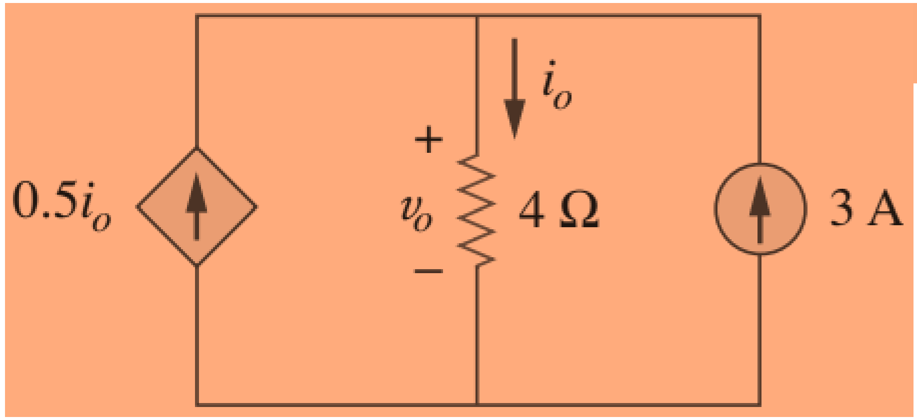
$$v_o = -6i$$

$$-16 + 10i - 12i = 0$$

$$\Rightarrow \qquad i = -8 \text{ A}$$

$$v_o = 48 \text{ V}.$$

continued…



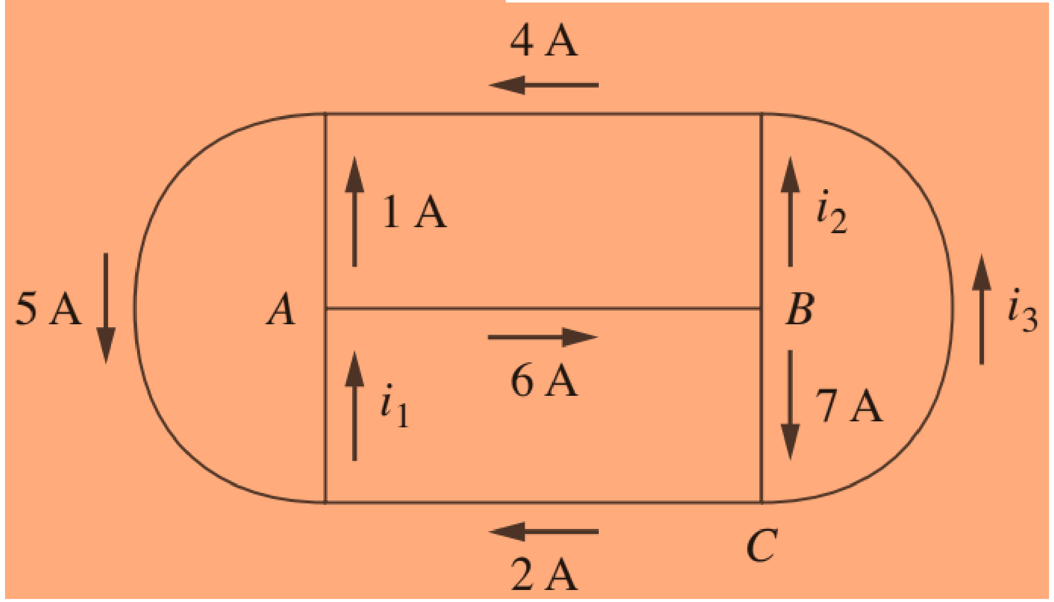
Find current i_o and voltage v_o in the circuit

$$3 + 0.5i_o = i_o$$

$$\Rightarrow \quad i_o = 6\text{ A}$$

$$v_o = 4i_o = 24\text{ V}$$

Find i_1, i_2, and i_3



At A, $1+6-i_1 = 0$

or $i_1 = 1+6 = \mathbf{7\ A}$

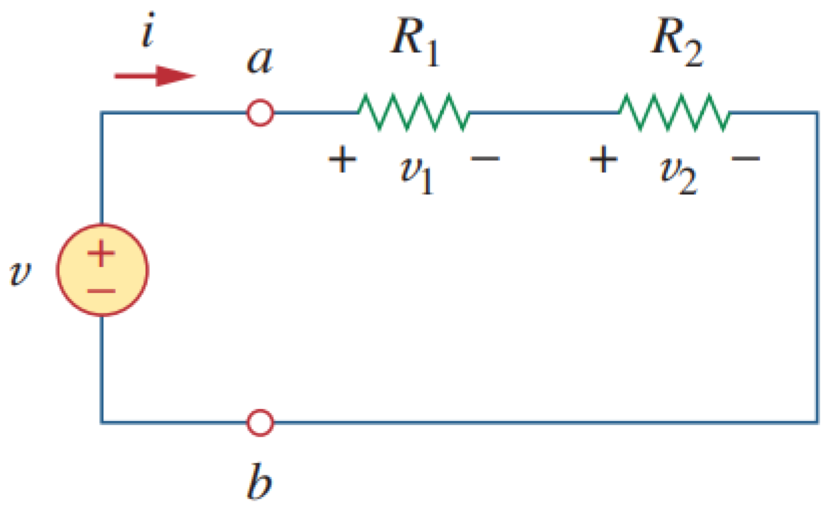
At B, $-6+i_2+7 = 0$

or $i_2 = 6-7 = \mathbf{-1\ A}$

At C, $2+i_3-7 = 0$

or $i_3 = 7-2 = \mathbf{5\ A}$

Series Resistors and Voltage Division

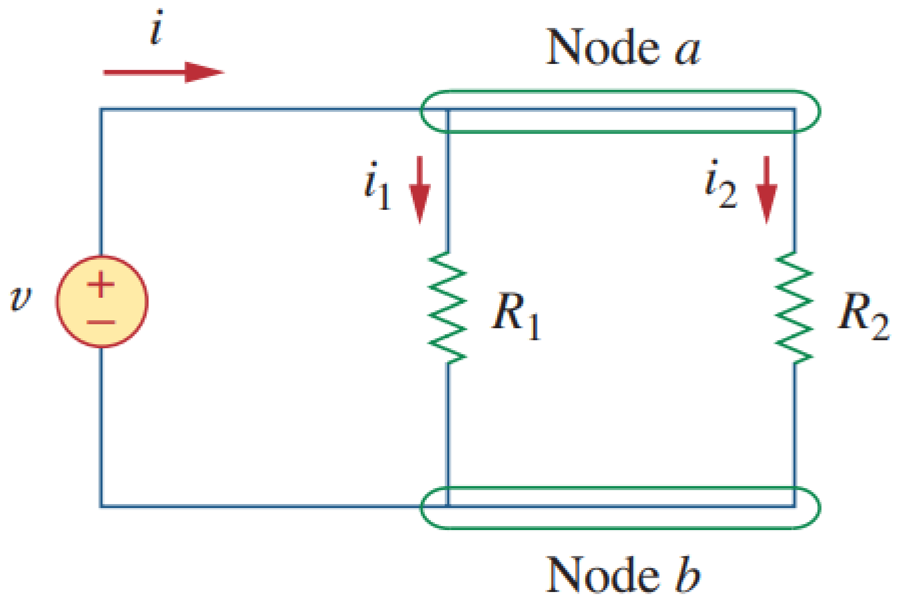


$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

Parallel Resistors and Current Division



$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$

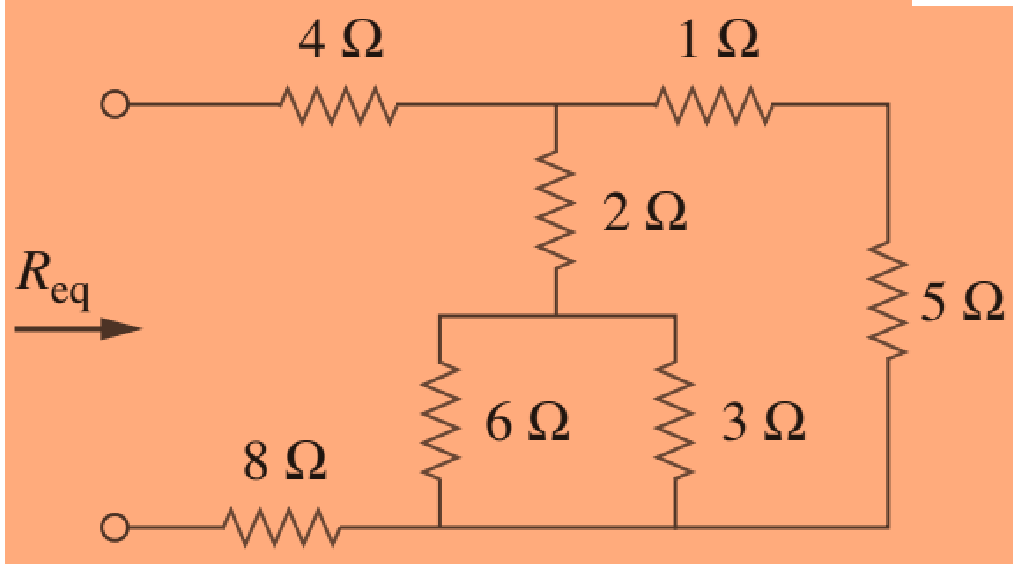
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

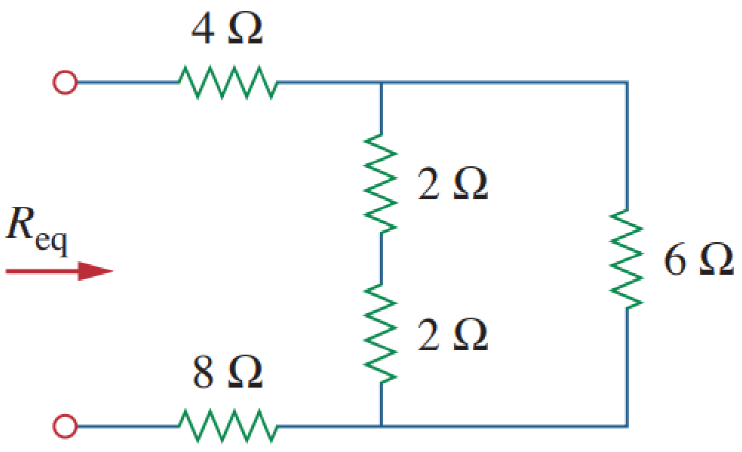
$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

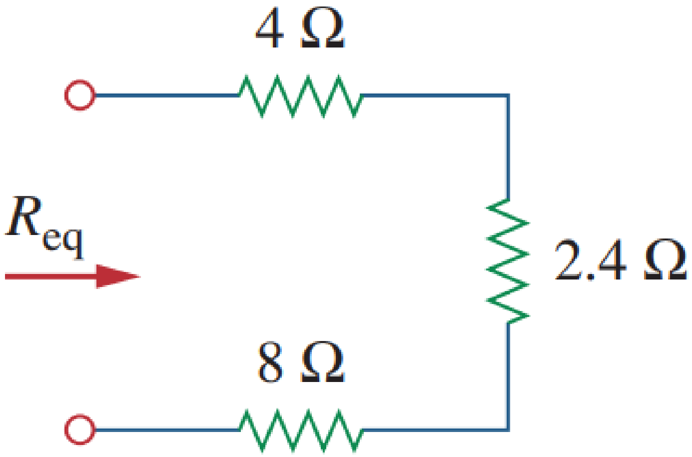
$$i_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} i$$

Example

Find R_{eq} for the circuit shown







$$6\ \Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

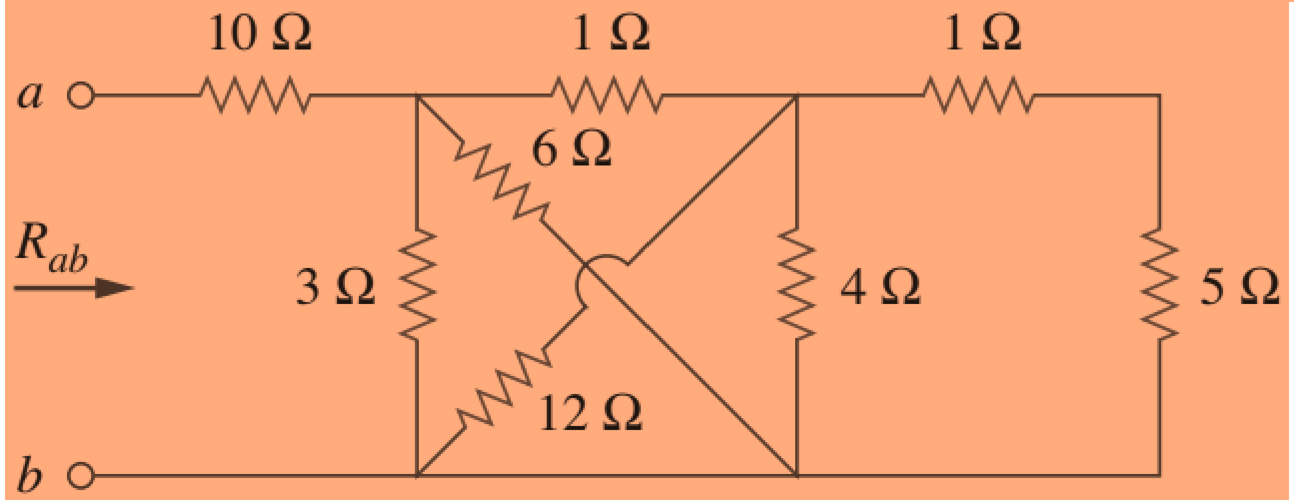
$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

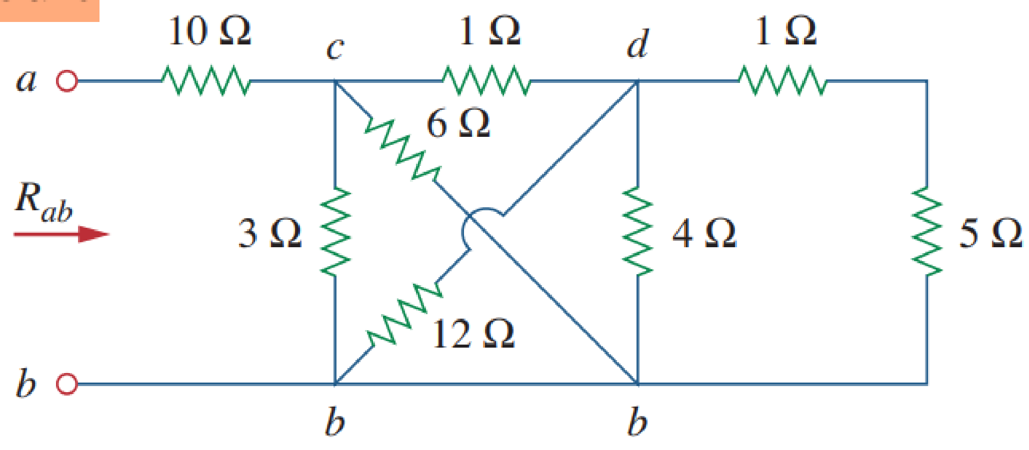
$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$

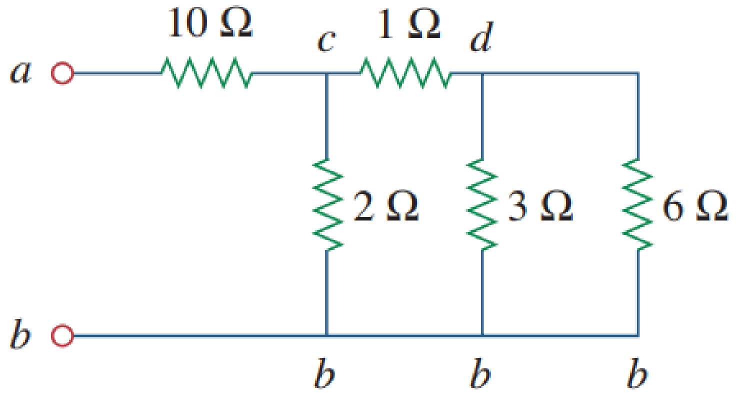
$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

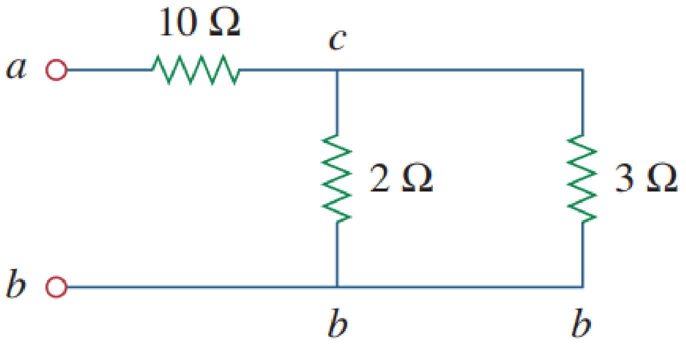
continued...

Calculate the equivalent resistance R_{ab} in the circuit









$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3 + 6} = 2\ \Omega$$

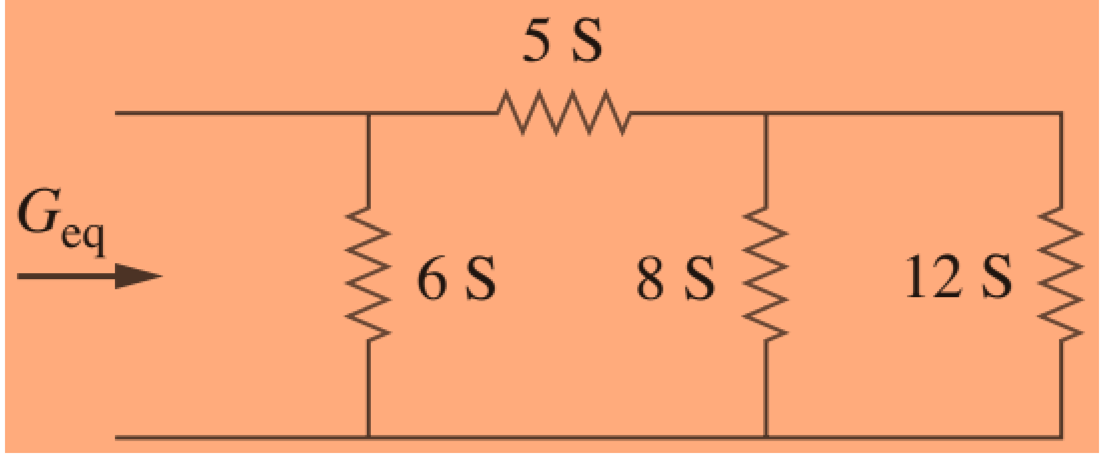
$$12\ \Omega \parallel 4\ \Omega = \frac{12 \times 4}{12 + 4} = 3\ \Omega$$

$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$

continued...

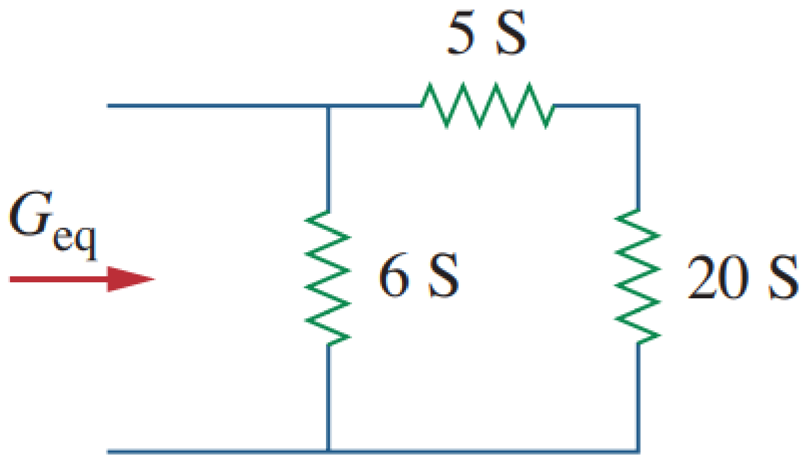
Find the equivalent conductance G_{eq} for the circuit



$$8\ \text{S} + 12\ \text{S} = 20\ \text{S}$$

$$\frac{20 \times 5}{20 + 5} = 4\ \text{S}$$

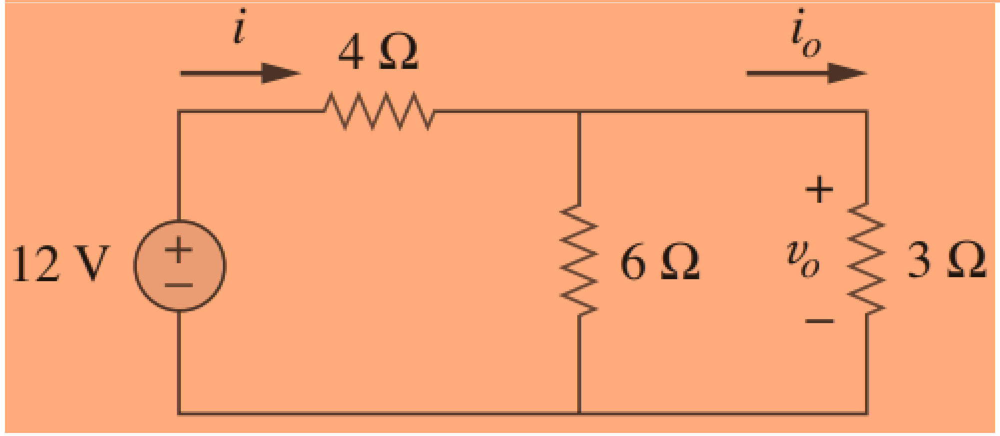
$$G_{eq} = 6 + 4 = 10\ \text{S}$$



continued…

Find i_o and v_o in the circuit

Calculate the power dissipated in the 3-Ω resistor.



$$6\ \Omega \parallel 3\ \Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

$$v_o = \frac{2}{2 + 4}(12\ \text{V}) = 4\ \text{V}$$

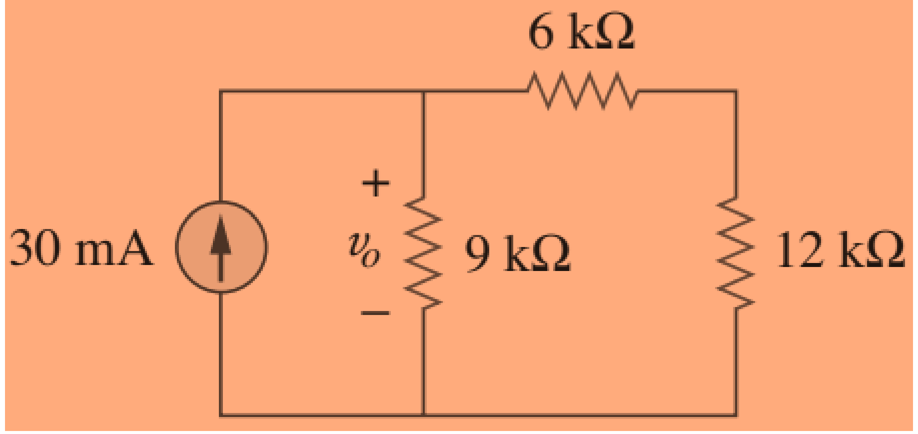
$$v_o = 3i_o = 4$$

$$\Rightarrow \qquad i_o = \frac{4}{3}\ \text{A}$$

$$p_o = v_o i_o = 4\left(\frac{4}{3}\right) = 5.333\ \text{W}$$

continued…

For the circuit shown determine: (a) the voltage v_o, (b) the power supplied by the current source, (c) the power absorbed by each resistor.



(a) $$i_1 = \frac{18{,}000}{9{,}000 + 18{,}000}(30\text{ mA}) = 20\text{ mA}$$

$$i_2 = \frac{9{,}000}{9{,}000 + 18{,}000}(30\text{ mA}) = 10\text{ mA}$$

$$v_o = 9{,}000 i_1 = 180\text{ V}$$

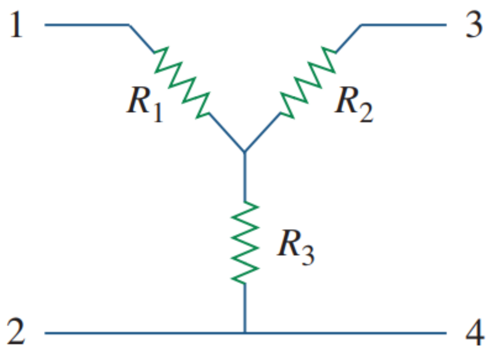
(b) $p_o = v_o i_o = 180(30)\text{ mW} = 5.4\text{ W}$

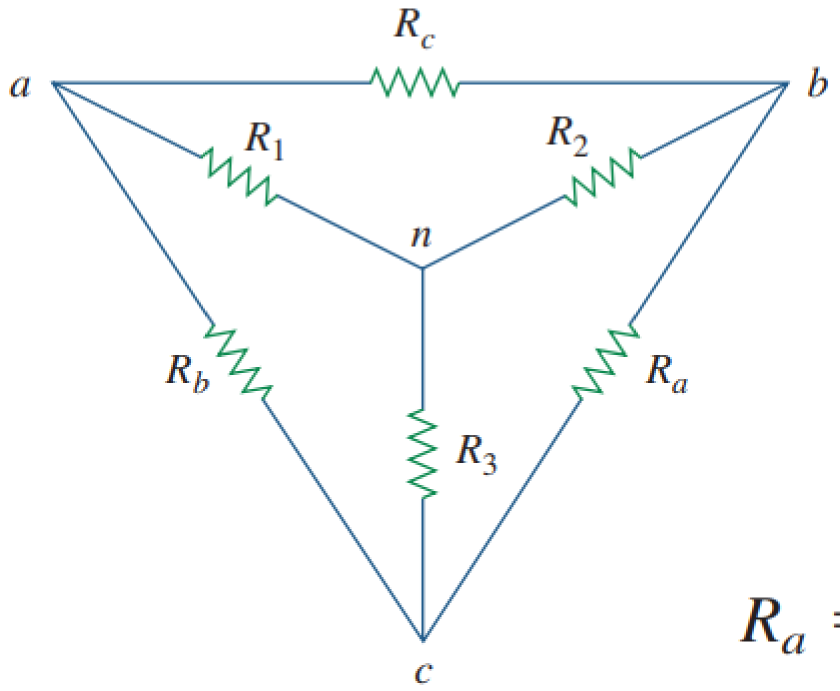
(c) Power absorbed by the 12-kΩ resistor $= i_2^2 R = (10 \times 10^{-3})^2\,(12{,}000) = 1.2\text{ W}$

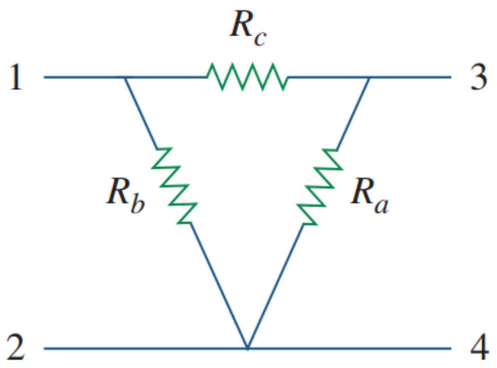
Power absorbed by the 6-kΩ resistor $= i_2^2 R = (10 \times 10^{-3})^2\,(6{,}000) = 0.6\text{ W}$

Power absorbed by the 9-kΩ resistor $= v_o i_1 = 180(20)\text{ mW} = 3.6\text{ W}$

Wye-Delta Transformations







$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

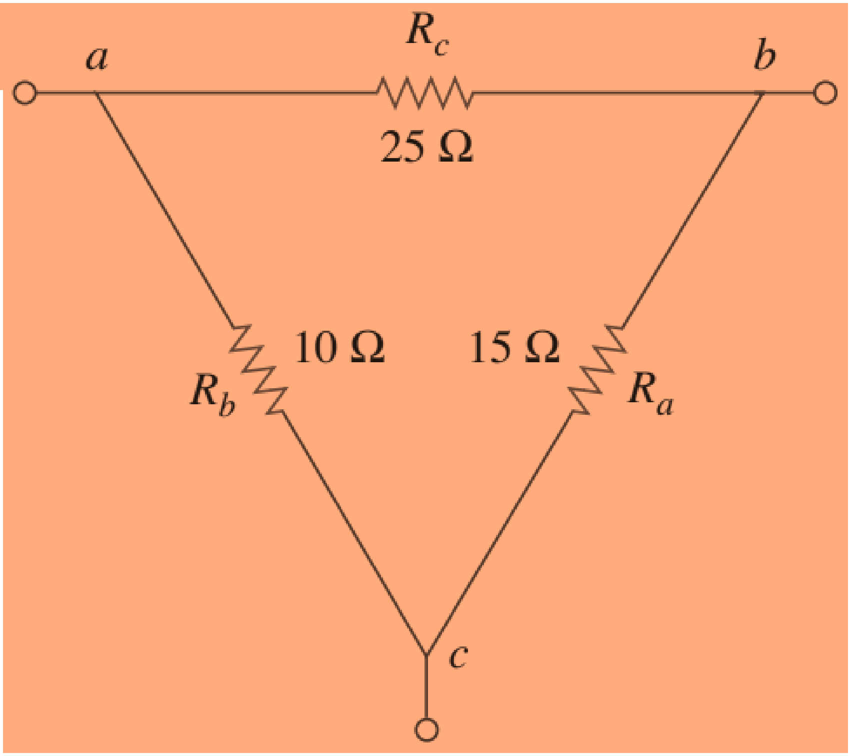
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Example

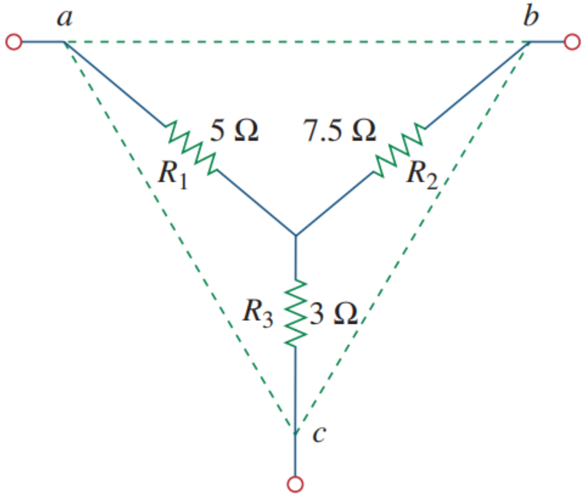
Convert the Δ network to an equivalent Y network.



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

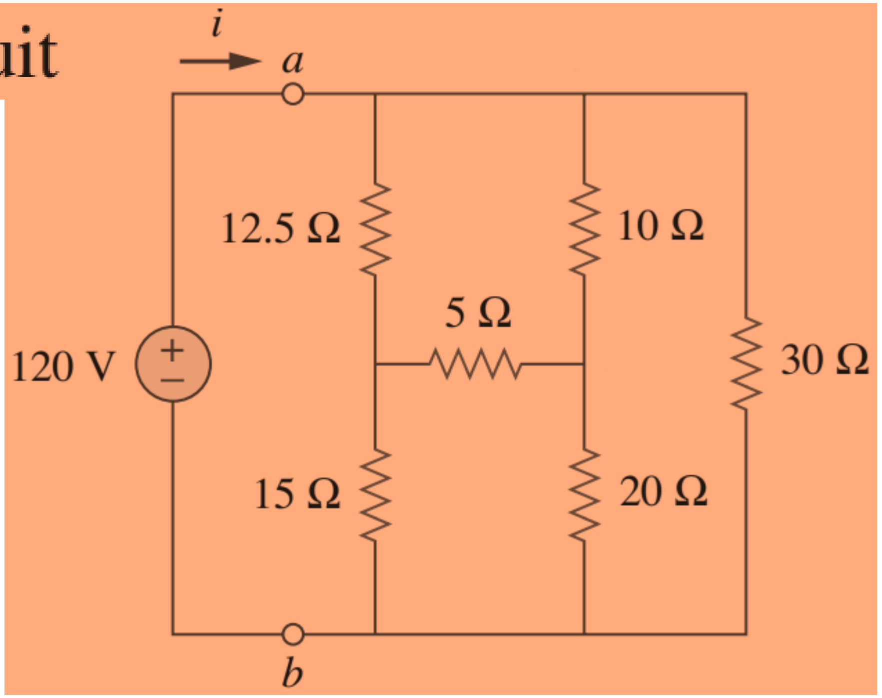
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$



continued…

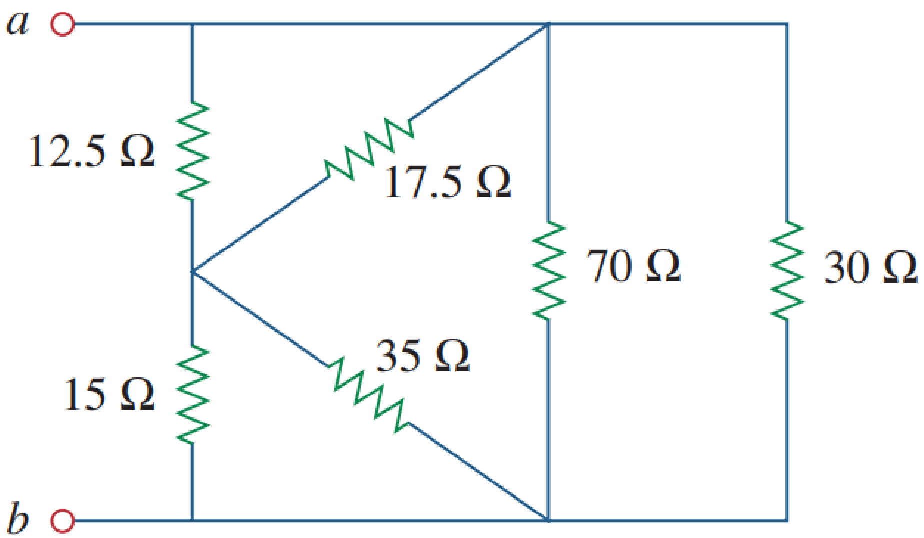
Obtain the equivalent resistance R_{ab} for the circuit find current i.



$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10} = \frac{350}{10} = 35\ \Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$



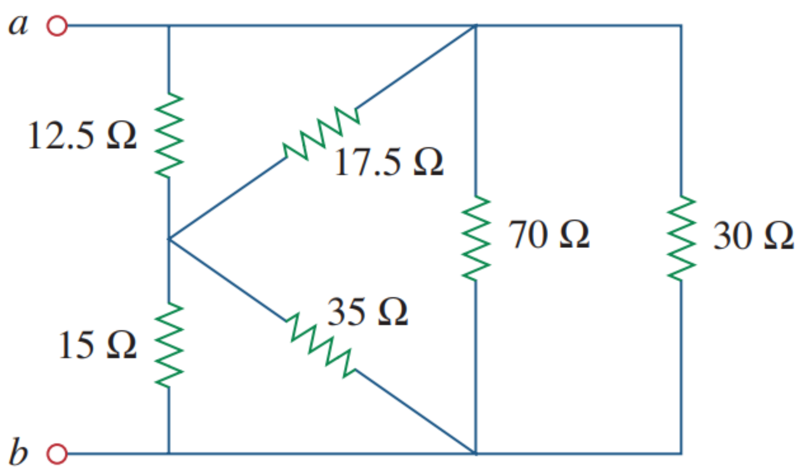
continued…

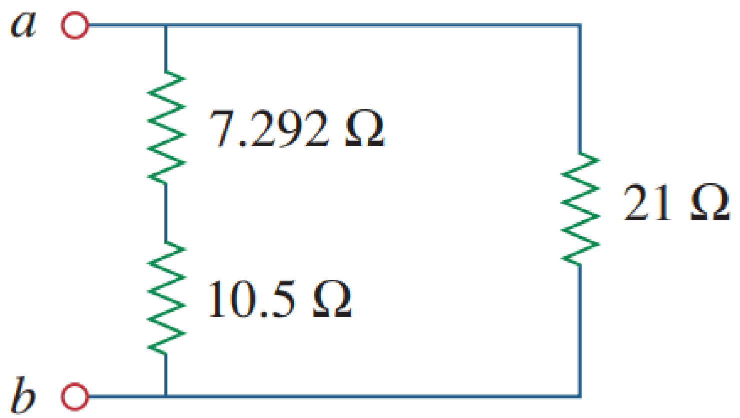
$$70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21\ \Omega$$

$$12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292\ \Omega$$

$$15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632\ \Omega}$$

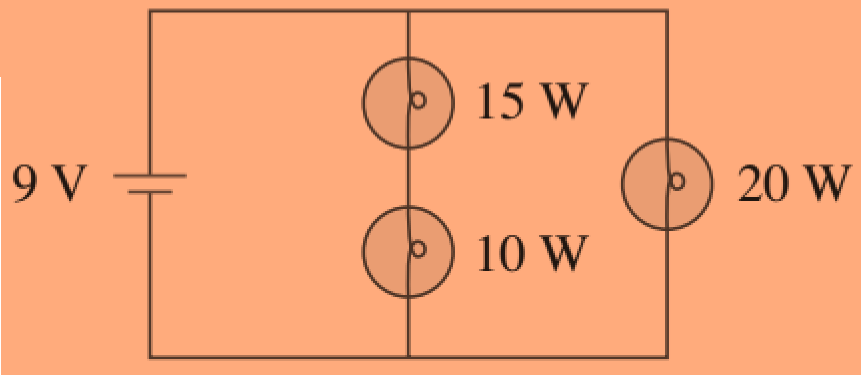




$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458\ A}$$

continued…

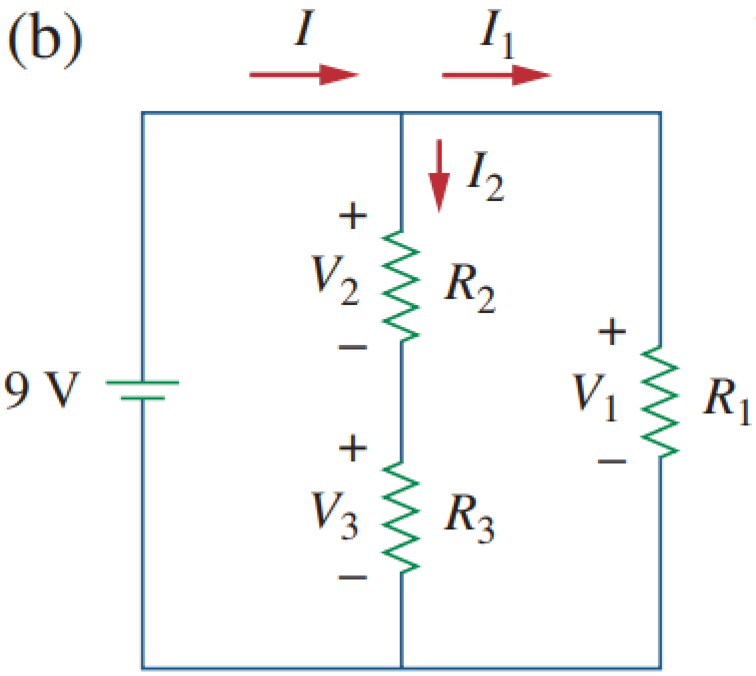
Calculate: (a) the total current supplied by the battery, (b) the current through each bulb, (c) the resistance of each bulb.



(a) $p = 15 + 10 + 20 = 45 \text{ W}$

$p = VI, \quad I = \dfrac{p}{V} = \dfrac{45}{9} = 5 \text{ A}$

(b)



$V_1 = V_2 + V_3 = 9 \text{ V}$

$I_1 = \dfrac{p_1}{V_1} = \dfrac{20}{9} = 2.222 \text{ A}$

$I_2 = I - I_1 = 5 - 2.222 = 2.778 \text{ A}$

(c) Since $p = I^2R$, $R_1 = \dfrac{p_1}{I_1^2} = \dfrac{20}{2.222^2} = 4.05\ \Omega$

$R_2 = \dfrac{p_2}{I_2^2} = \dfrac{15}{2.777^2} = 1.945\ \Omega$

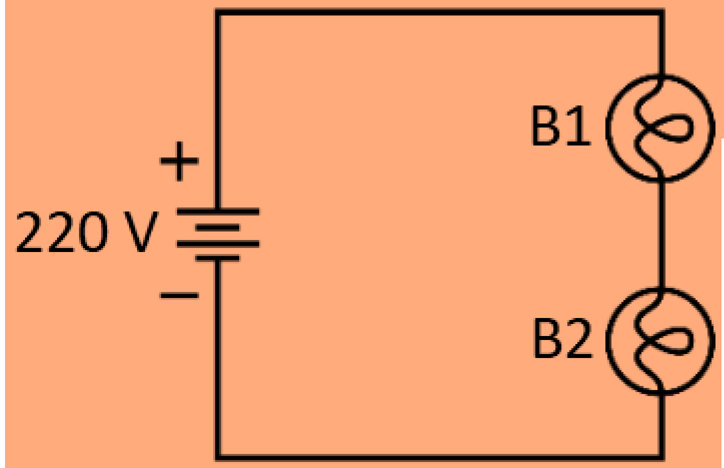
$R_3 = \dfrac{p_3}{I_3^2} = \dfrac{10}{2.777^2} = 1.297\ \Omega$

continued...

Which bulb is brighter? [in this series combination]

B1 rated: 220 V – 60 W

B2 rated: 220 V – 100 W



$$P = {V^2}/{R}$$

$$R_{60} = 220^2/60 = 807\ \Omega$$

$$R_{100} = 220^2/100 = 484\ \Omega$$

$$I = {V}/{R} = {220}/{(807 + 484)} = 0.17\ A$$

Power dissipated in Bulb1, $P_{60} = I^2 R_{60} = 0.17^2 \cdot 807 = 23.3\ W$

Power dissipated in Bulb2, $P_{100} = I^2 R_{100} = 0.17^2 \cdot 484 = 13.9\ W$

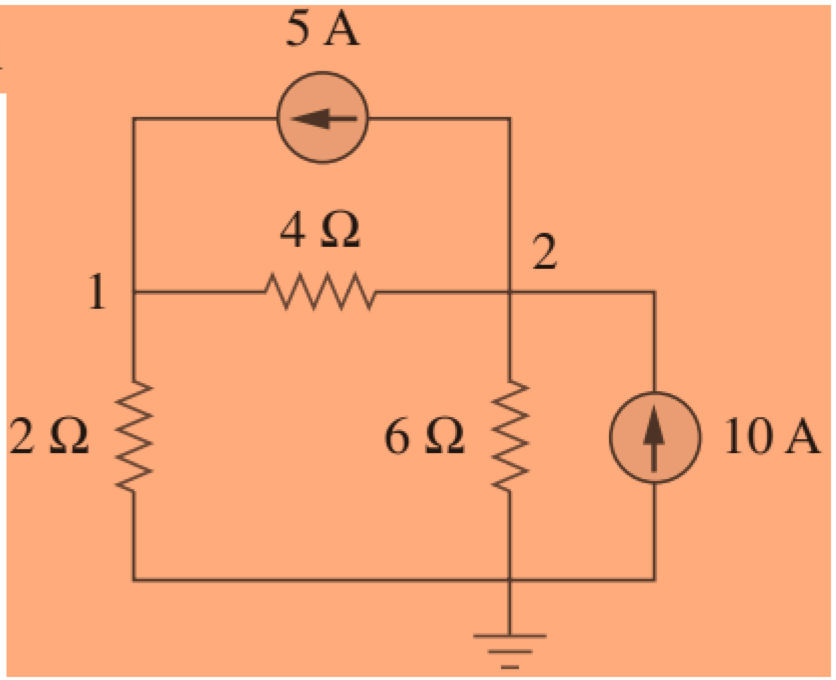
Nodal Analysis

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

1. Select a node as the reference node. Assign voltages v_1, $v_2, \ldots, v_{n-1}$ to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Example

Calculate the node voltages in the circuit shown



At node 1,

$$5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

$$20 = v_1 - v_2 + 2v_1$$

$$3v_1 - v_2 = 20$$

At node 2,

$$\frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

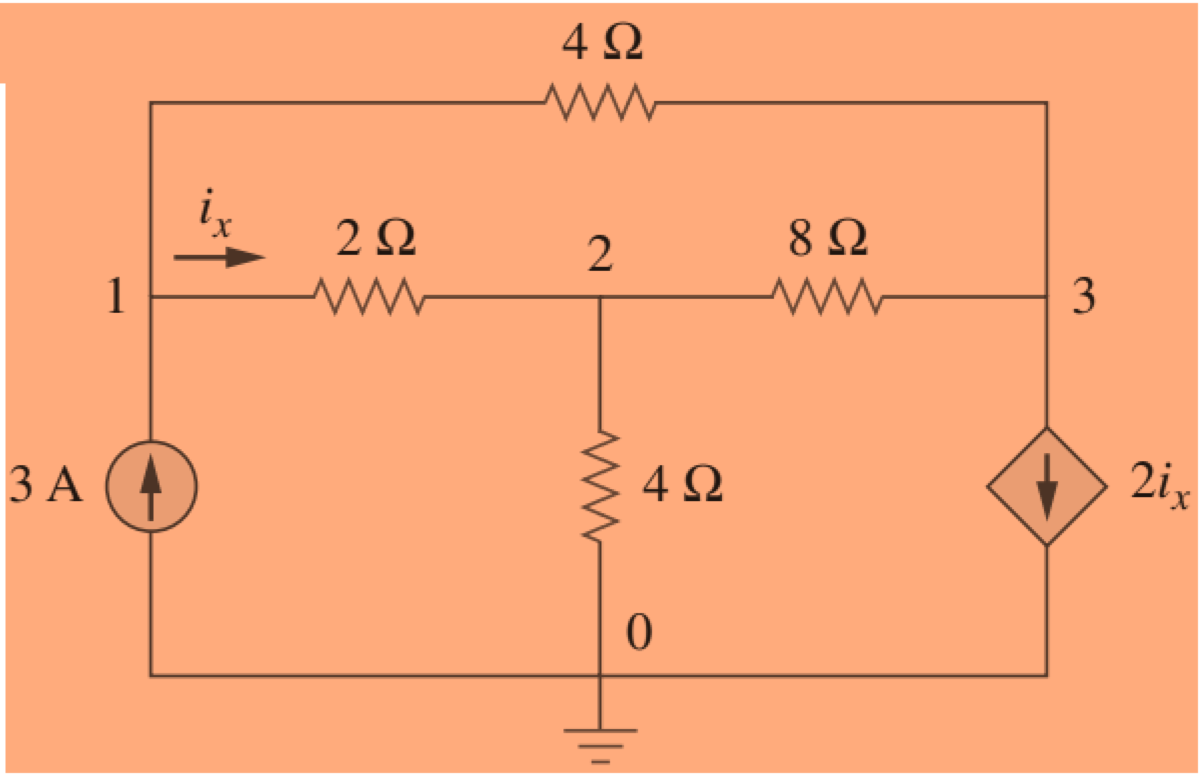
$$3v_1 - 3v_2 + 120 = 60 + 2v_2$$

$$-3v_1 + 5v_2 = 60$$

$$v_1 = \frac{40}{3} = 13.333 \text{ V} \qquad v_2 = 20 \text{ V}$$

continued…

Determine the voltages at the nodes



At node 1,

$$3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

$$3v_1 - 2v_2 - v_3 = 12$$

At node 2,

$$\frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

$$-4v_1 + 7v_2 - v_3 = 0$$

At node 3,

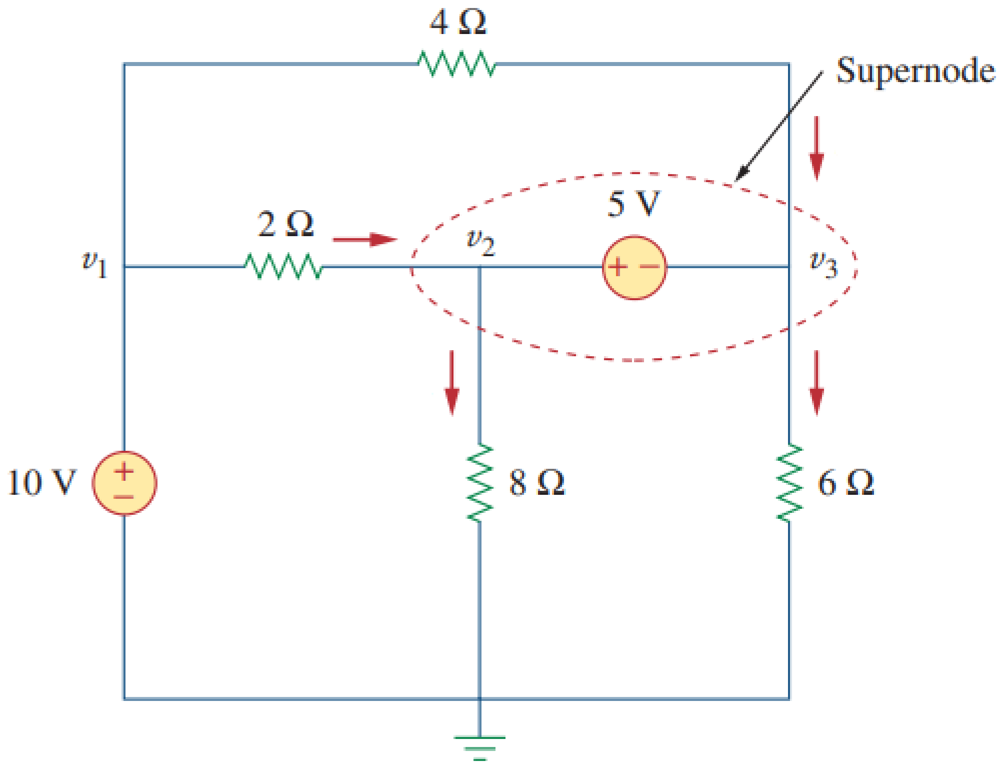
$$\frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

$$2v_1 - 3v_2 + v_3 = 0$$

$$v_1 = 4.8 \text{ V}, \qquad v_2 = 2.4 \text{ V}, \qquad v_3 = -2.4 \text{ V}$$

Supernode

If the voltage source (dependent or independent) is connected between two **nonreference** nodes, the two nonreference nodes form a generalized node or Supernode.



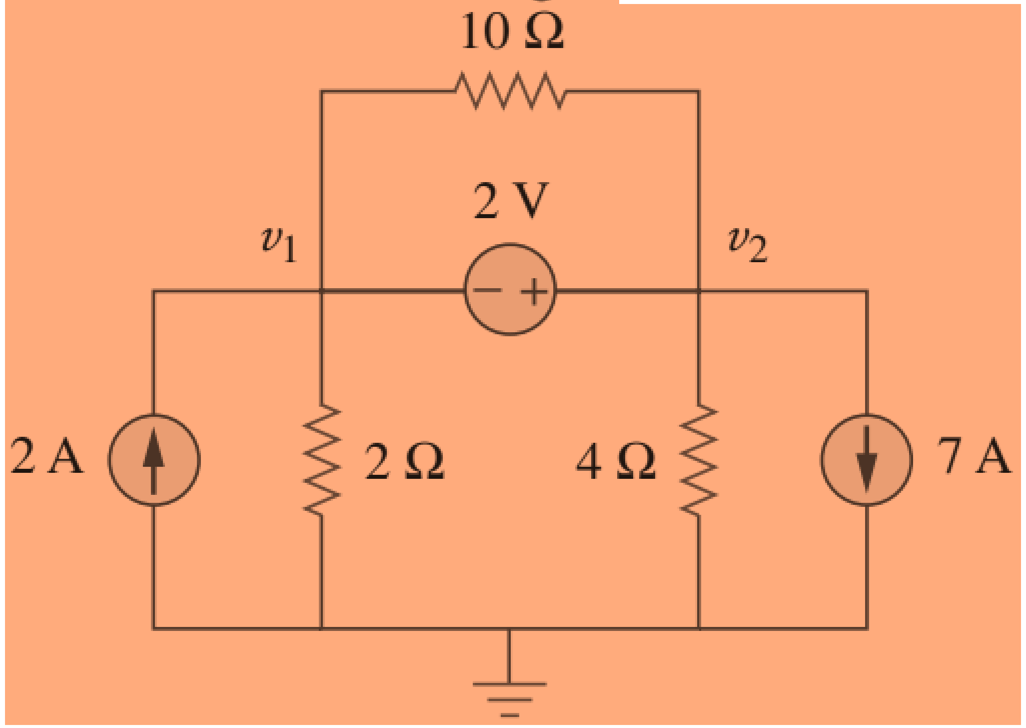
$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$$

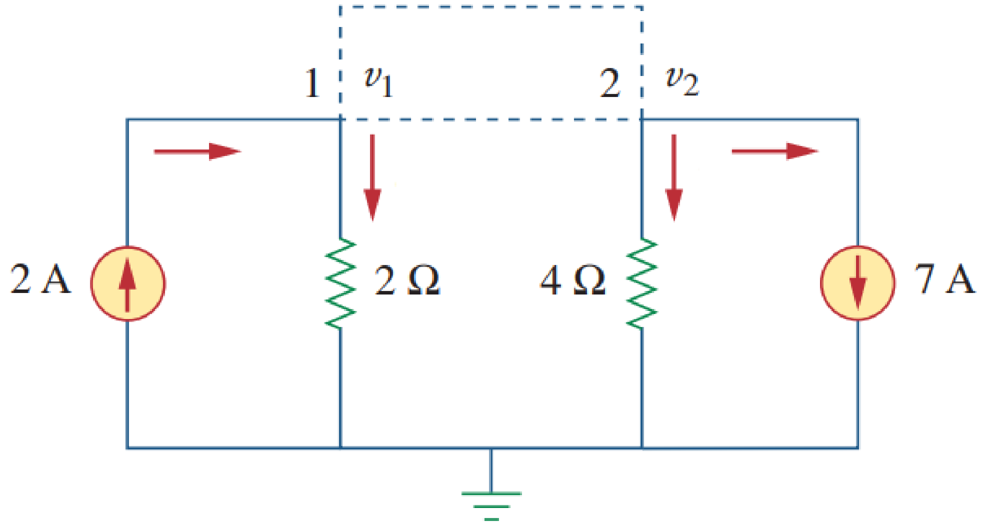
$$v_2 - v_3 = 5$$

$$v_1 = 10 \text{ V}$$

Example

find the node voltages





$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7$$

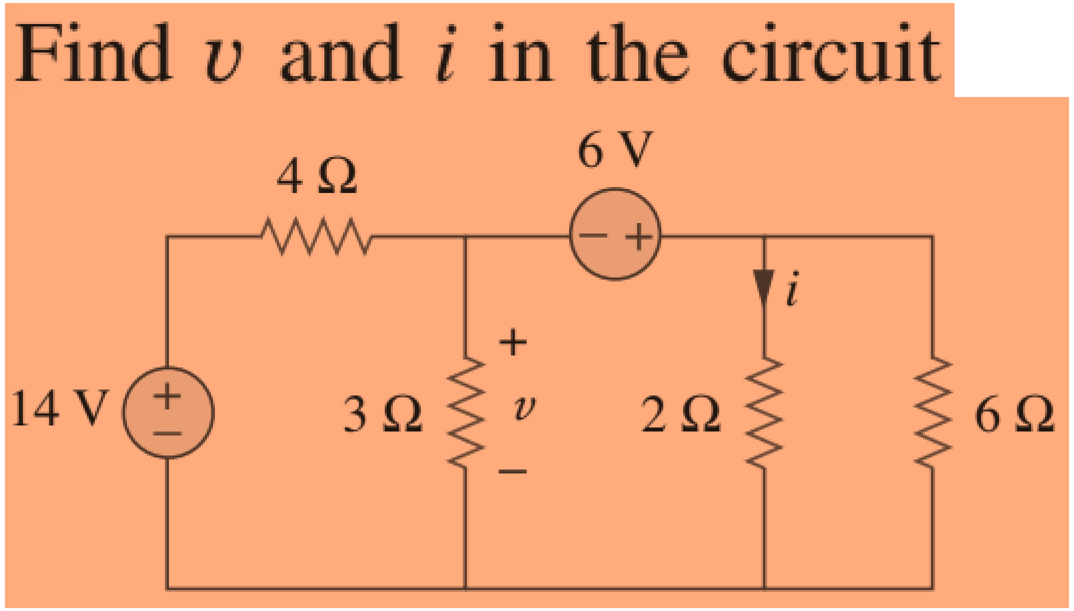
$$\Rightarrow \quad 8 = 2v_1 + v_2 + 28$$

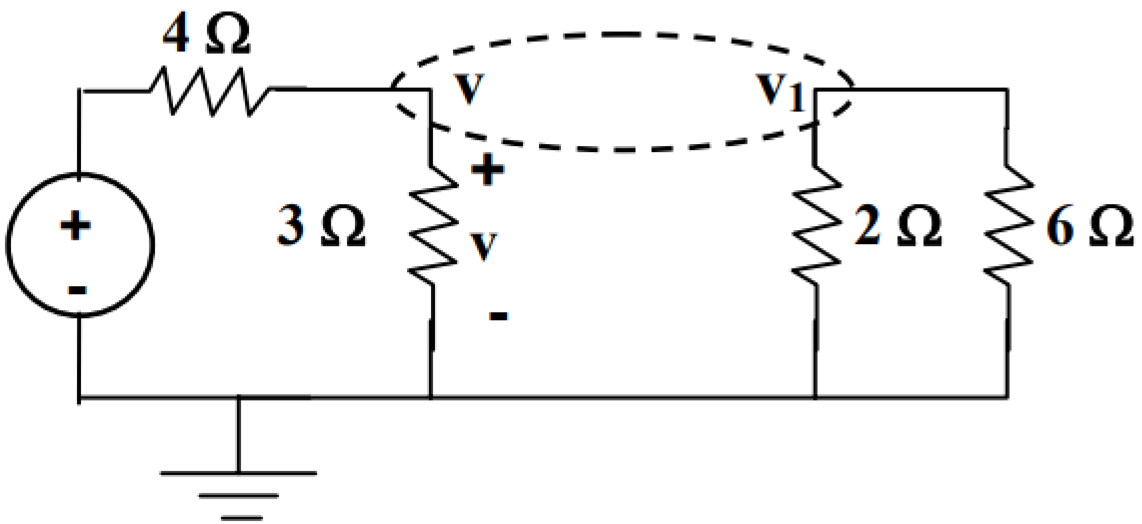
$$v_2 = v_1 + 2$$

$$v_1 = -7.333 \text{ V}$$

$$v_2 = v_1 + 2 = -5.333 \text{ V}$$

continued...





$$\frac{v-14}{4}+\frac{v-0}{3}+\frac{v_1-0}{2}+\frac{v_1-0}{6}=0$$

$$\Longrightarrow 3v-42+4v+6v_1+2v_1=0$$

$$\Longrightarrow 8v_1+7v=42$$

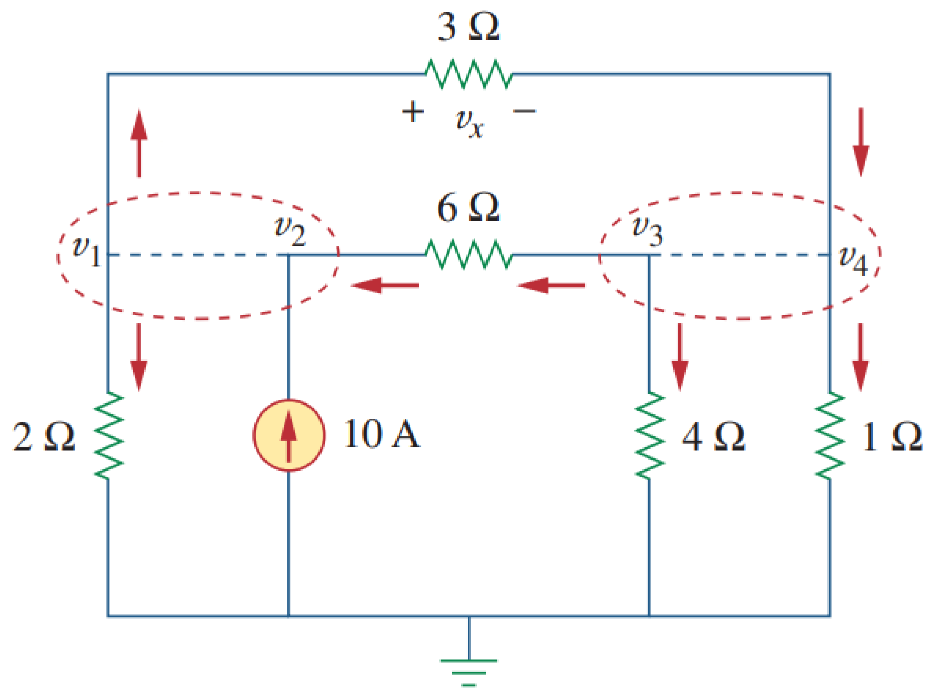
$$v_1-v=6$$

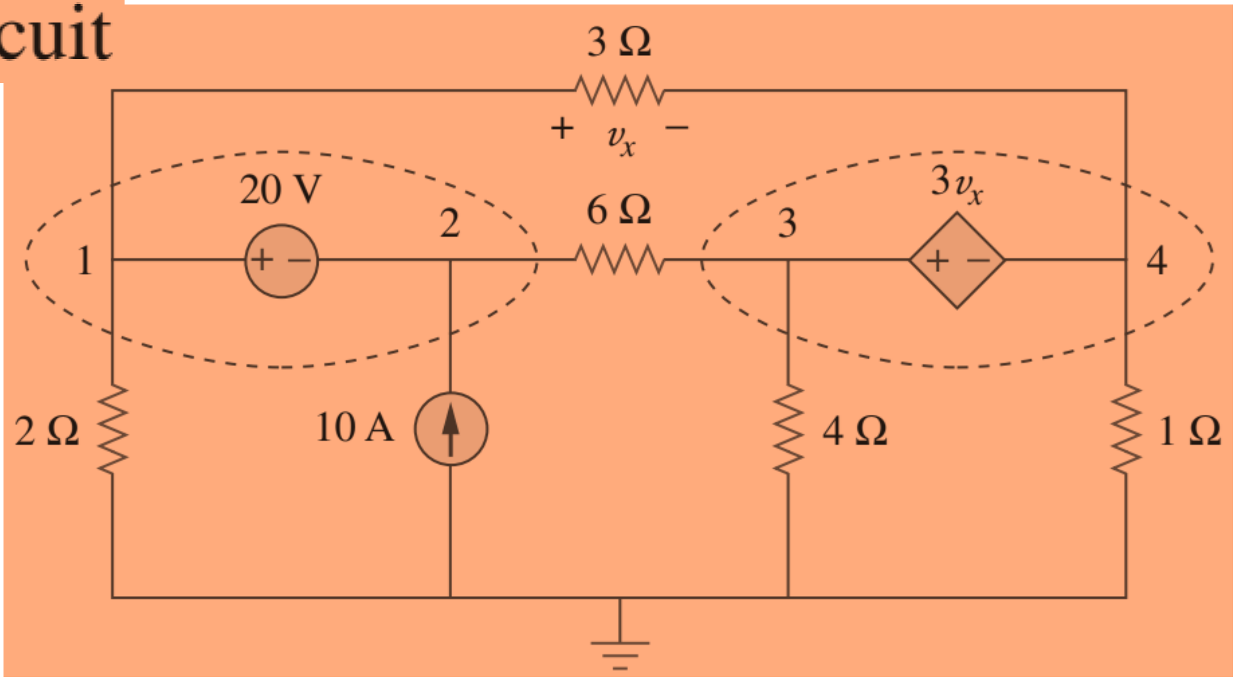
$$v_1=5.6V\,;\,v=-0.4V$$

$$\therefore i=5.6/2=2.8A$$

continued…

Find the node voltages in the circuit

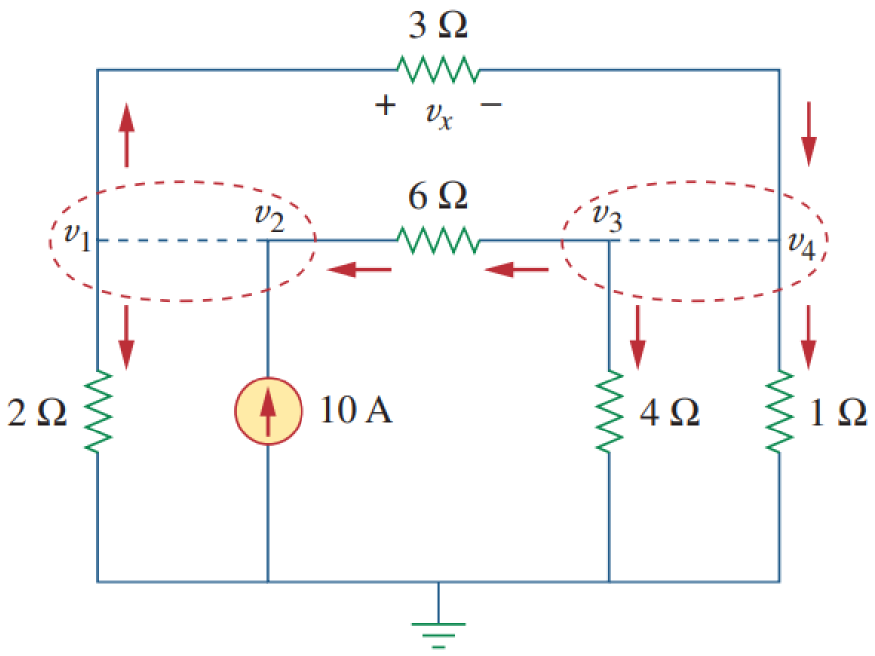




At supernode 1-2, $\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$

$$5v_1 + v_2 - v_3 - 2v_4 = 60$$

continued…



At supernode 3-4,

$$\frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}$$

$$4v_1 + 2v_2 - 5v_3 - 16v_4 = 0$$

$$v_1 - v_2 = 20$$

$$-v_3 + 3v_x + v_4 = 0 \qquad \text{But } v_x = v_1 - v_4$$

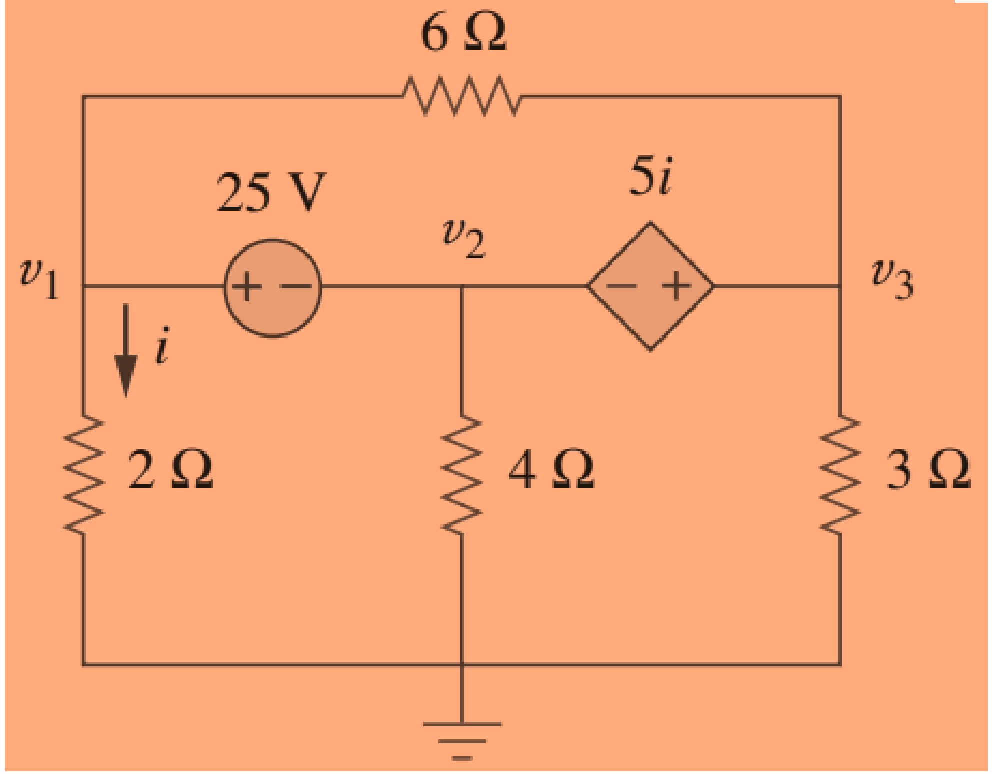
$$3v_1 - v_3 - 2v_4 = 0$$

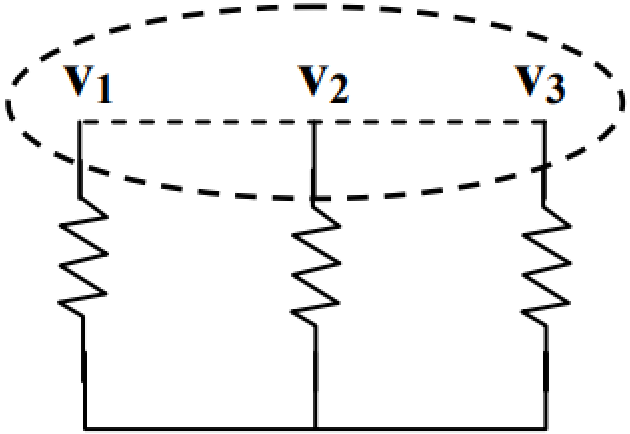
$v_1 = 26.67$ V $v_2 = 6.667$ V

$v_3 = 173.33$ V $v_4 = -46.67$ V

continued…

Find v_1, v_2, and v_3 in the circuit





$$\frac{v_1}{2} + \frac{v_2}{4} + \frac{v_3}{3} = 0$$

$$6v_1 + 3v_2 + 4v_3 = 0$$

$$v_1 - v_2 = 25$$

$$v_3 - v_2 = 5i$$

But, $i = v_1/2$

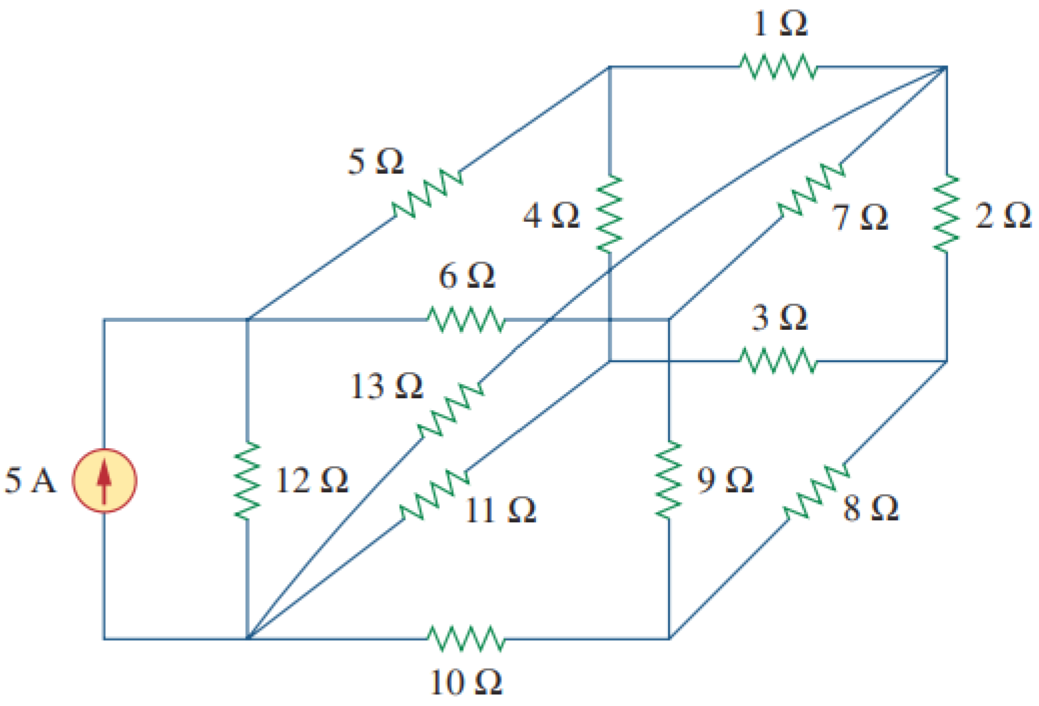
$$\therefore v_3 - v_2 = {}^{5v_1}/_{2}$$

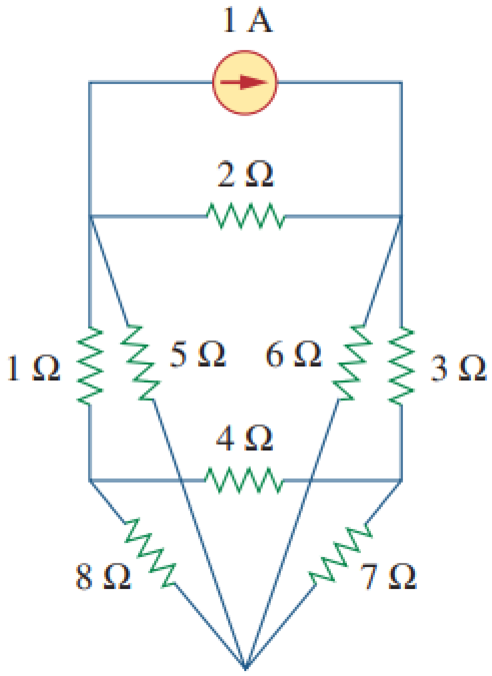
$$\Rightarrow -2.5v_1 - v_2 + v_3 = 0$$

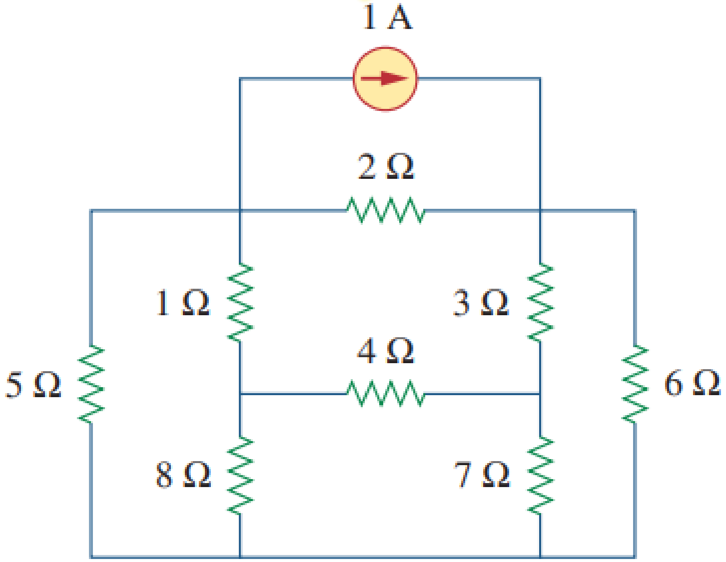
$$v_1 = 7.608 \text{ V}, \; v_2 = -17.39 \text{ V}, \; v_3 = 1.6305 \text{ V}$$

Mesh Analysis

- It is ***only*** applicable to a circuit that is planar. A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is nonplanar.

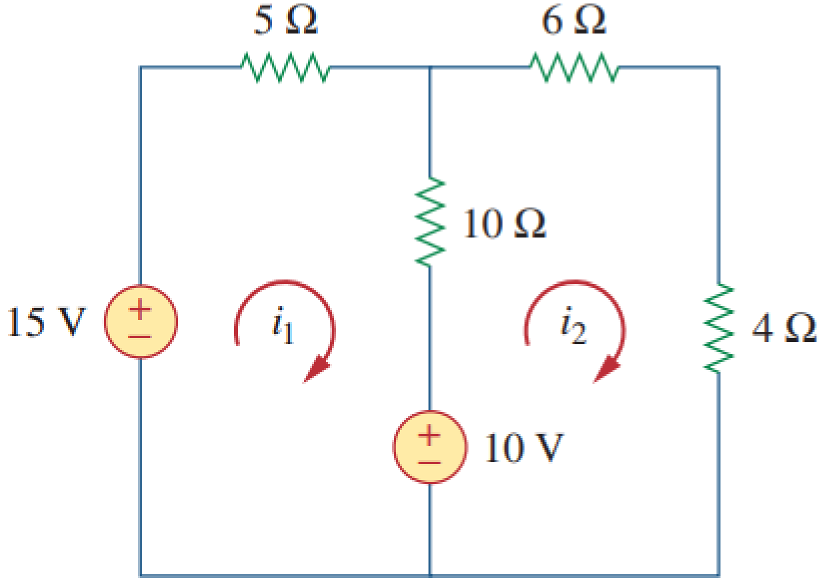


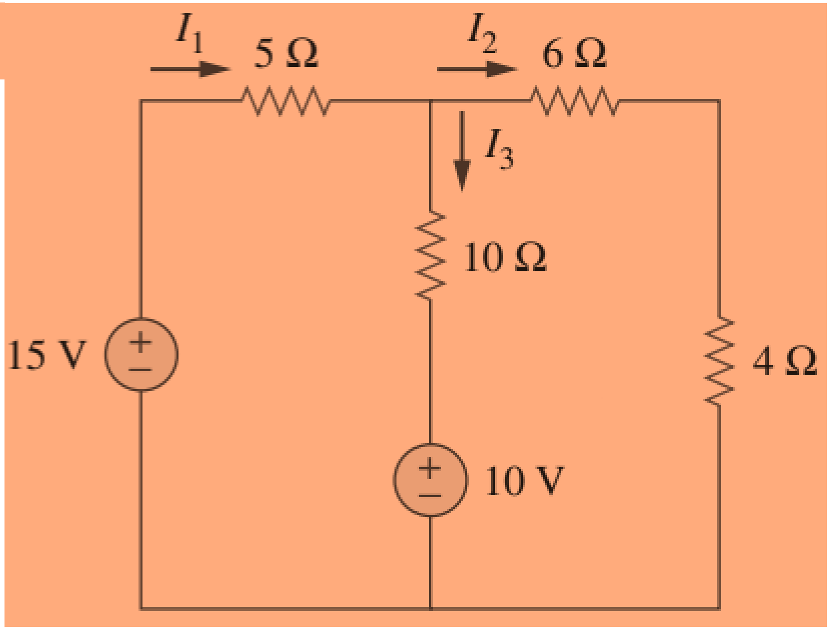




Example

find the branch currents I_1, I_2, and I_3





$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0 \longrightarrow 3i_1 - 2i_2 = 1$$

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0 \longrightarrow i_1 = 2i_2 - 1$$

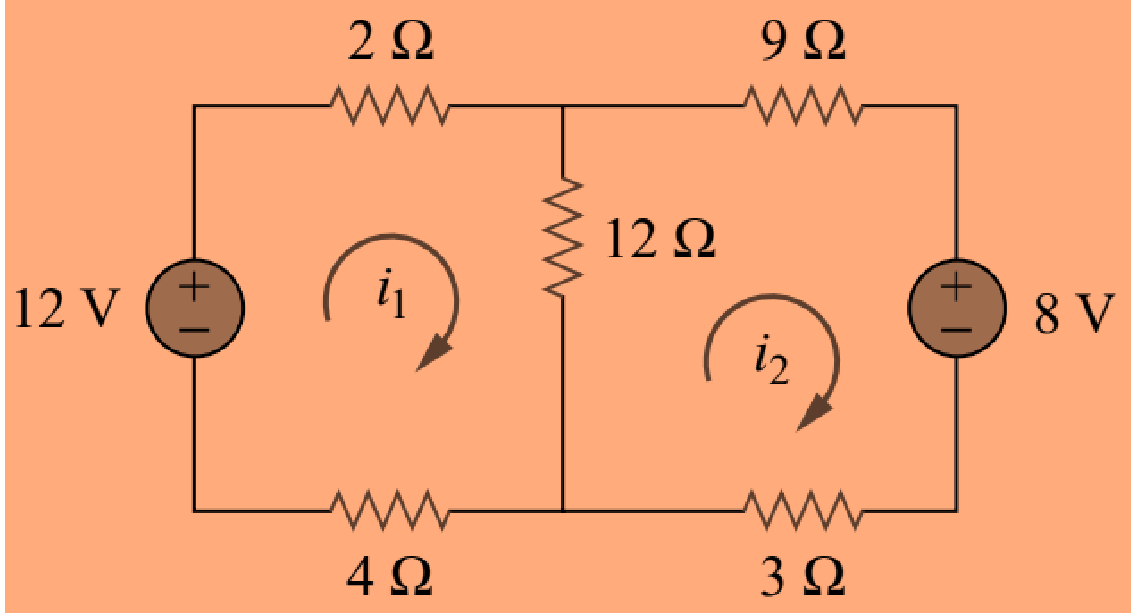
$i_1 = 1$ A $\qquad i_2 = 1$ A

$$I_1 = i_1 = 1 \text{ A}, \quad I_2 = i_2 = 1 \text{ A},$$

$$I_3 = i_1 - i_2 = 0$$

continued…

Calculate the mesh currents i_1 and i_2 of the circuit



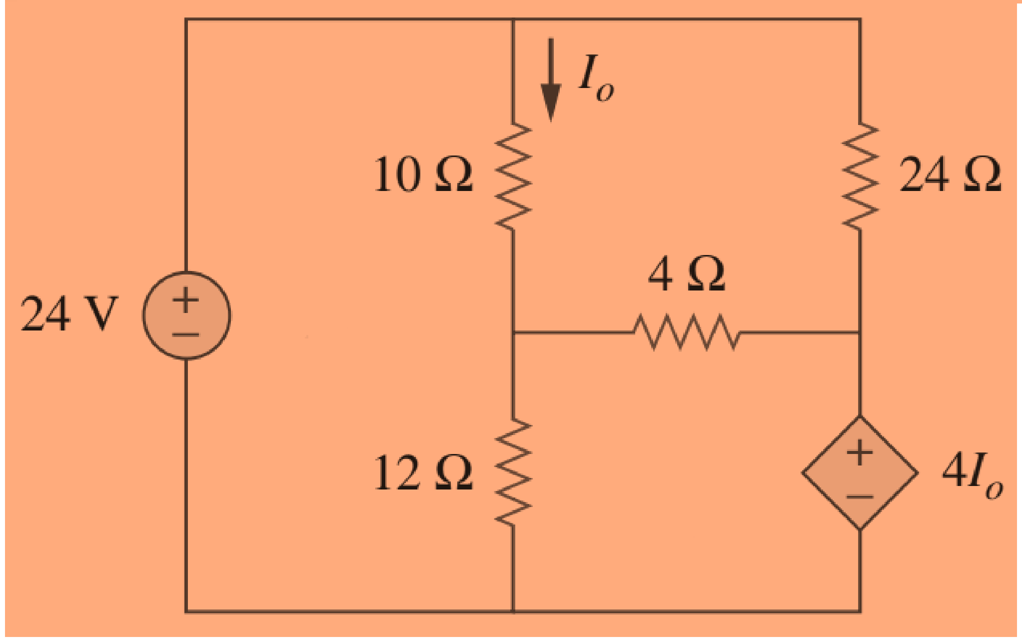
$- 12 + 18i_i - 12i_2 = 0 \longrightarrow 3i_i - 2i_2 = 2$

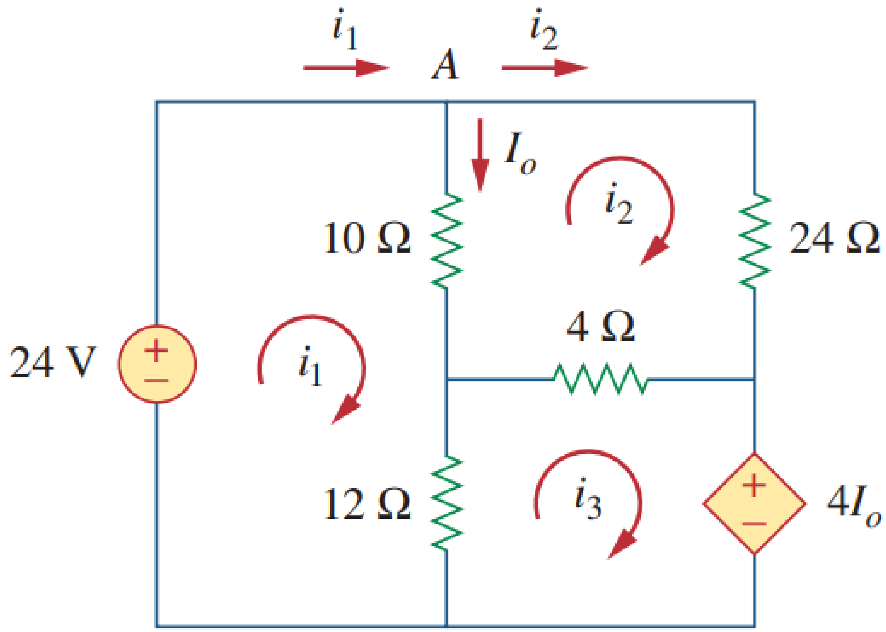
$8 + 24i_2 - 12i_1 = 0 \longrightarrow - 3i_1 + 6i_2 = -2$

$$i_1 = \frac{2}{3}\ \text{A},\ i_2 = 0\ \text{A}.$$

continued…

Use mesh analysis to find the current I_o in the circuit





$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0 \longrightarrow 11i_1 - 5i_2 - 6i_3 = 12$$

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0 \longrightarrow -5i_1 + 19i_2 - 2i_3 = 0$$

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

continued…

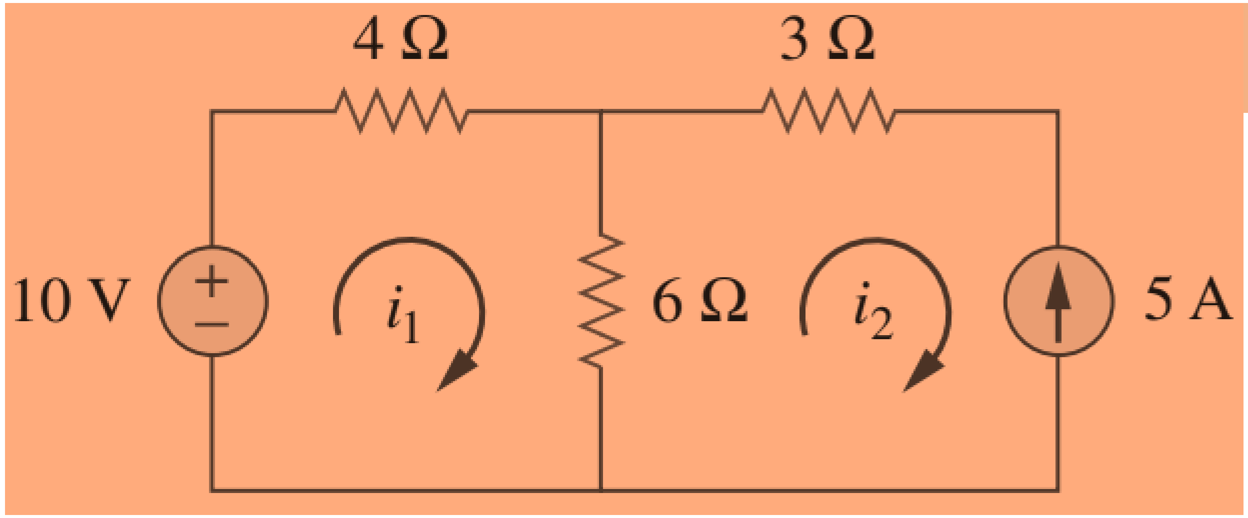
$$11i_1 - 5i_2 - 6i_3 = 12$$

$$-5i_1 + 19i_2 - 2i_3 = 0$$

$$-i_1 - i_2 + 2i_3 = 0$$

$i_1 = 2.25$ A, $i_2 = 0.75$ A, $i_3 = 1.5$ A

$$I_o = i_1 - i_2 = 1.5 \text{ A}.$$



Find i_1 and i_2.

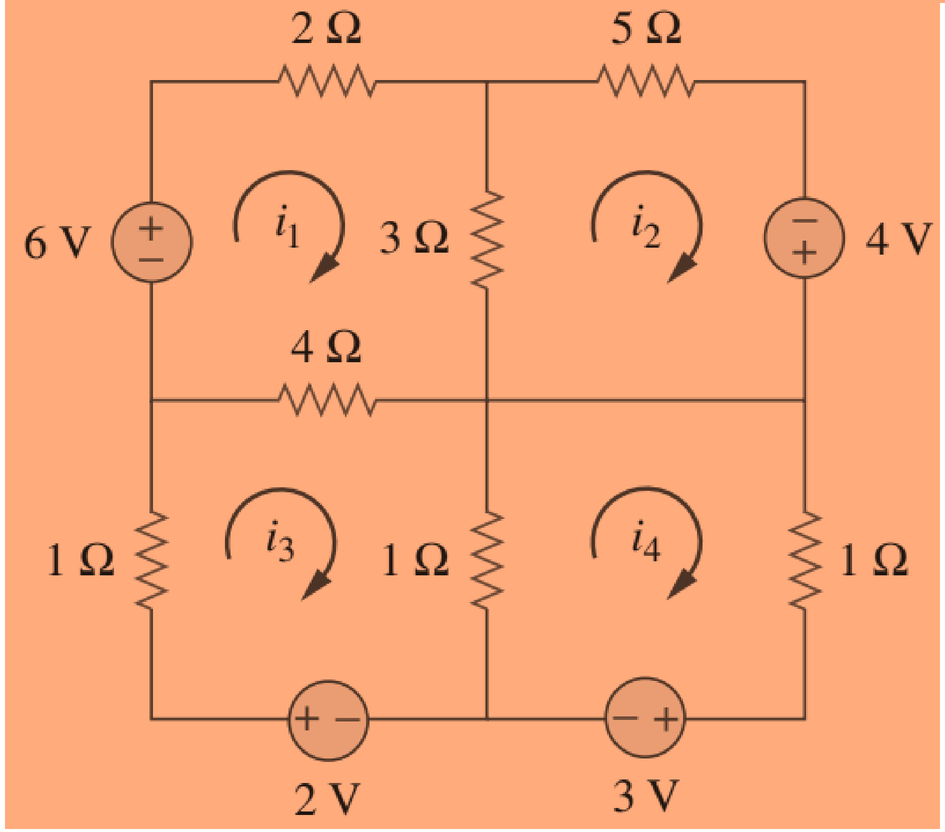
$$i_2 = -5 \text{ A}$$

$$-10 + 4i_1 + 6(i_1 - i_2) = 0$$

$$\Rightarrow \quad i_1 = -2 \text{ A}$$

continued…

Write the mesh-current equations



$$-6 + 9i_1 - 3i_2 - 4i_3 = 0$$

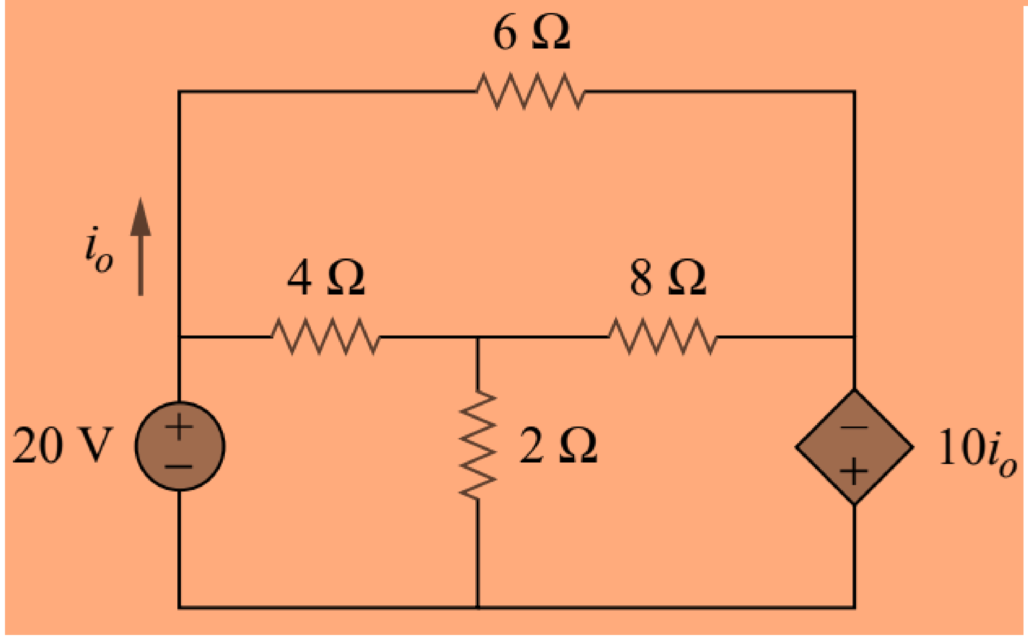
$$-4 - 3i_1 + 8i_2 = 0$$

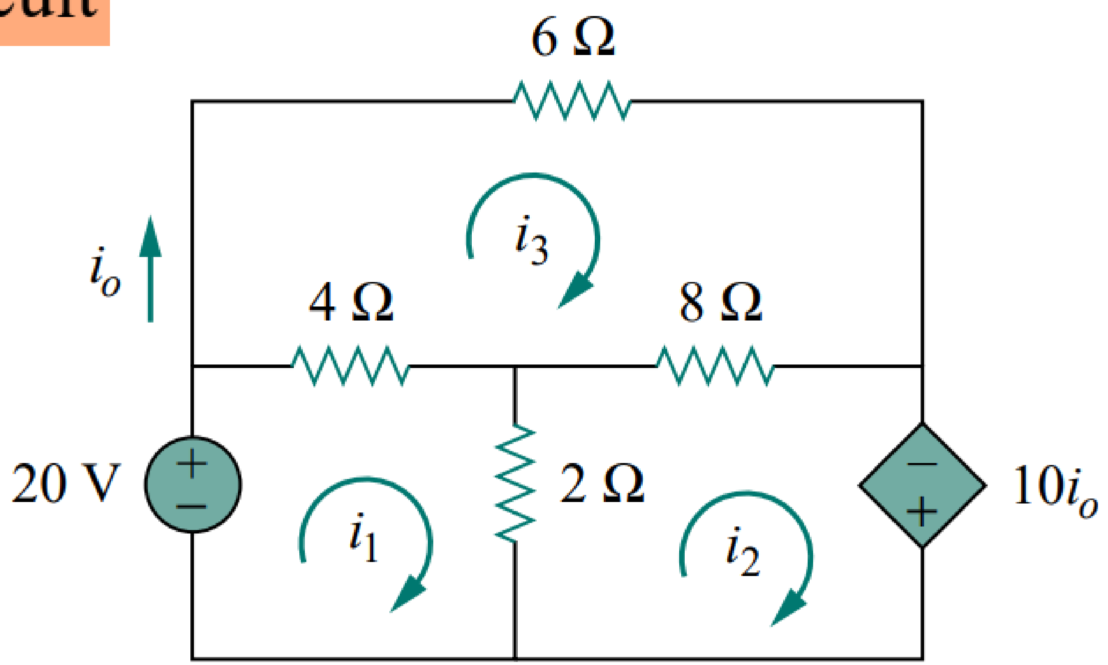
$$-2 - 4i_1 + 6i_3 - 1i_4 = 0$$

$$+3 - 1i_3 + 2i_4 = 0$$

continued…

Using mesh analysis, find i_o in the circuit





$$-20 + 6i_1 - 2i_2 - 4i_3 = 0 \longrightarrow 3i_1 - i_2 - 2i_3 = 10$$

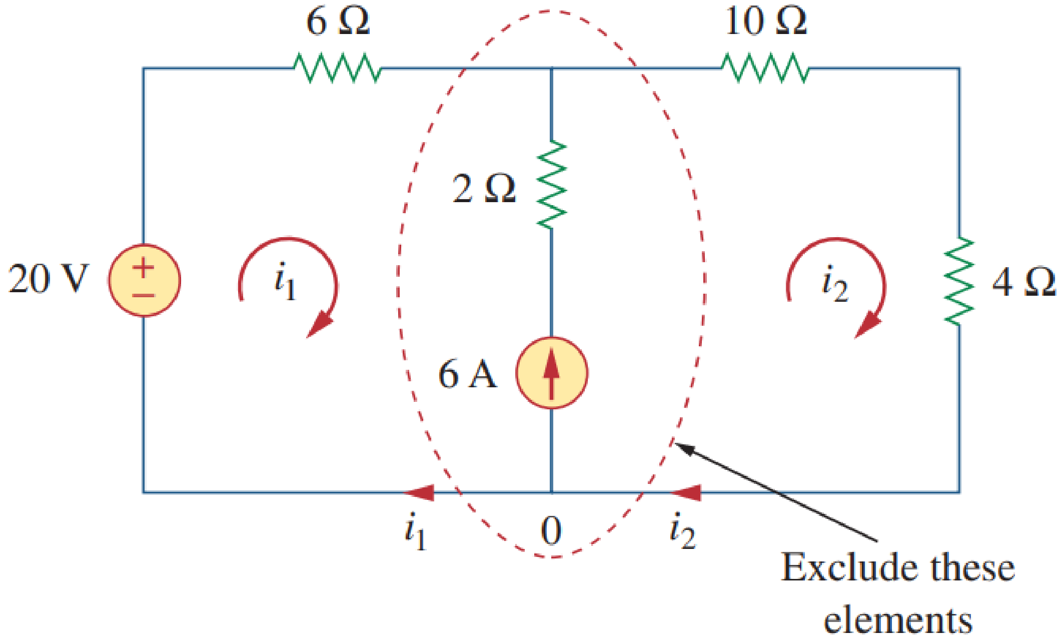
$$10i_2 - 2i_1 - 8i_3 - 10i_0 = 0 \longrightarrow -i_1 + 5i_2 - 9i_3 = 0 \qquad \because i_0 = i_3$$

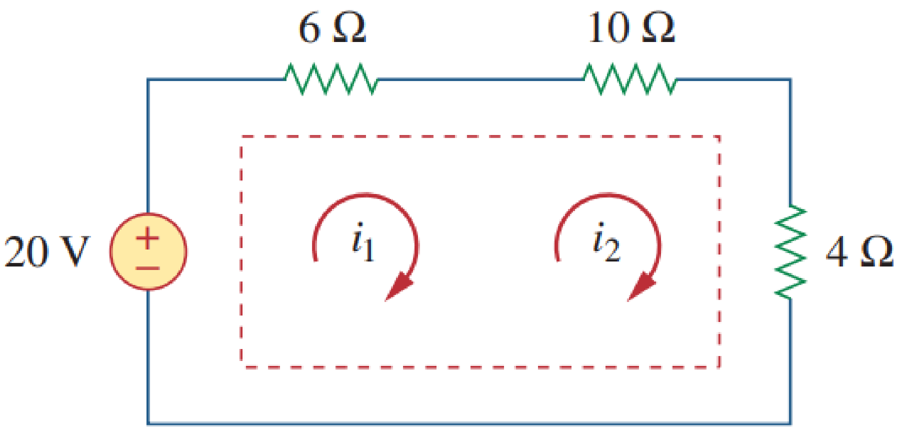
$$18i_3 - 4i_1 - 8i_2 = 0 \longrightarrow -2i_1 - 4i_2 + 9i_3 = 0$$

$$i_1 = -3.214, \quad i_2 = -9.643, \quad i_3 = -5\text{A} \qquad i_0 = -5\text{ A.}$$

Supermesh

Supermesh results when two meshes have a (dependent or independent) current source in **common**.





$$i_2 = i_1 + 6$$

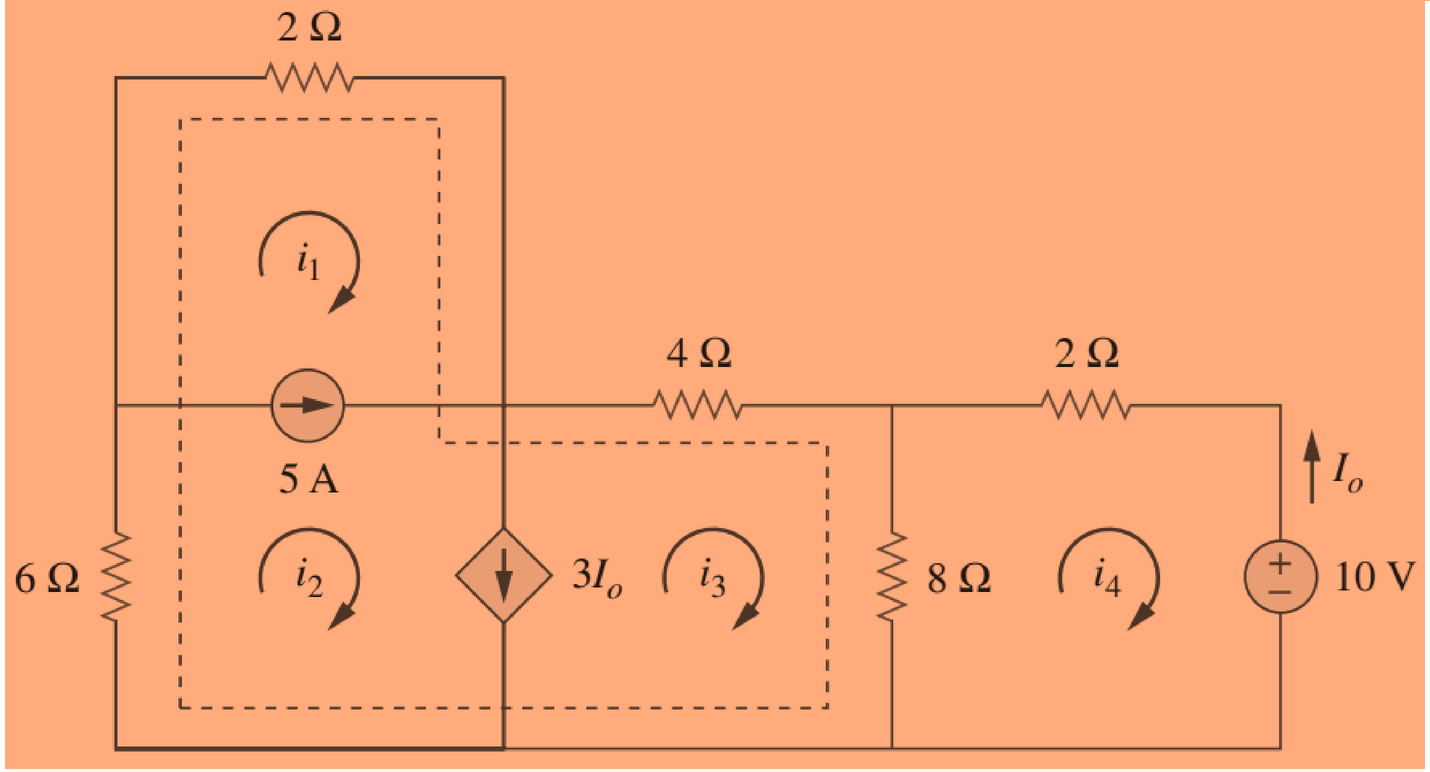
$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$\Rightarrow 6i_1 + 14i_2 = 20$$

$$i_1 = -3.2 \text{ A}, \qquad i_2 = 2.8 \text{ A}$$

Example

For the circuit find i_1 to i_4 using mesh analysis.



$$i_2 = i_1 + 5$$

$$i_2 = i_3 + 3I_o$$

But $I_o = -i_4$,

$$i_2 = i_3 - 3i_4$$

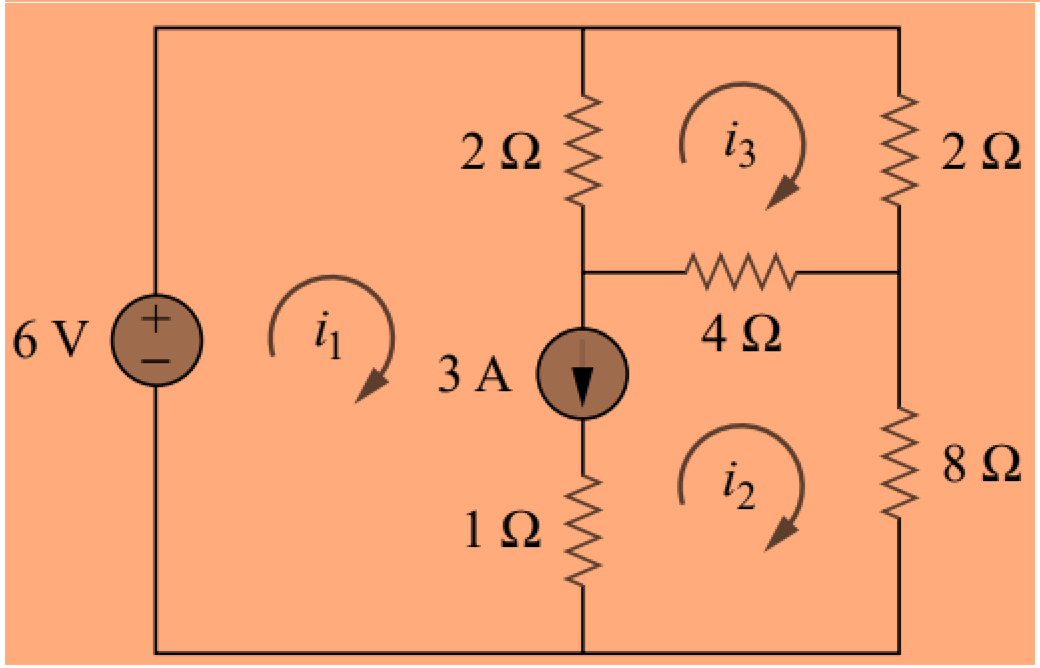
$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0 \rightarrow i_1 + 3i_2 + 6i_3 - 4i_4 = 0$$

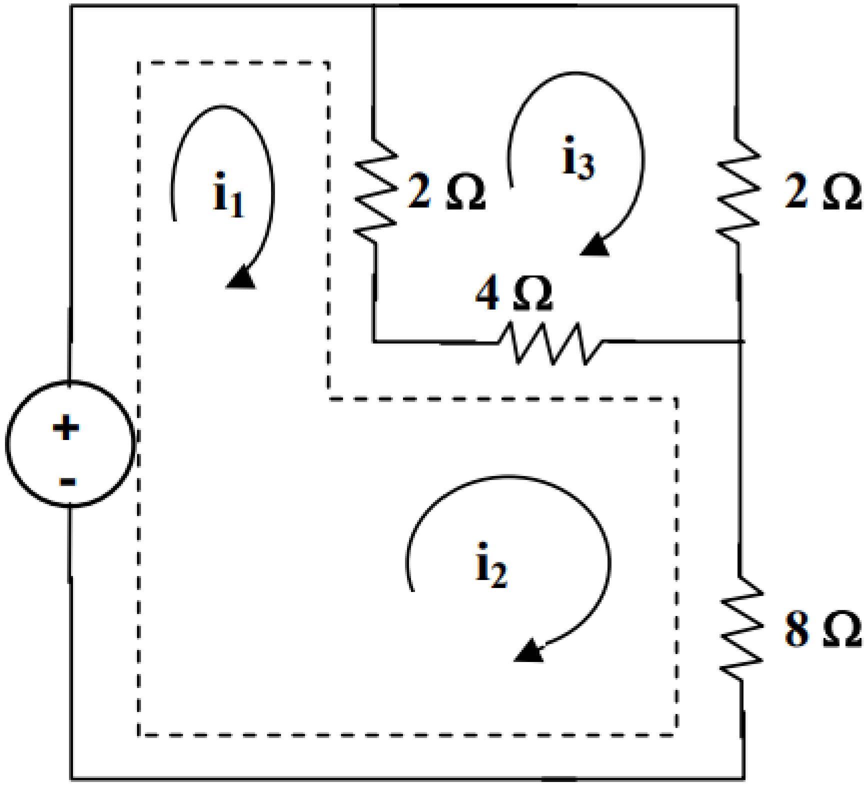
$$2i_4 + 8(i_4 - i_3) + 10 = 0 \rightarrow 5i_4 - 4i_3 = -5$$

$i_1 = -7.5$ A, $i_2 = -2.5$ A, $i_3 = 3.93$ A, $i_4 = 2.143$ A

continued…

Use mesh analysis to determine i_1, i_2, and i_3





$- 6 + 2i_1 - 2i_3 + 12i_2 - 4i_3 = 0 \rightarrow i_1 + 6i_2 - 3i_3 = 3$

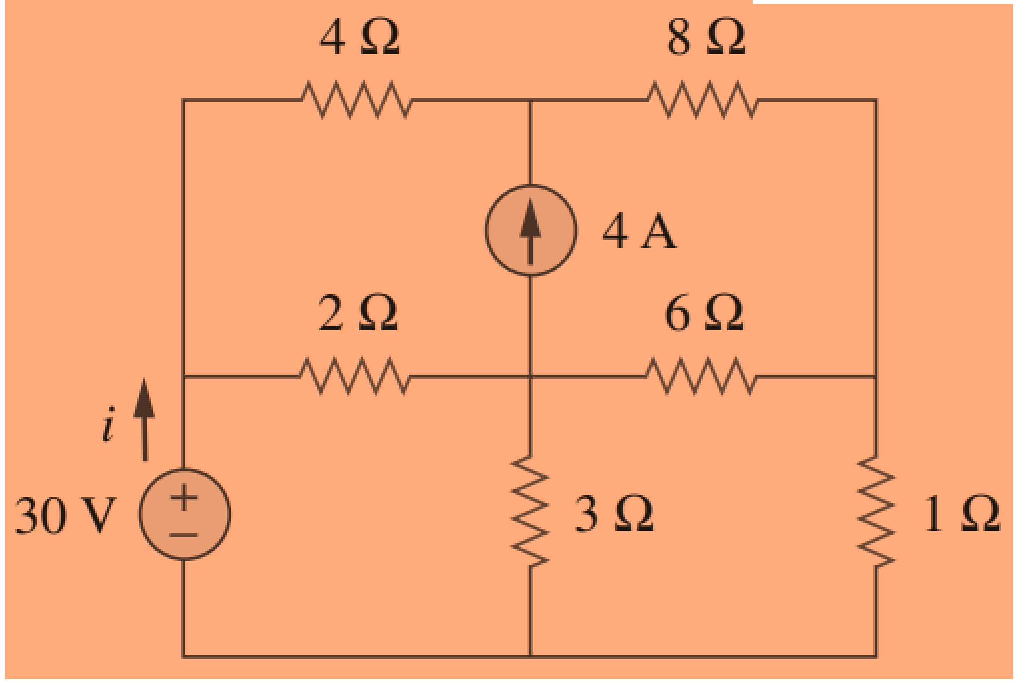
$8i_3 - 2i_1 - 4i_2 = 0 \rightarrow - i_1 - 2i_2 + 4i_3 = 0$

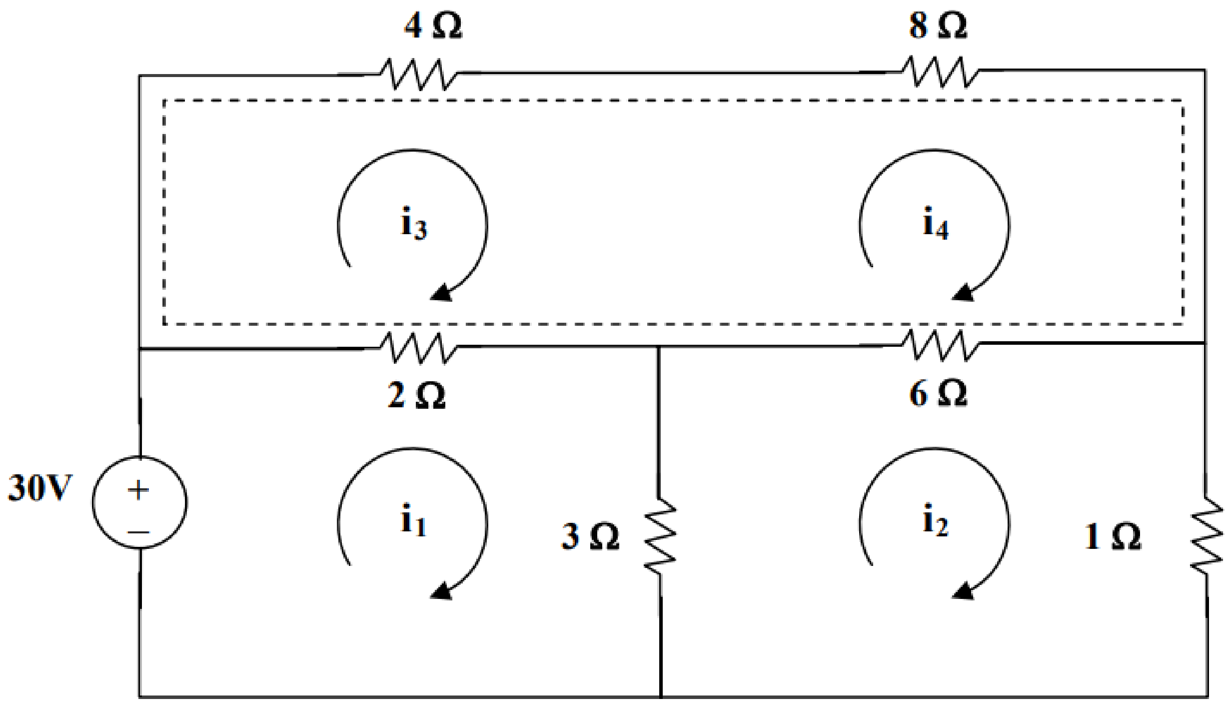
$i_1 = 3 + i_2 \rightarrow i_1 - i_2 = 3$

$$i_1 = 3.474 \text{ A}, i_2 = 0.4737 \text{ A}, i_3 = 1.1052 \text{ A}.$$

continued…

Find current i in the circuit





For the supermesh, $6i_3 + 14i_4 - 2i_1 - 6i_2 = 0$

For loop 1, $30 = 5i_1 - 3i_2 - 2i_3$

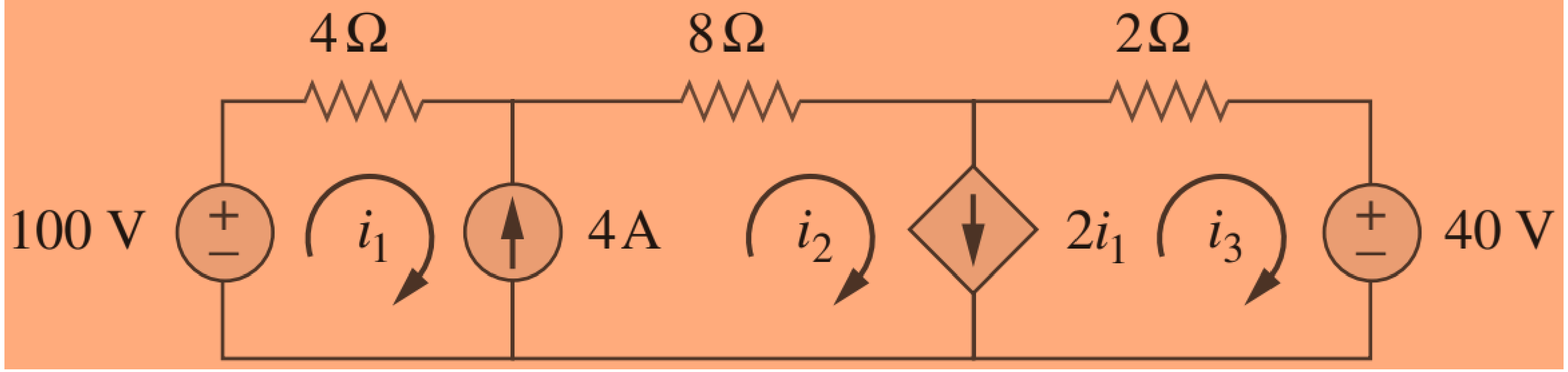
For loop 2, $10i_2 - 3i_1 - 6i_4 = 0$

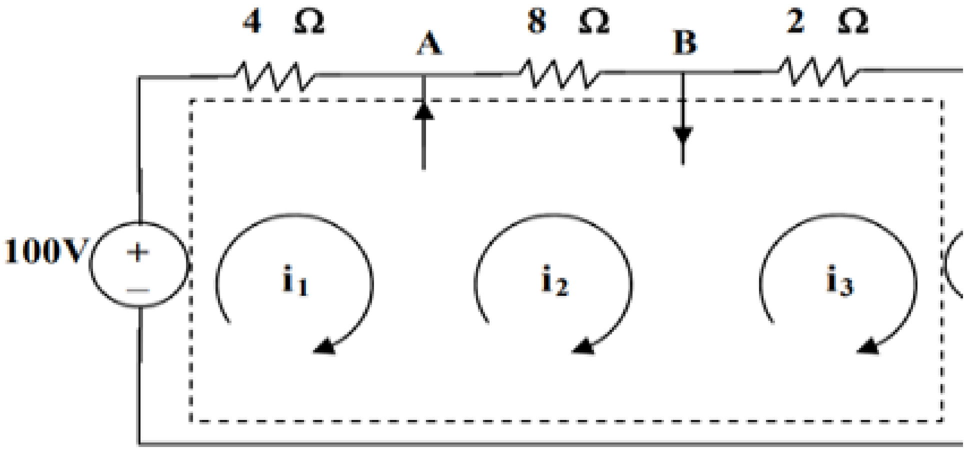
$i_4 = i_3 + 4$

$$i = i_1 = \mathbf{8.561\ A.}$$

continued…

Find the mesh currents i_1, i_2, and i_3





At node A, $i_1 + 4 = i_2$

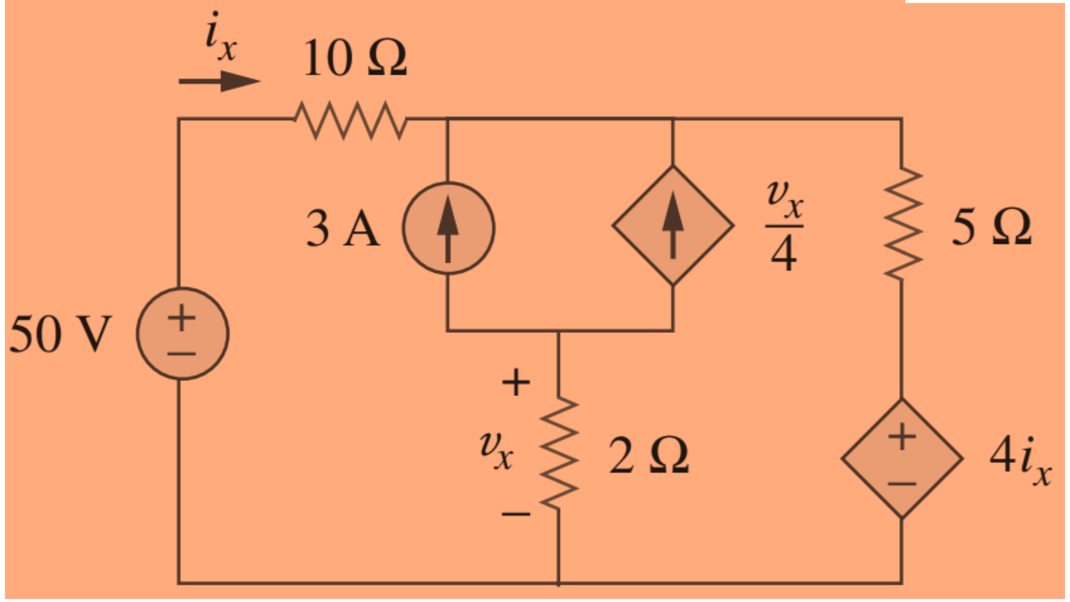
At node B, $i_2 = 2i_1 + i_3$

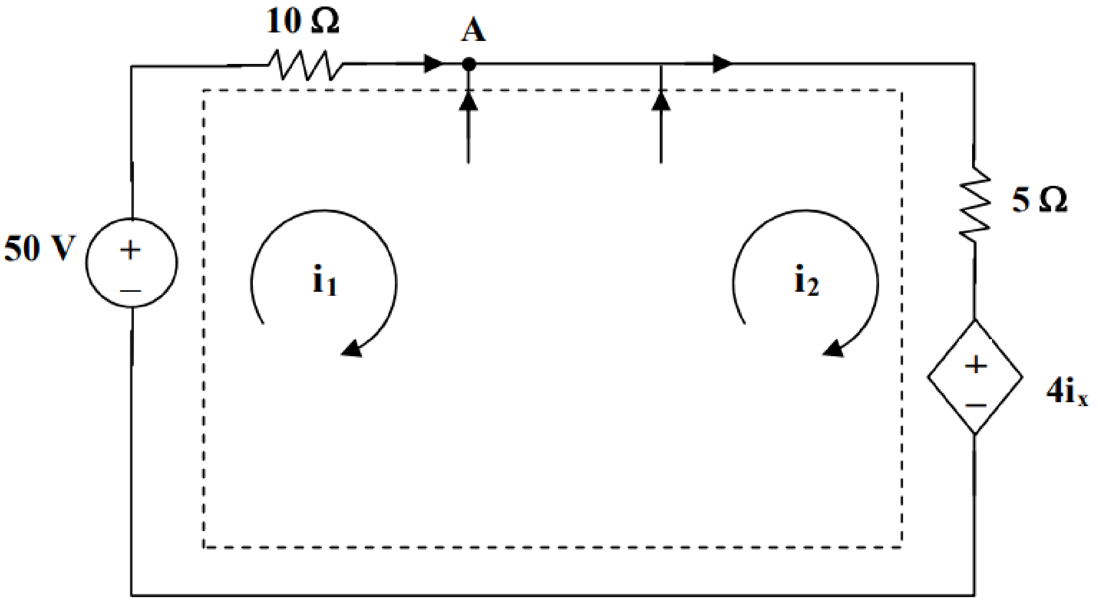
For the supermesh, $-100 + 4i_1 + 8i_2 + 2i_3 + 40 = 0$

$i_1 = \mathbf{2A}$, $i_2 = \mathbf{6A}$, and $i_3 = \mathbf{2A}$.

continued…

Find v_x and i_x in the circuit





For the supermesh, $-50 + 10i_1 + 5i_2 + 4i_x = 0$

but $i_x = i_1$ → $50 = 14i_1 + 5i_2$

At node A, $i_1 + 3 + (v_x/4) = i_2$

but $v_x = 2(i_1 - i_2)$ → $i_1 + 2 = i_2$

$i_1 = 2.105$ A
$i_2 = 4.105$ A

$v_x = 2(i_1 - i_2) = -4$ $\quad$ $i_x = 2.105$

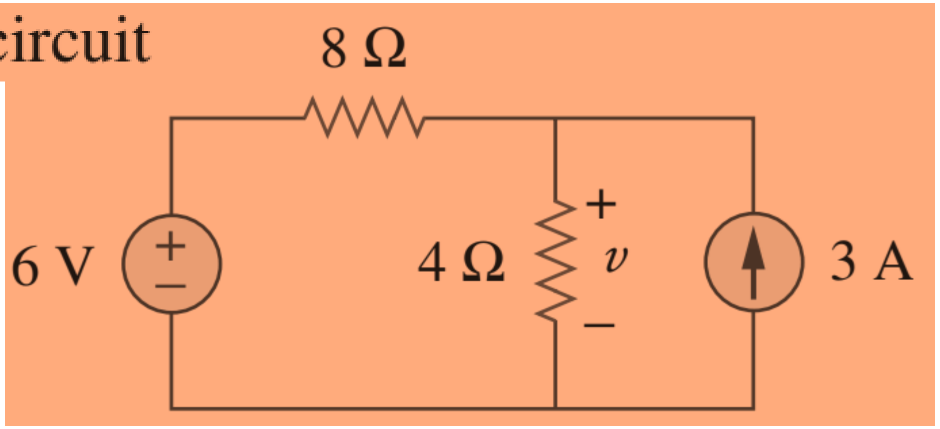
Superposition

Voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

- Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques previously covered.
- Repeat previous step for each of the other independent sources.
- Find the total contribution by adding algebraically all the contributions due to the independent sources.

Example

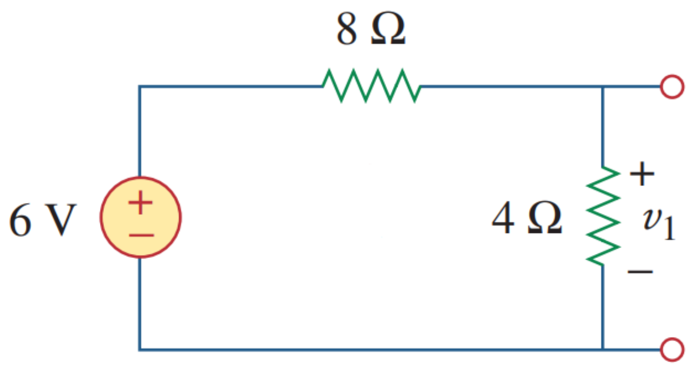
Use the superposition theorem to find v in the circuit



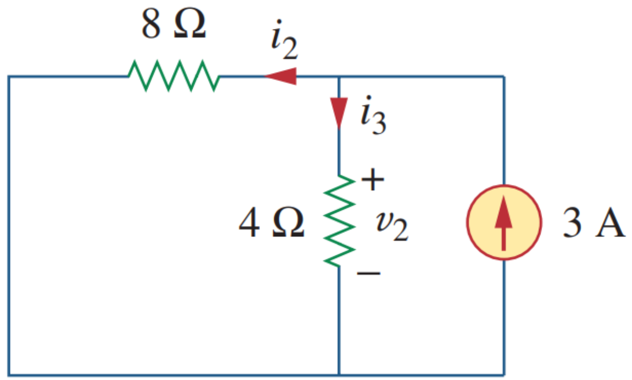
$$v = v_1 + v_2$$

v_1 due to 6-V voltage source alone.

v_2 due to 3-A current source alone.



$$v_1 = \frac{4}{4+8}(6) = 2\text{ V}$$



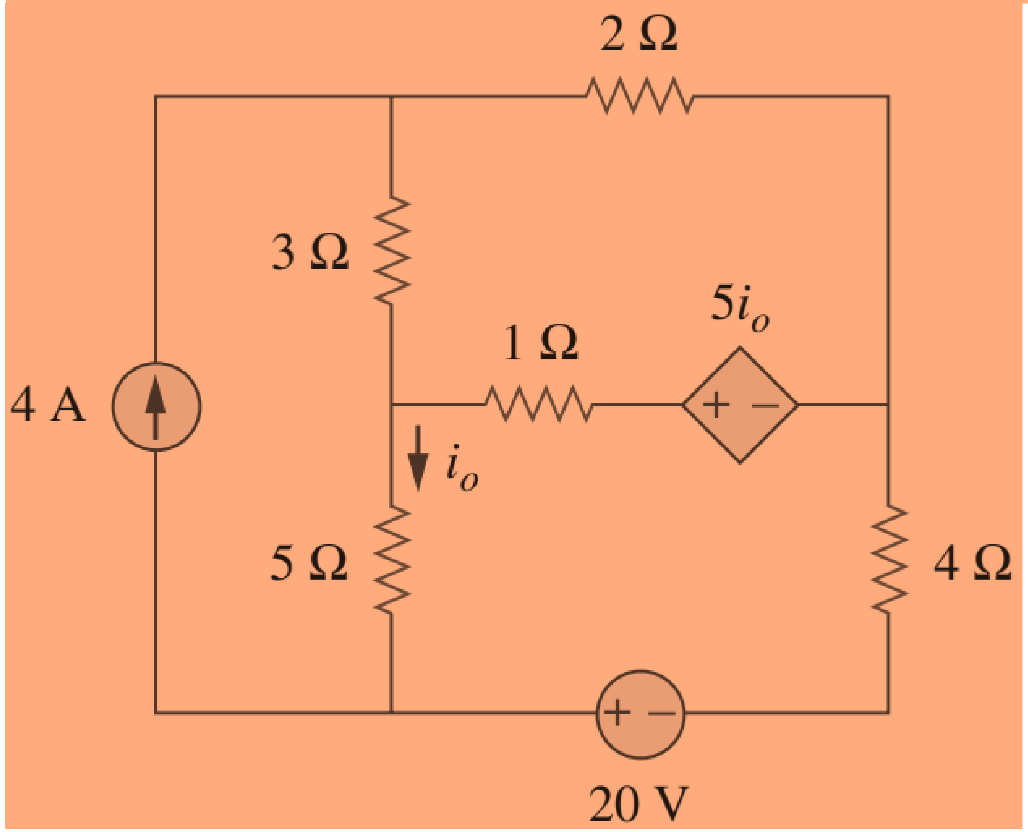
$$i_3 = \frac{8}{4+8}(3) = 2\text{ A}$$

$$v_2 = 4i_3 = 8\text{ V}$$

$$v = v_1 + v_2 = 2 + 8 = 10\text{ V}$$

continued…

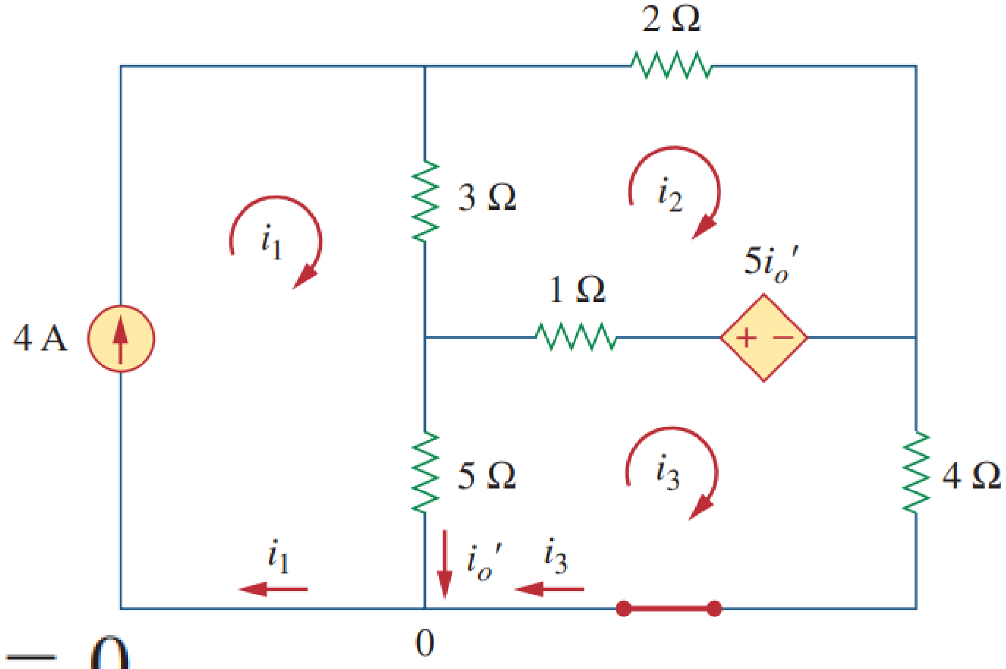
Find i_o in the circuit using superposition.



$$i_o = i'_o + i''_o$$

i_o' due to 4-A current source alone.

i_o" due to 20-V voltage source alone.



For loop 1, $i_1 = 4$ A

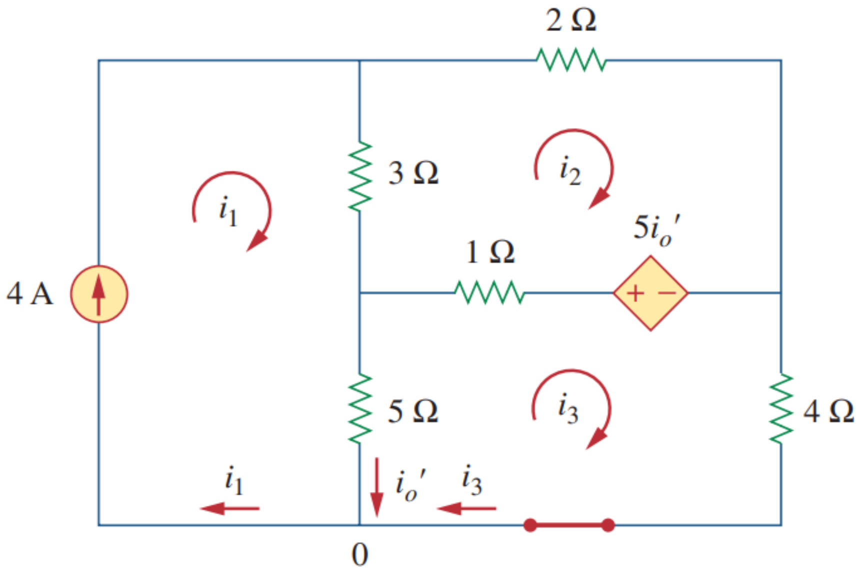
For loop 2, $-3i_1 + 6i_2 - 1i_3 - 5i'_o = 0$

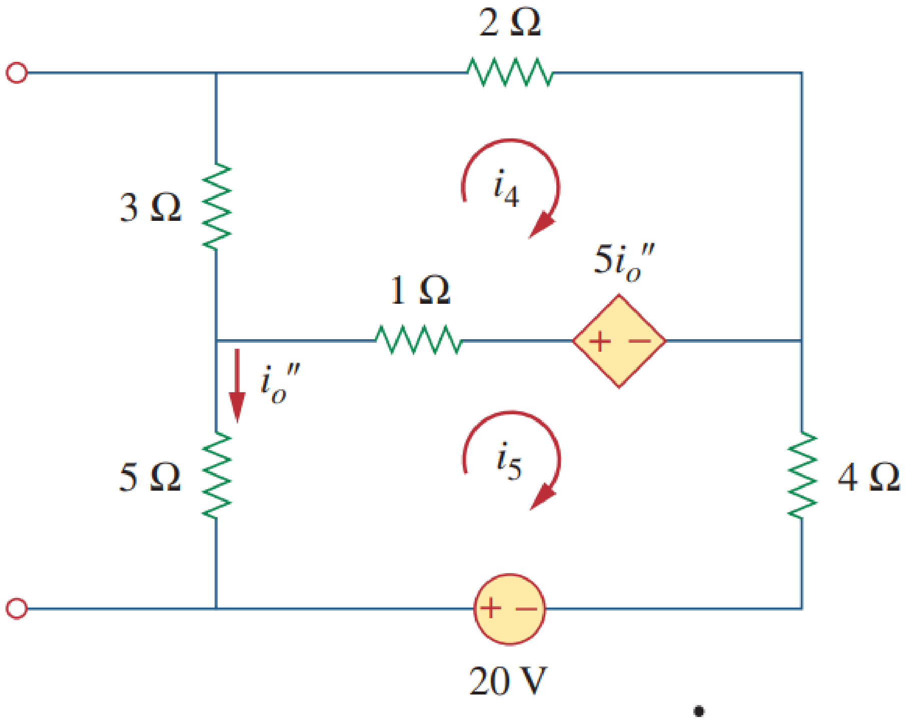
For loop 3, $-5i_1 - 1i_2 + 10i_3 + 5i'_o = 0$

continued…

at node 0, $i_3 = i_1 - i'_o = 4 - i'_o$

$$\left.\begin{aligned} 6i_2 - 1i_3 - 5i'_o &= 12 \\ -1i_2 + 10i_3 + 5i'_o &= 20 \\ 0i_2 + 1i_3 + 1i'_o &= 4 \end{aligned}\right\} \quad i'_o = \frac{52}{17}\ \text{A}$$





For loop 4, $6i_4 - i_5 - 5i''_o = 0$

for loop 5, $-i_4 + 10i_5 - 20 + 5i''_o = 0$

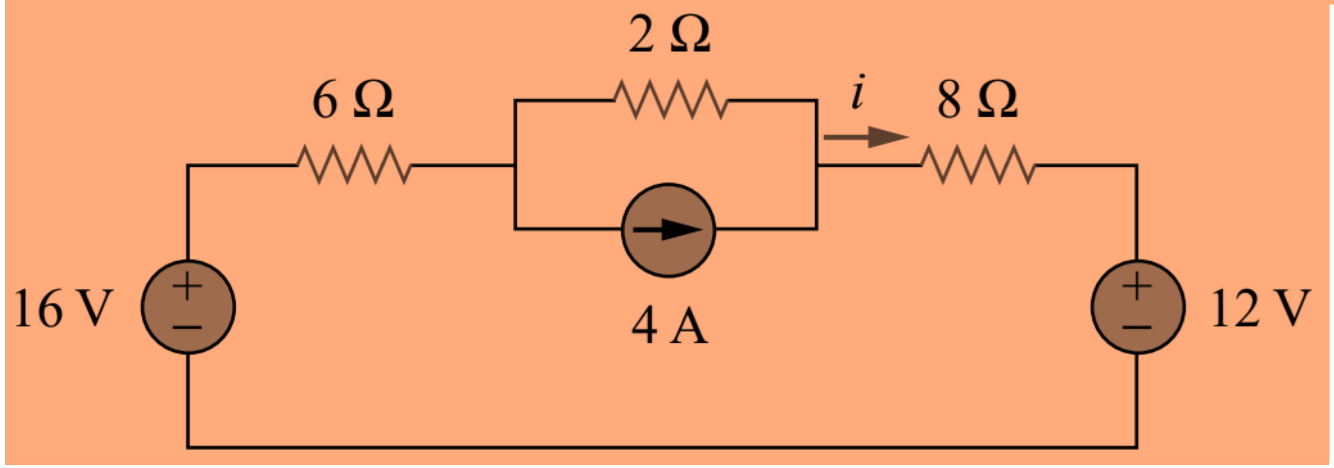
But $i_5 = -i''_o$.

$$\left.\begin{aligned} 6i_4 - 4i''_o &= 0 \\ i_4 + 5i''_o &= -20 \end{aligned}\right\} \quad i''_o = -\frac{60}{17}\ \text{A}$$

$$i_o = i'_o + i''_o = -0.4706\ \text{A}$$

continued…

Find i in the circuit using the superposition principle.

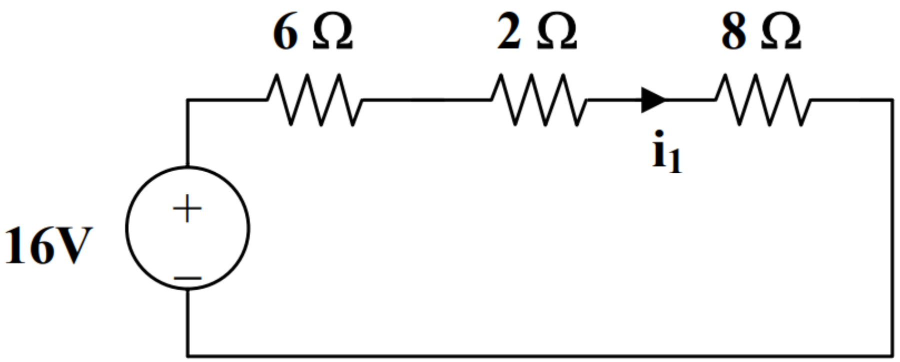


$$i = i_1 + i_2 + i_3$$

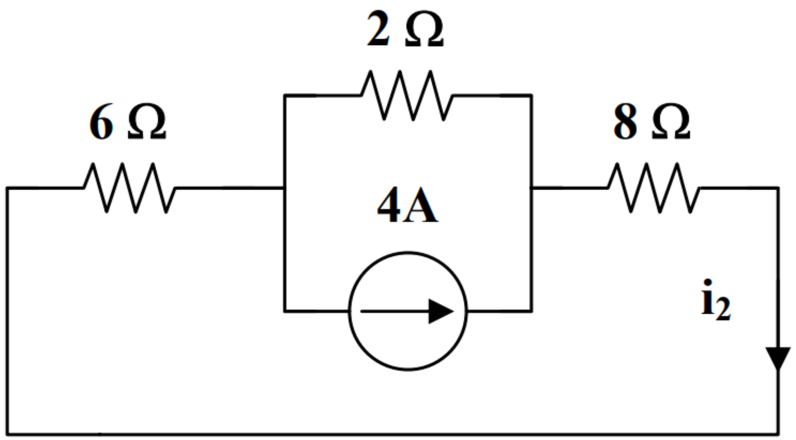
i_1 due to 16-V voltage source alone.

i_2 due to 4-A current source alone.

i_3 due to 12-V voltage source alone.

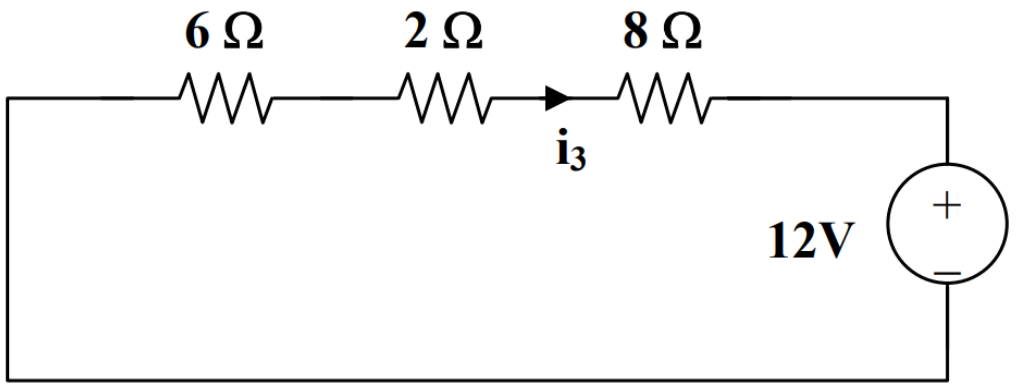


$$i_1 = \frac{16}{6+2+8} = 1A$$



$$i_2 = \frac{2}{2+14}(4) = 0.5$$

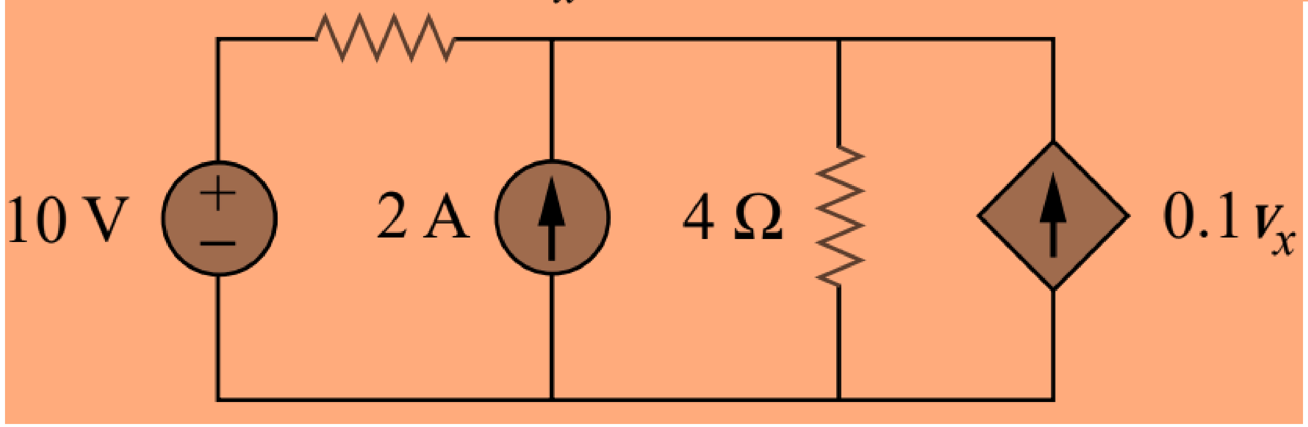
continued…



$$i_3 = \frac{-12}{16} = -0.75\text{A}$$

$$i = i_1 + i_2 + i_3 = 1 + 0.5 - 0.75 = 0.75 \text{ A}$$

Use superposition to find v_x in the circuit

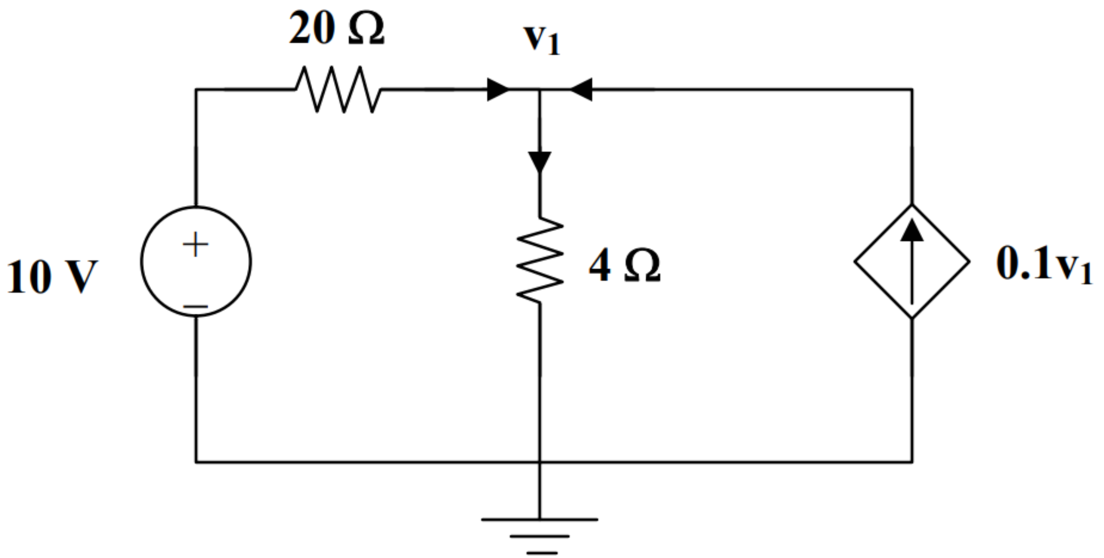


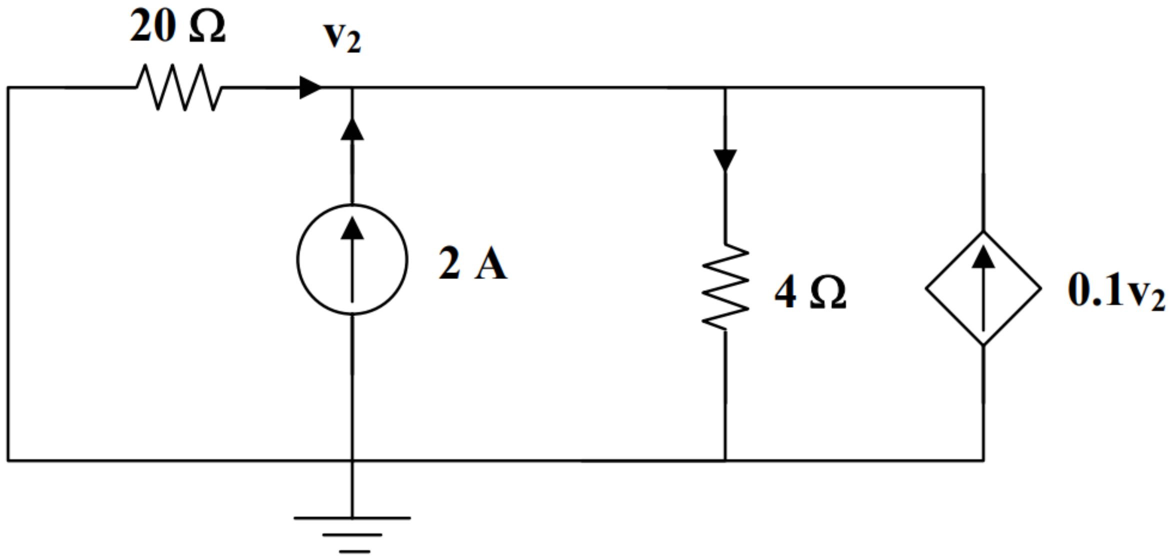
$$v_x = v_1 + v_2$$

v_1 due to 10-V voltage source alone.

v_2 due to 2-A current source alone.

continued…





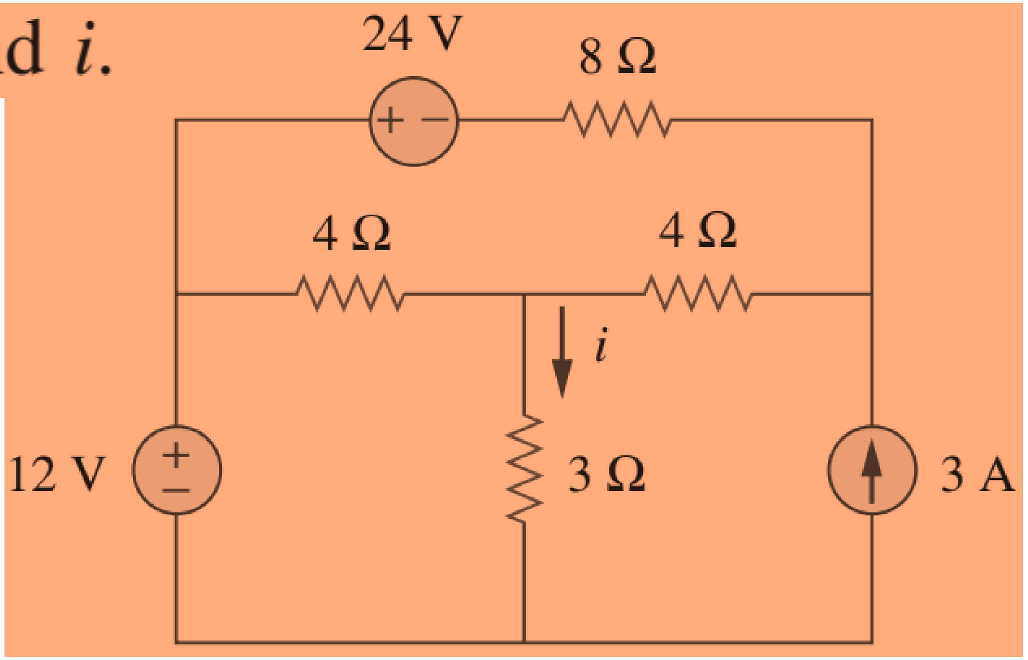
$$0.1v_1 + \frac{10 - v_1}{20} = \frac{v_1}{4} \quad \rightarrow \quad v_1 = 2.5$$

$$2 + 0.1v_2 + \frac{0 - v_2}{20} = \frac{v_2}{4} \quad \rightarrow \quad v_2 = 10$$

$$v_x = v_1 + v_2 = 12.5 \text{ V.}$$

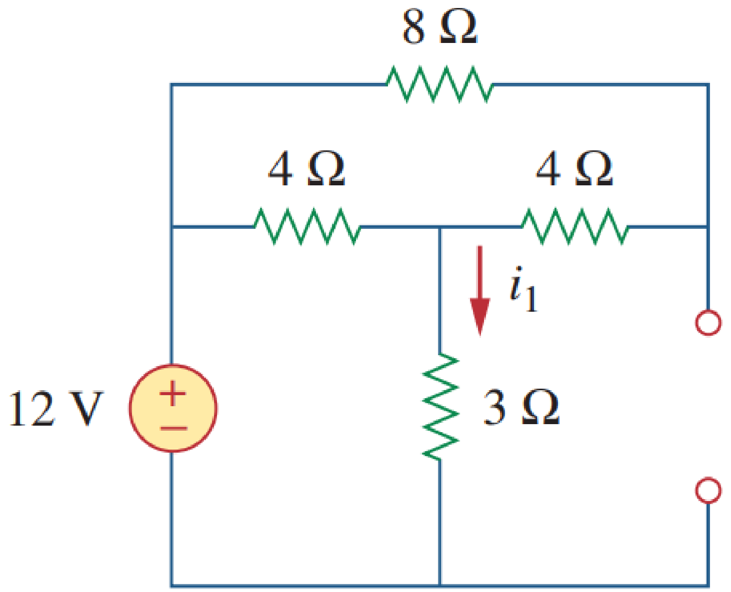
continued…

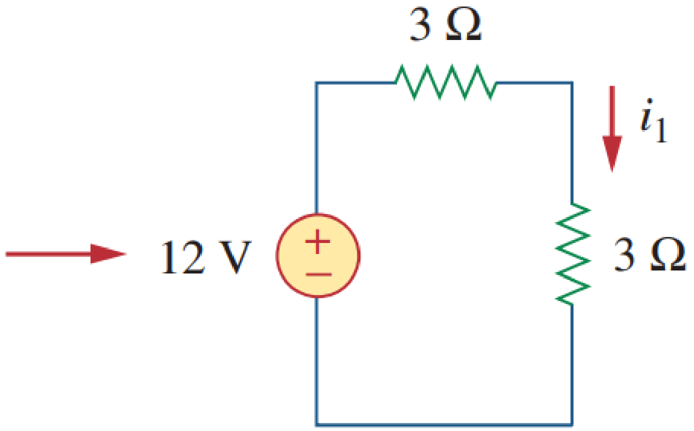
use the superposition theorem to find i.



$$i = i_1 + i_2 + i_3$$

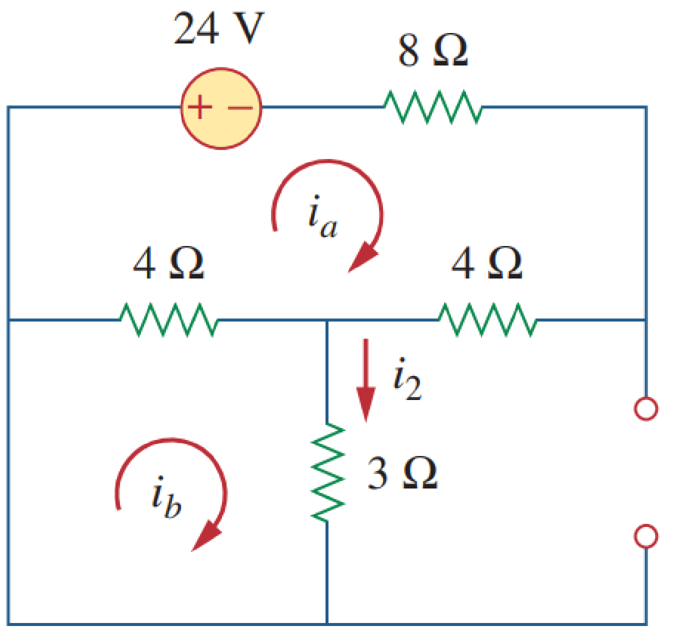
i_1 due to 12-V voltage source alone.
i_2 due to 24-V voltage source alone.
i_3 due to 3-A current source alone.





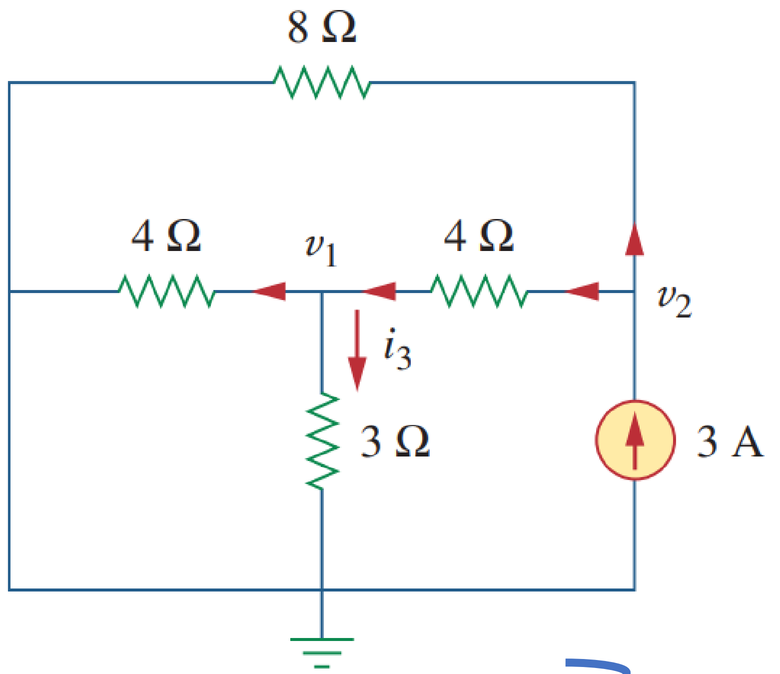
$$i_1 = \frac{12}{6} = 2\text{ A}$$

continued...



$$16i_a - 4i_b + 24 = 0$$
$$7i_b - 4i_a = 0$$
$$i_b = -1$$

$$i_2 = i_b = -1$$

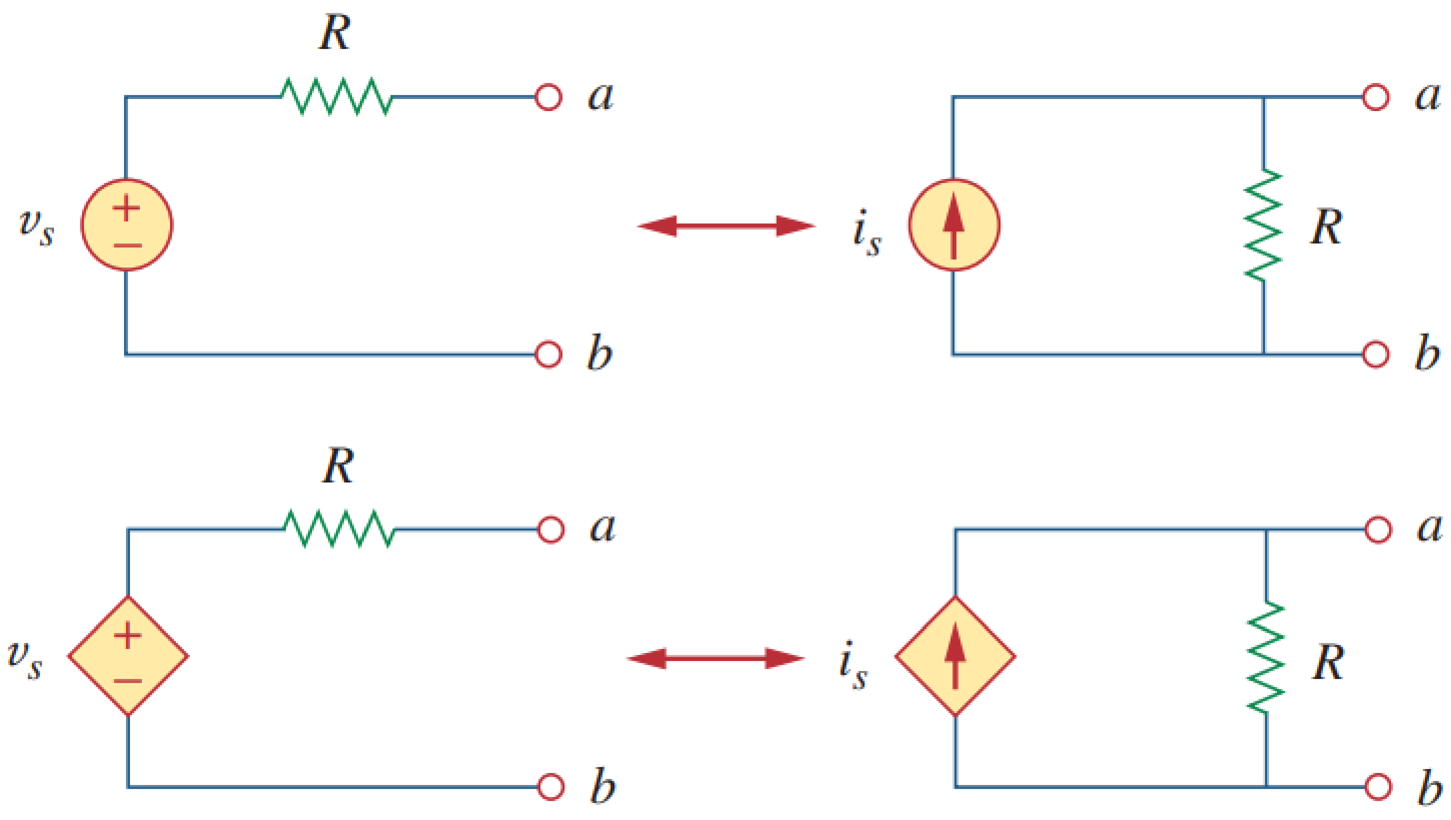


$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4}$$
$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3}$$
$$v_1 = 3$$

$$i_3 = \frac{v_1}{3} = 1 \text{ A}$$

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2 \text{ A}$$

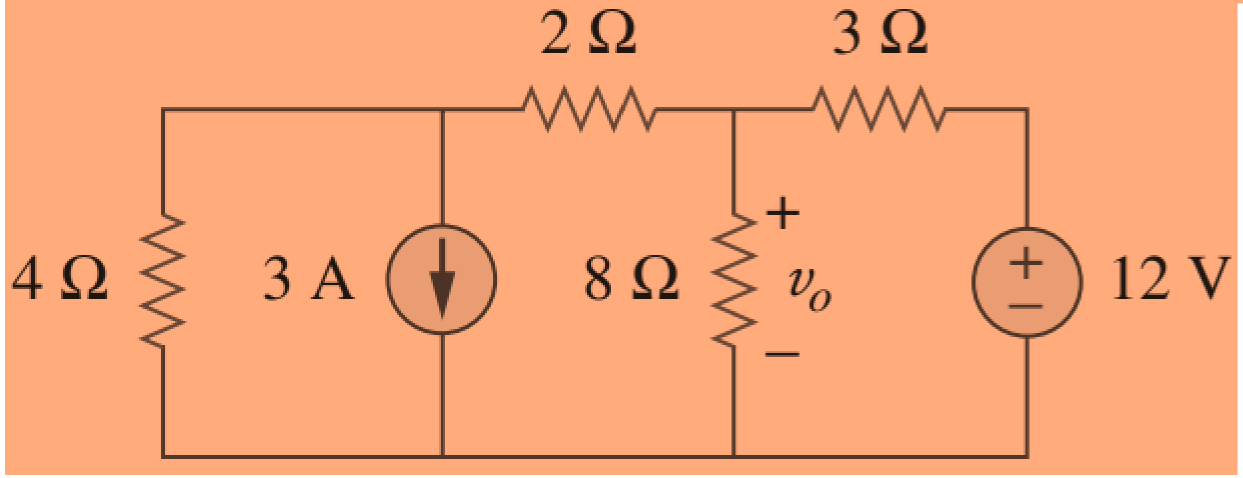
Source Transformation

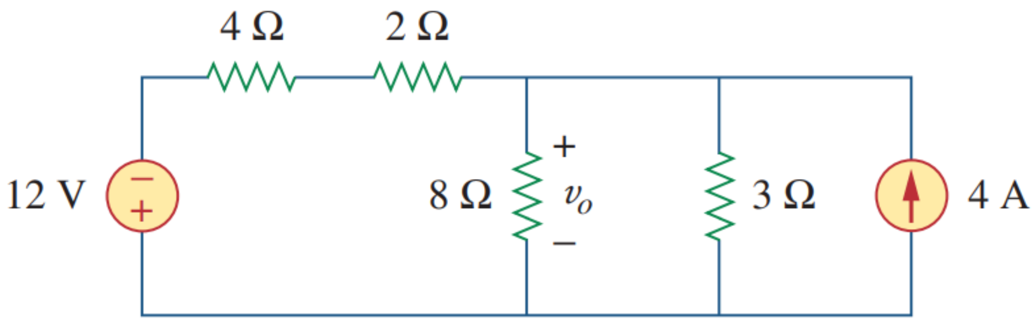


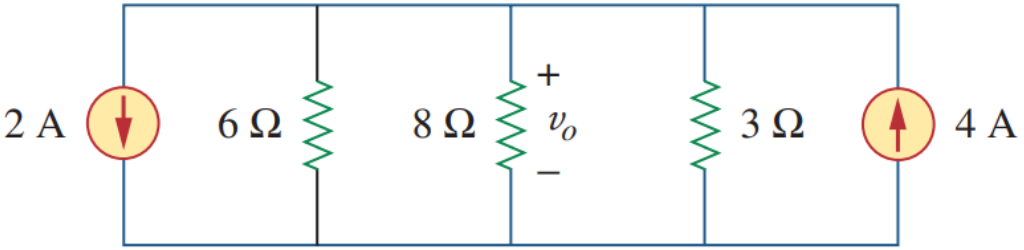
$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}$$

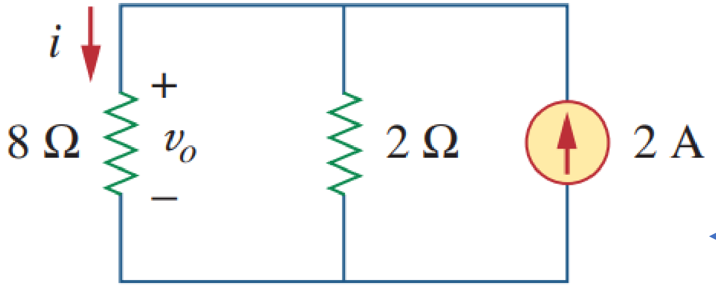
Example

Use source transformation to find v_o in the circuit







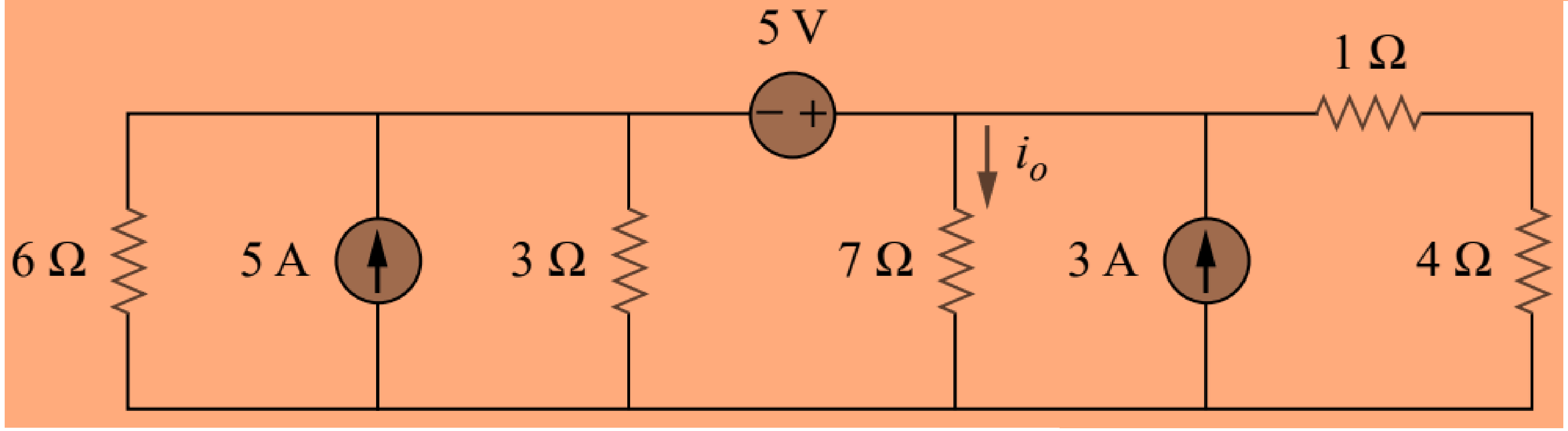


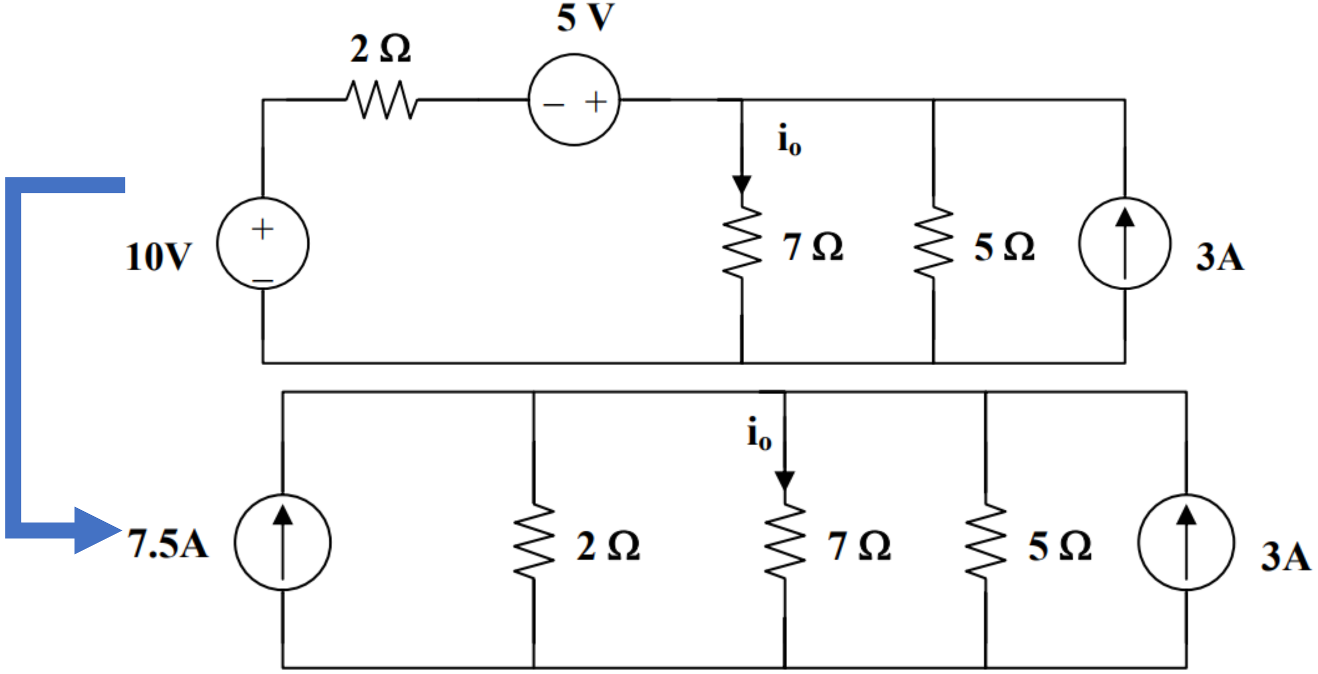
$$i = \frac{2}{2 + 8}(2) = 0.4 \text{ A}$$

$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

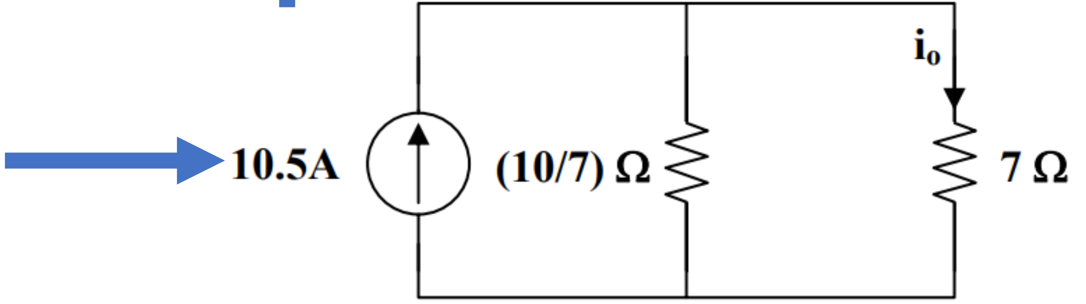
continued...

Find i_o in the circuit using source transformation.



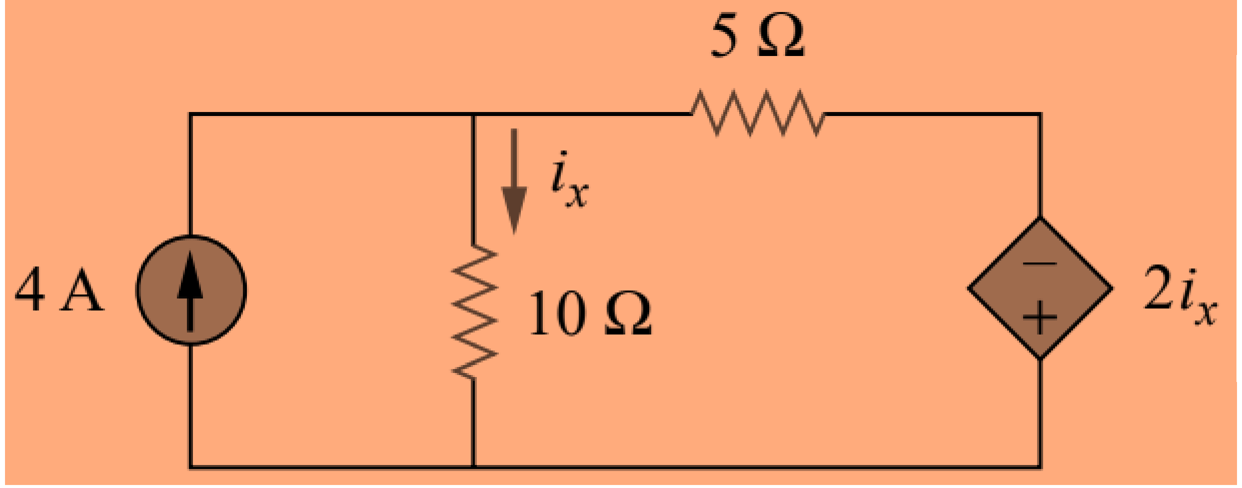


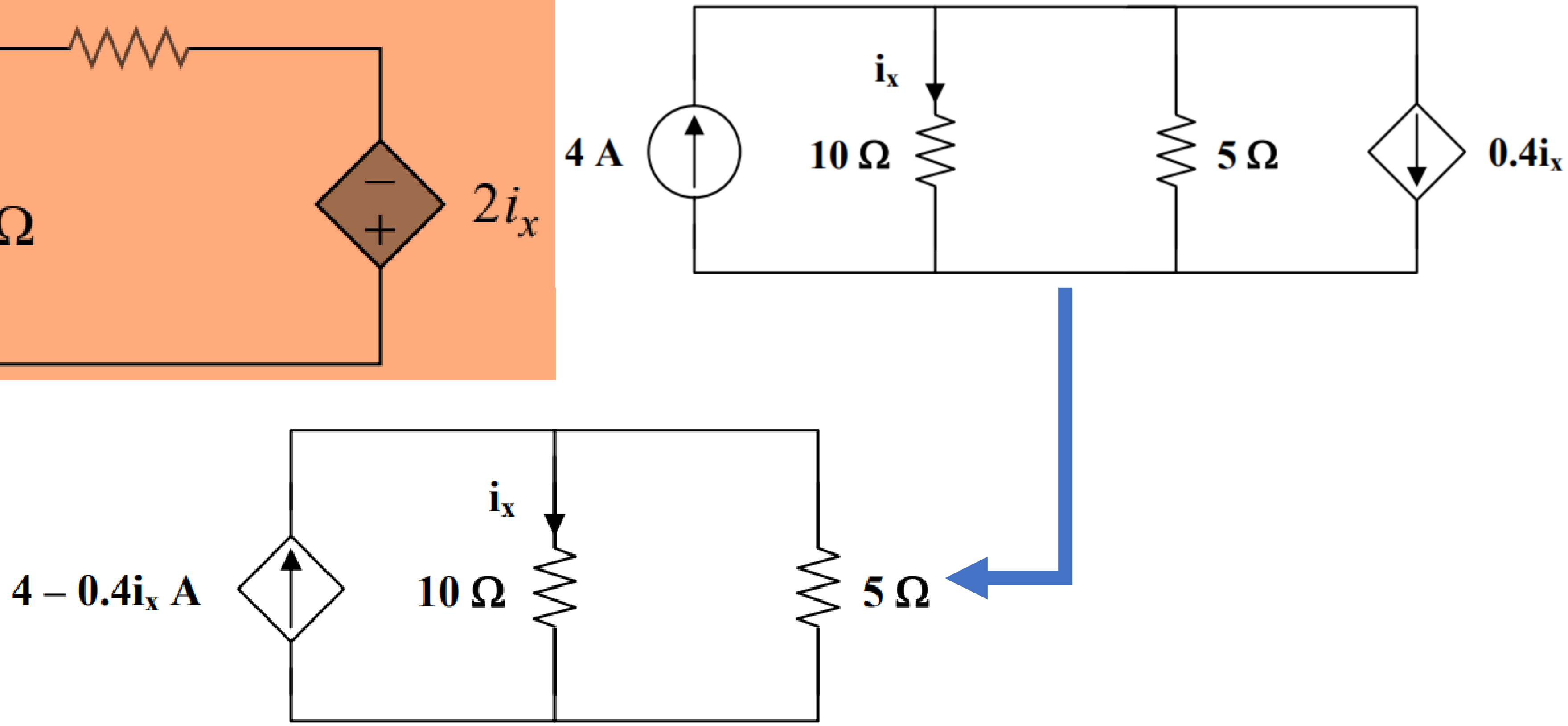
$$i_o = \frac{\frac{10}{7}}{\frac{10}{7}+7}(10.5) = 1.78\text{ A.}$$



continued…

Use source transformation to find i_x in the circuit

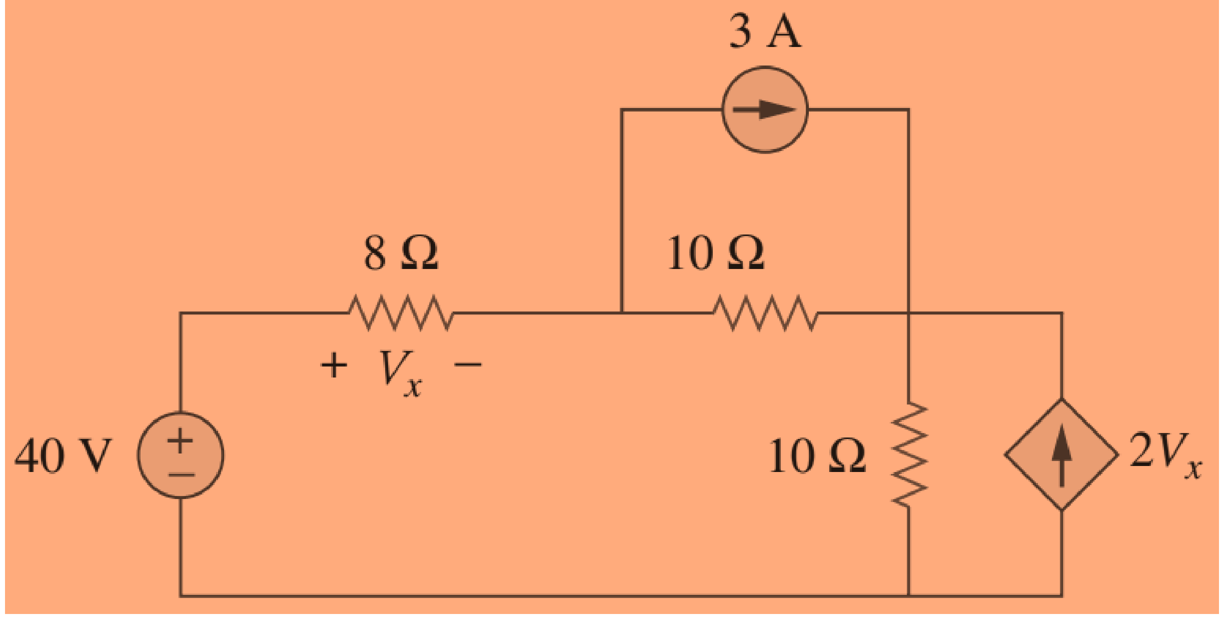


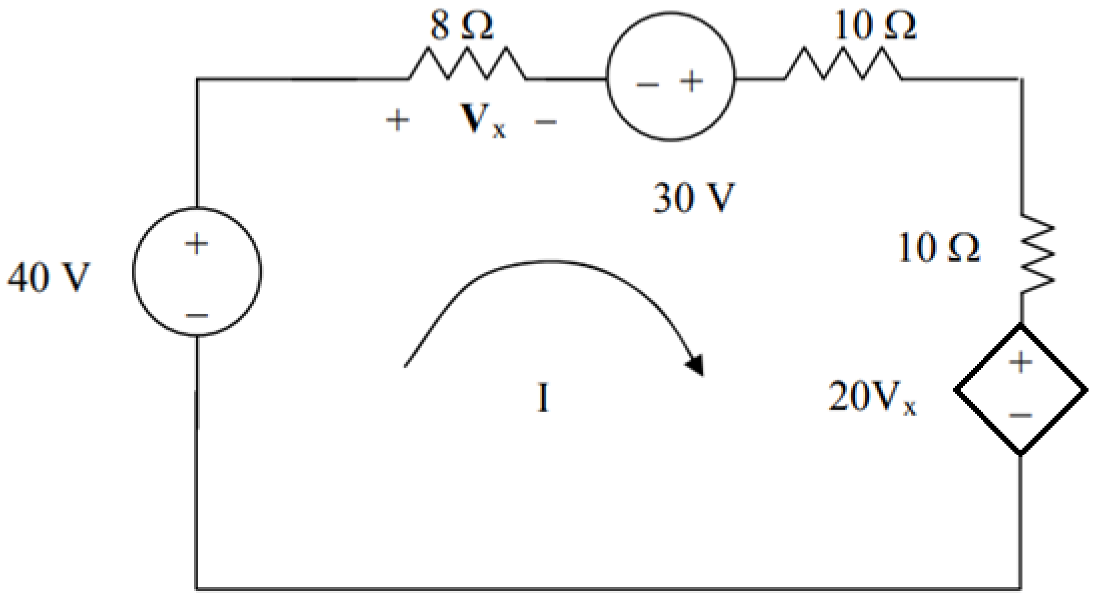


$$i_x = \frac{5}{15}(4 - 0.4i_x) \longrightarrow i_x = 1.176 \text{ A.}$$

continued...

Use source transformation to find the voltage V_x





$$28I - 70 + 20V_x = 0$$

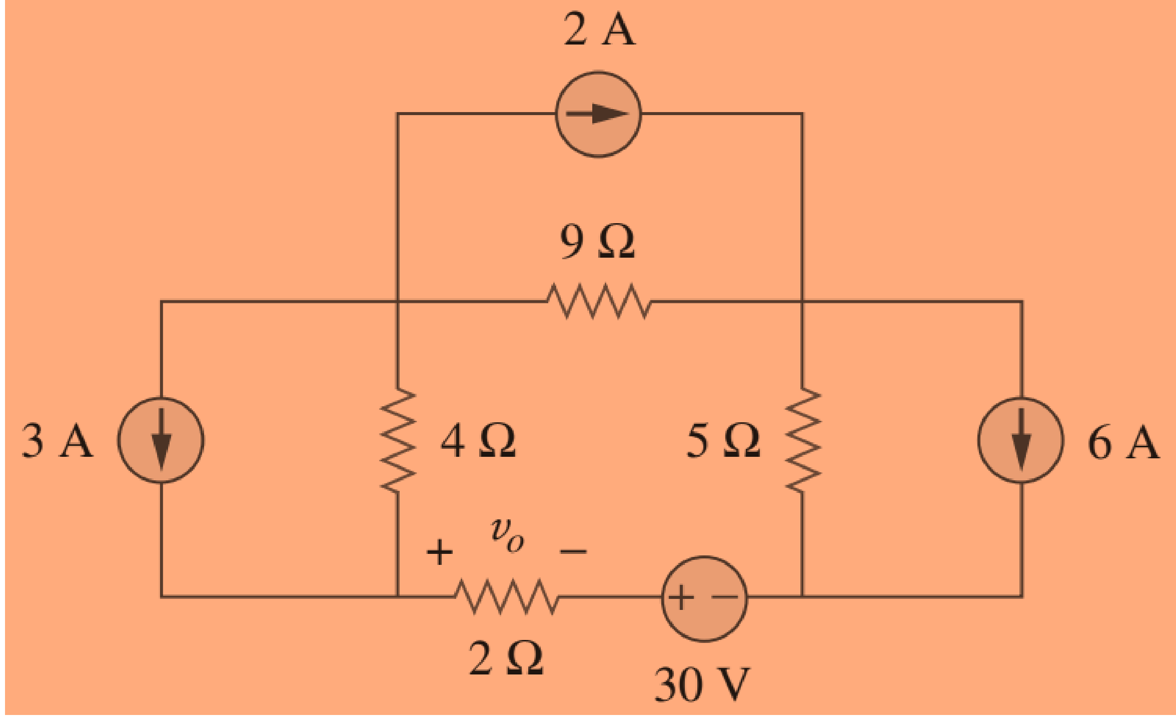
but $V_x = 8I$

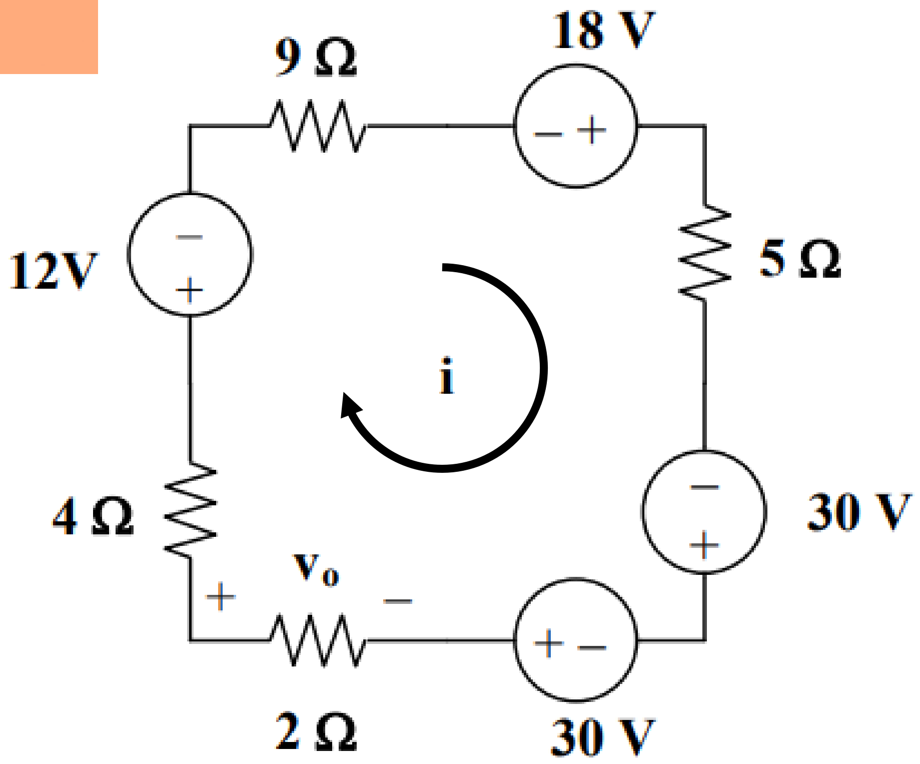
$$28I + 160I = 70 \rightarrow I = 0.3723 \text{ A}$$

$$V_x = \mathbf{2.978\ V}.$$

continued…

Obtain v_o in the circuit using source transformation.



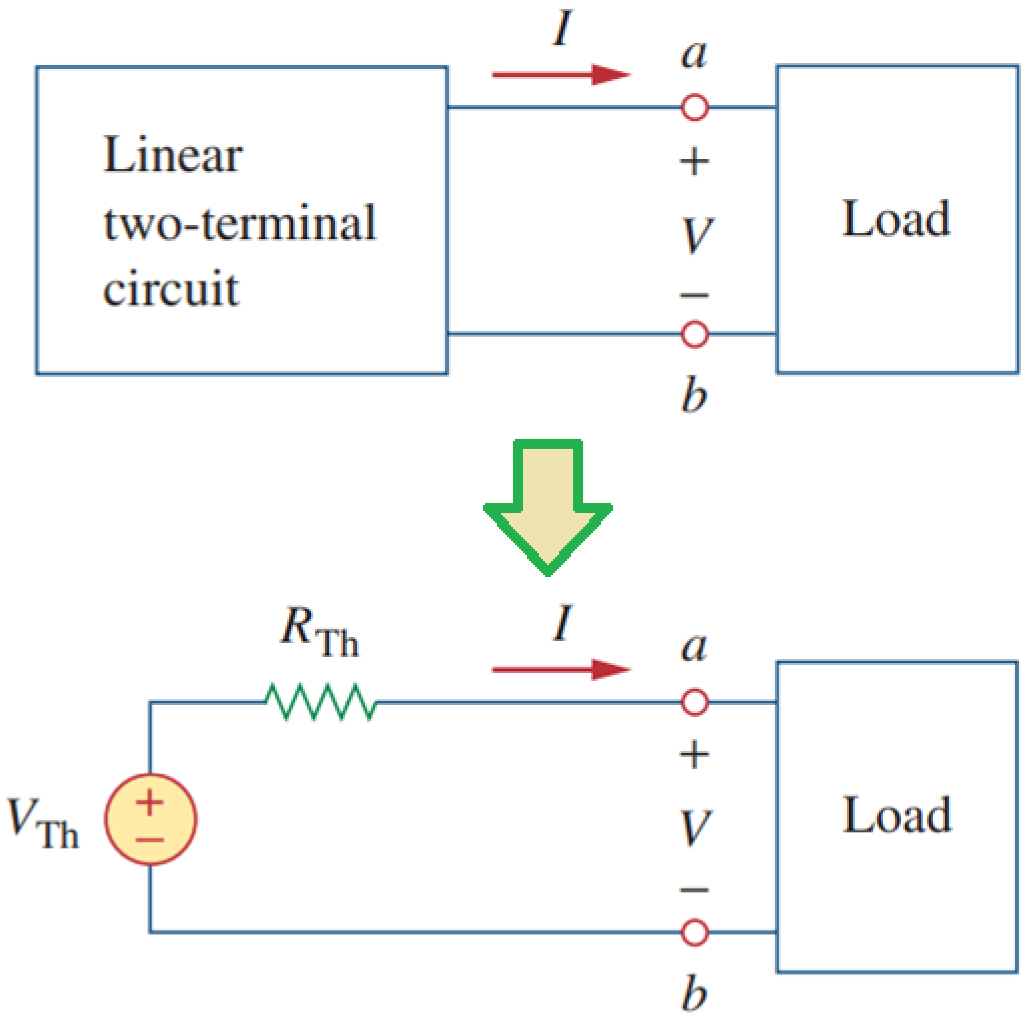


$$20i + 12 - 18 - 30 - 30 = 0$$

$$\Rightarrow i = 3.3$$

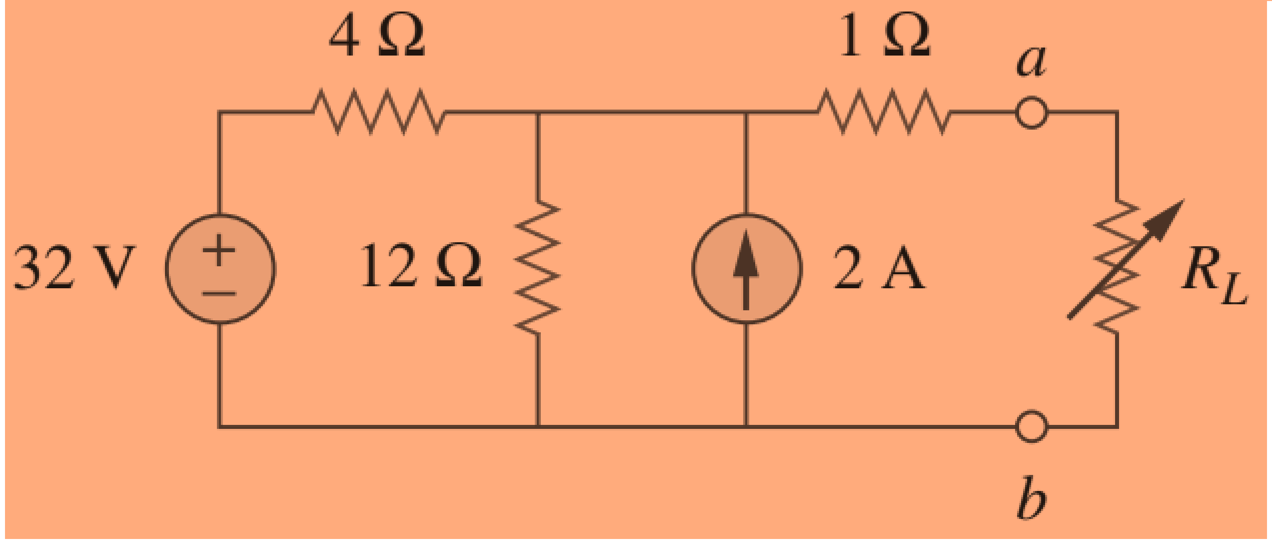
$$\therefore v_0 = -2i = -6.6 \text{ V}$$

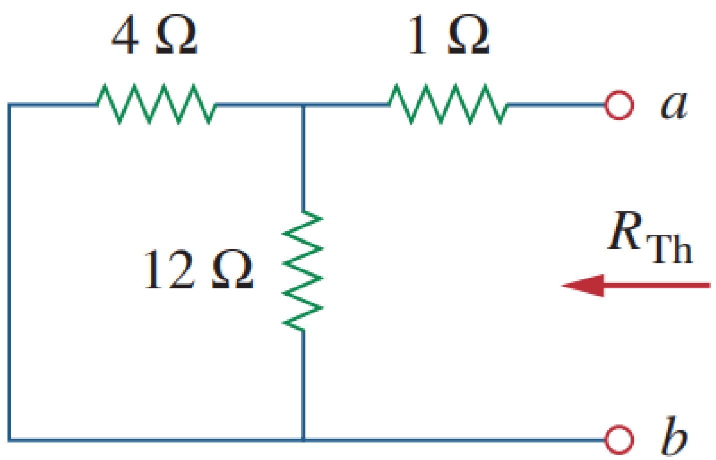
Thevenin's Theorem



Example

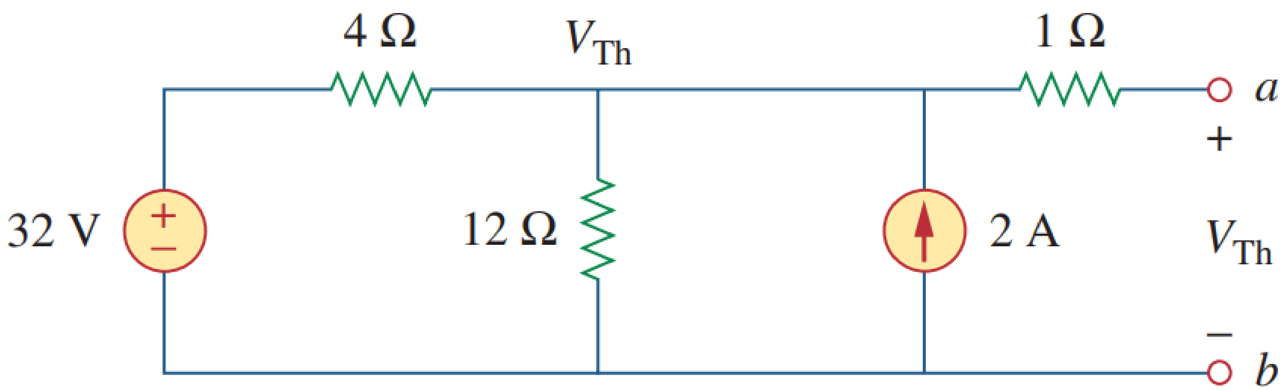
Find the Thevenin equivalent circuit of the circuit shown , to the left of the terminals *a*-*b*. Then find the current through $R_L = 6, 16,$ and $36\ \Omega$.





$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

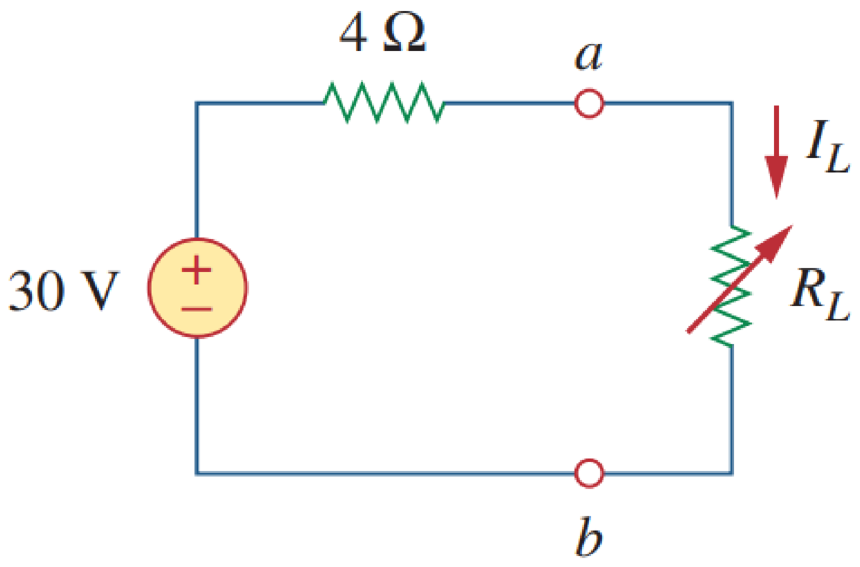
continued…



$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

$$96 - 3V_{Th} + 24 = V_{Th}$$

$$V_{Th} = 30\text{ V}$$



$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

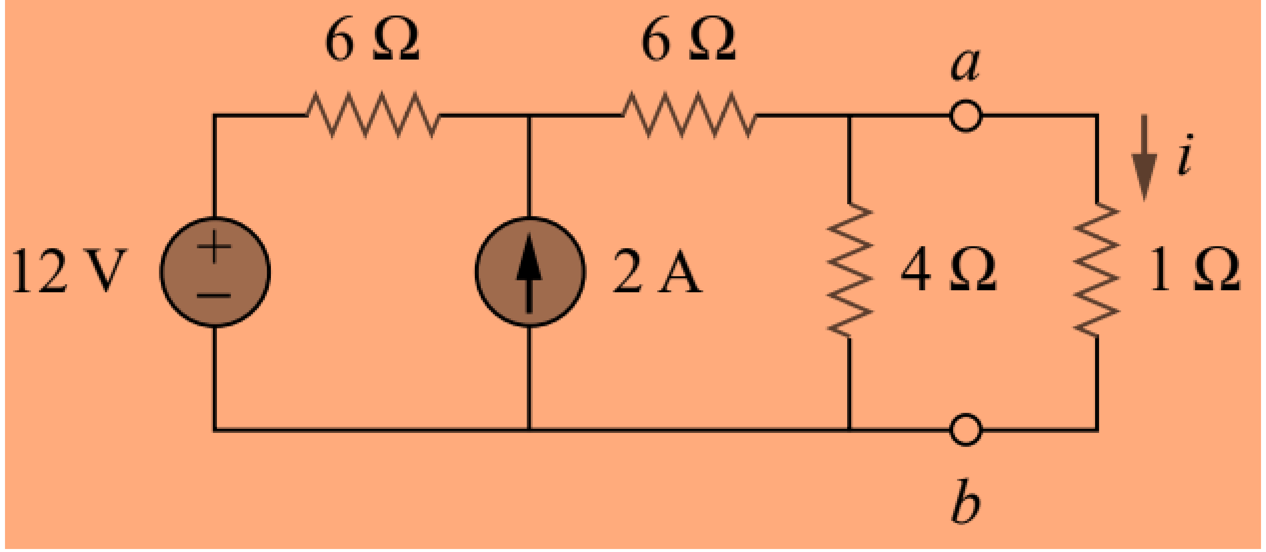
When $R_L = 6, \quad I_L = \frac{30}{10} = 3\text{ A}$

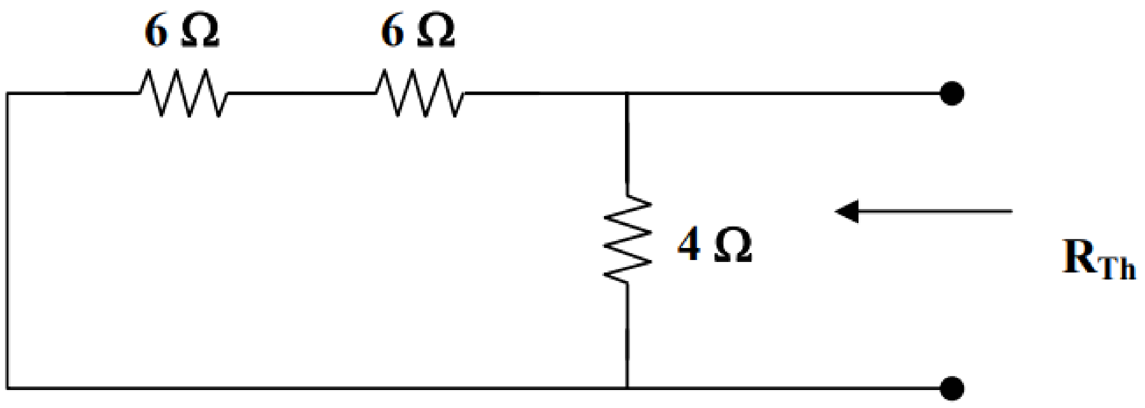
When $R_L = 16, \quad I_L = \frac{30}{20} = 1.5\text{ A}$

When $R_L = 36, \quad I_L = \frac{30}{40} = 0.75\text{ A}$

continued…

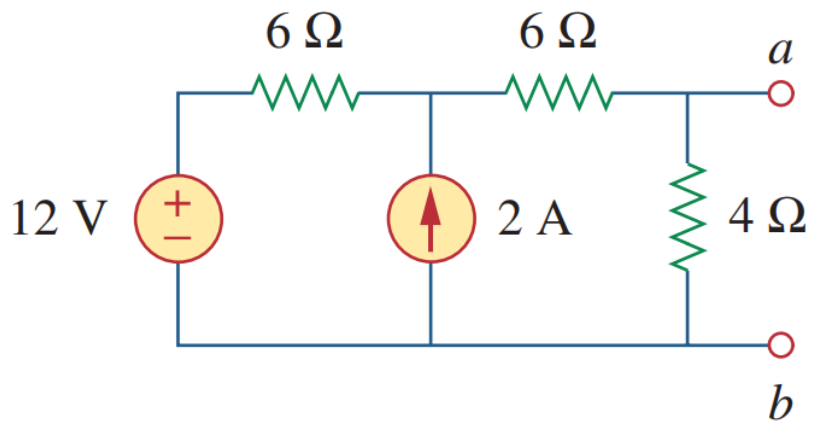
Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit . Then find i.

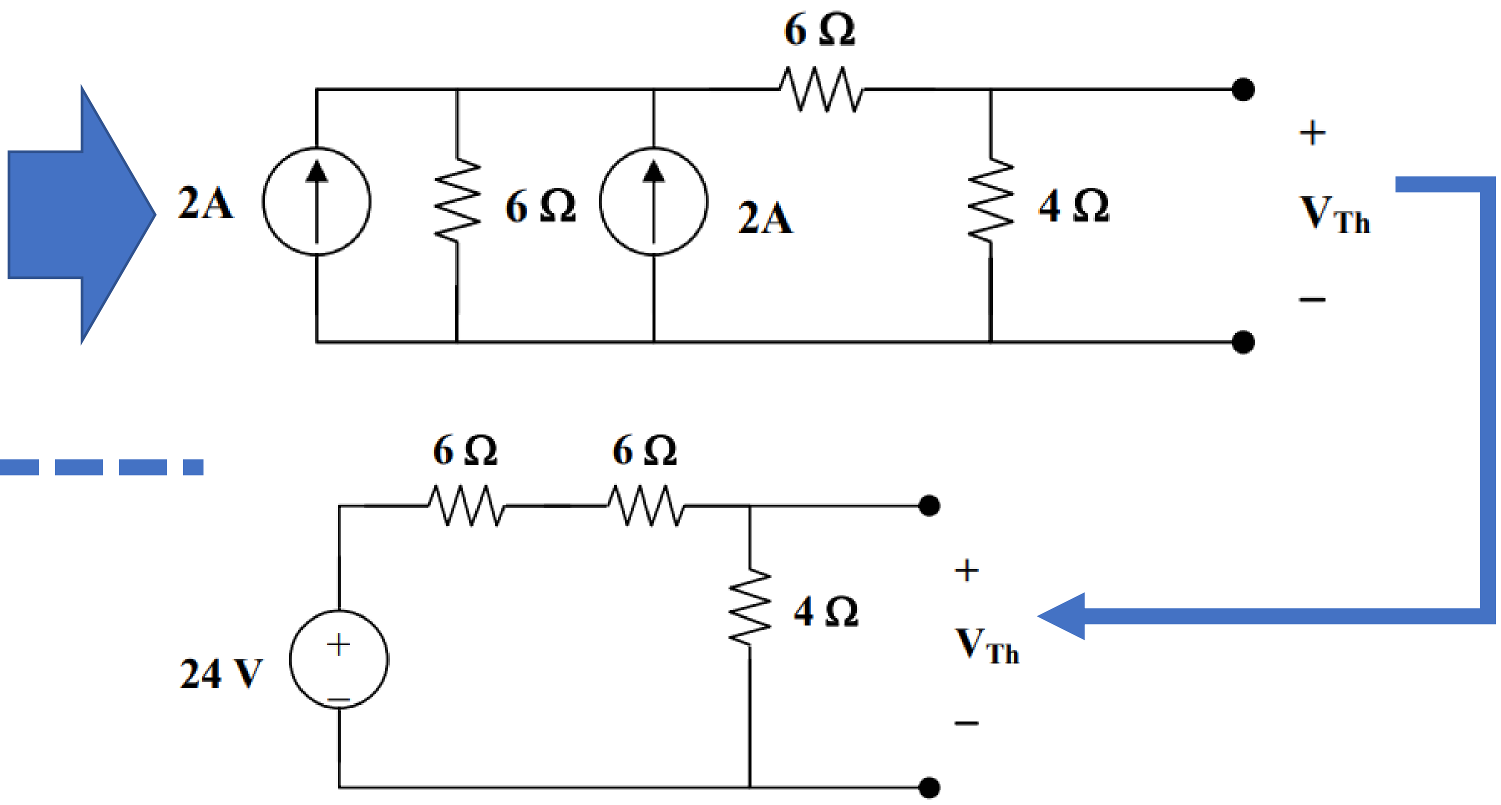




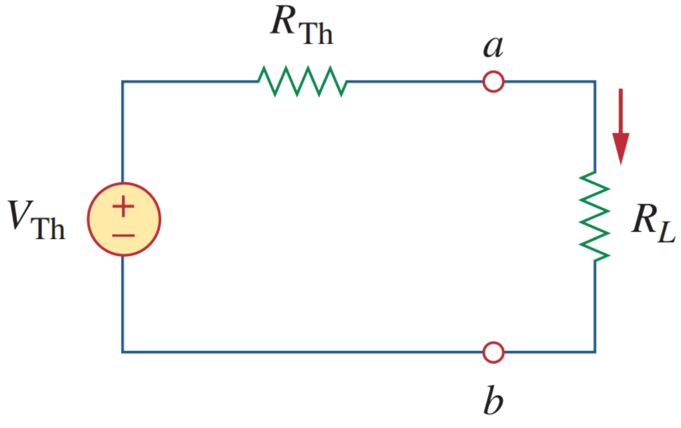
$$R_{Th} = (6+6)\|4 = \frac{12\text{x}4}{16} = 3\ \Omega$$

continued…





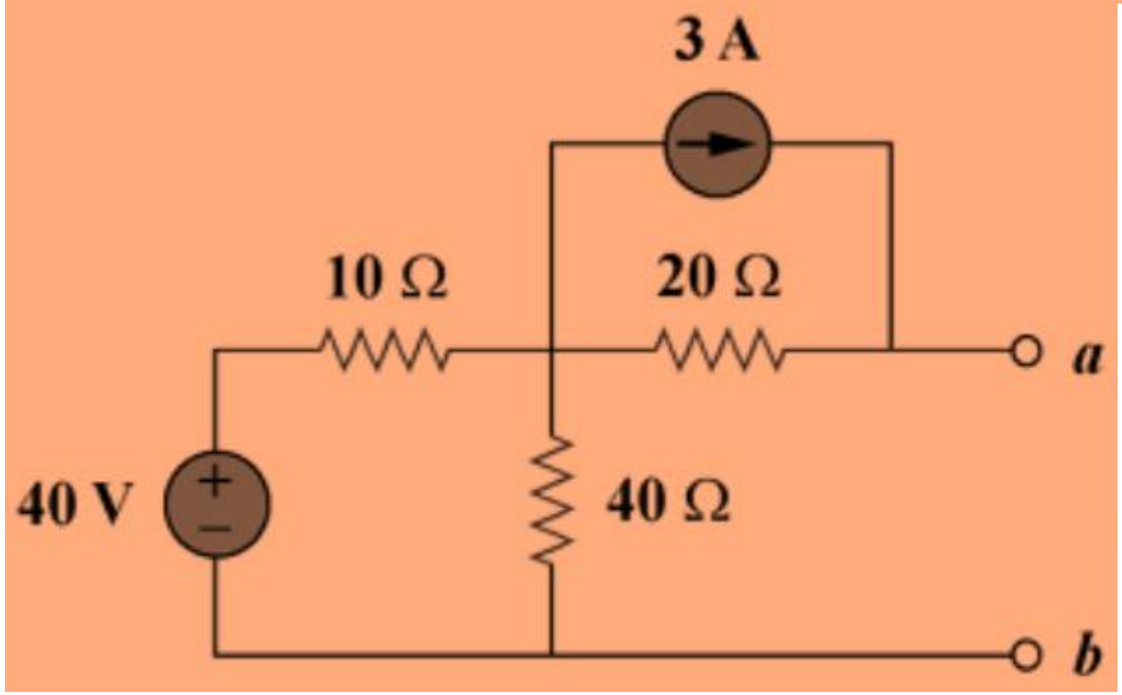
$$V_{Th} = \frac{4}{4+12}(24) = 6 \text{ V}$$

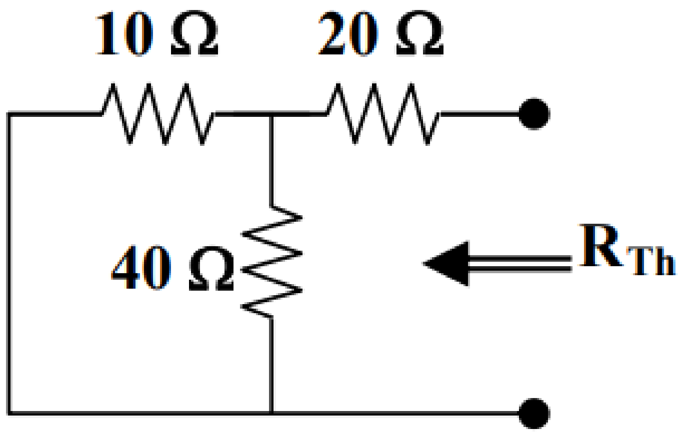


$$i = \frac{V_{Th}}{R_{Th}+1} = \frac{6}{3+1} = 1.5 \text{ A}$$

continued…

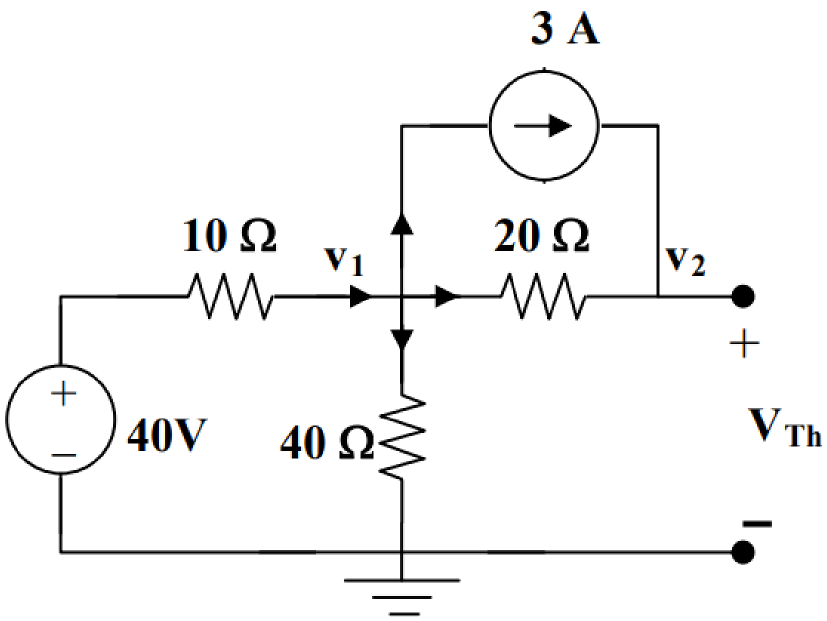
Find the Thevenin equivalent at terminals ***a-b*** of the circuit





$R_{Th} = 20 + 10||40 = \mathbf{28\ ohms}$

continued…



At node 1,

$$(40 - v_1)/10 = 3 + [(v_1 - v_2)/20] + v_1/40$$

$$\rightarrow 40 = 7v_1 - 2v_2$$

At node 2,

$$3 + (v_1 - v_2)/20 = 0$$

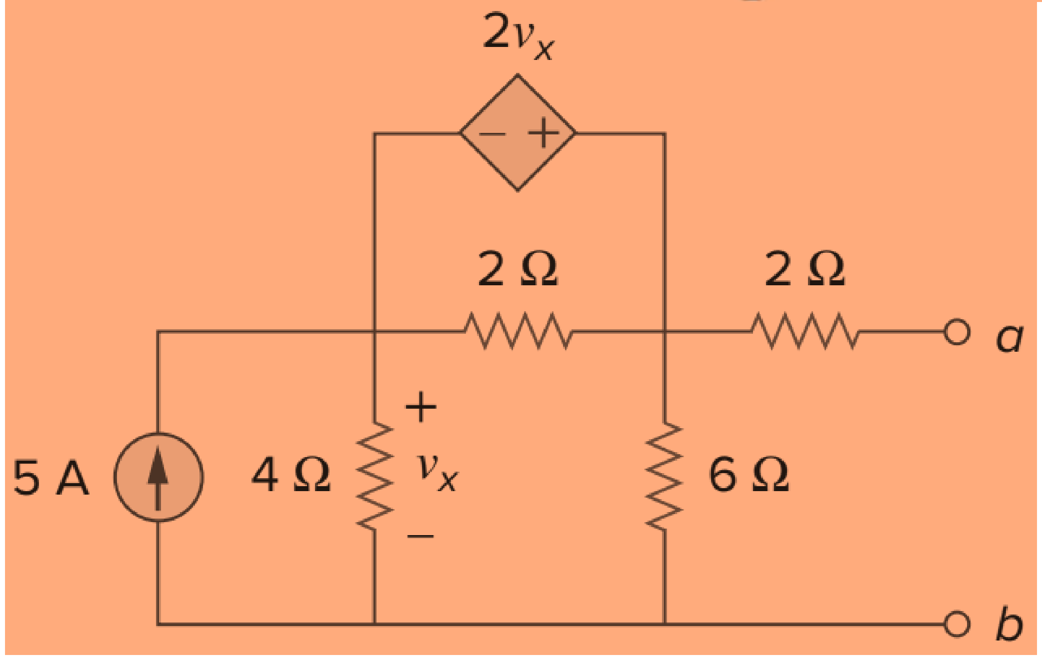
$$\rightarrow -60 = v_1 - v_2$$

$$v_1 = 32 \text{ V}, \; v_2 = 92 \text{ V}$$

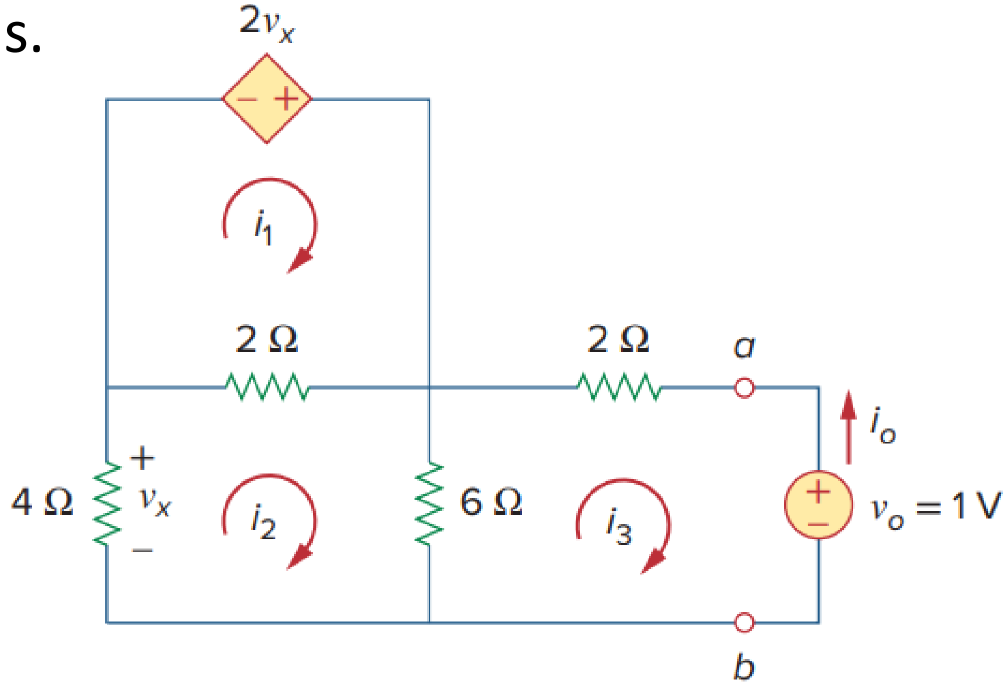
$$V_{Th} = v_2 = \mathbf{92\ V}$$

continued…

Obtain the Thevenin equivalent seen at terminals *a*-*b*



- To find R_{Th}, we set the independent source equal to zero but leave the dependent source alone.
- Because of the presence of the dependent source, however, we excite the network with a voltage source v_o connected to the terminals.



Applying mesh analysis to loop 1

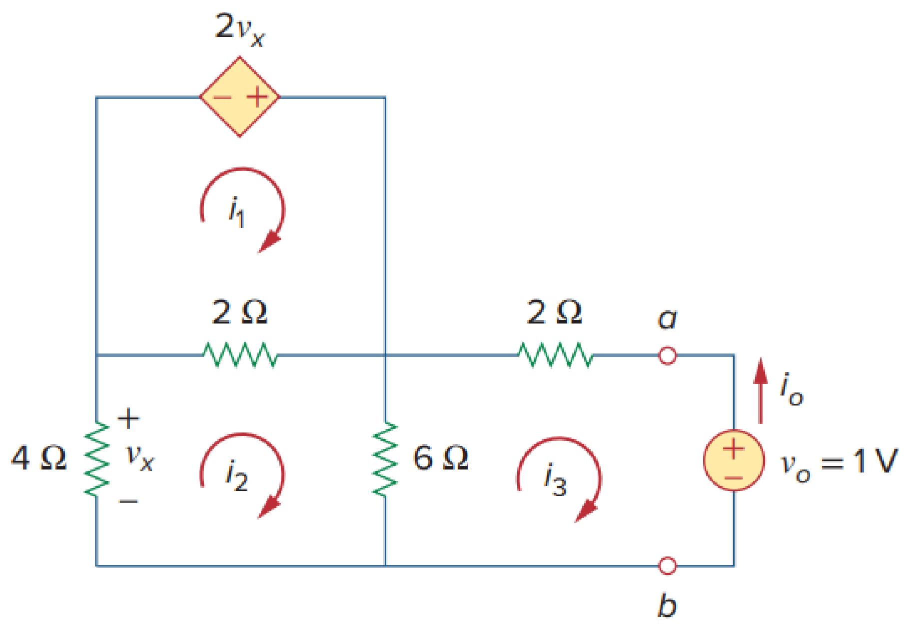
$$-2v_x + 2(i_1 - i_2) = 0$$

$$v_x = i_1 - i_2$$

But $-4i_2 = v_x$

$$i_1 = -3i_2$$

continued…



For loops 2 and 3,

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

$$6(i_3 - i_2) + 2i_3 + 1 = 0$$

Solving these equations ⟶ $i_3 = -\frac{1}{6}$ A

But $i_o = -i_3 = 1/6$ A.

$$R_{Th} = \frac{1\text{ V}}{i_o} = 6\ \Omega$$

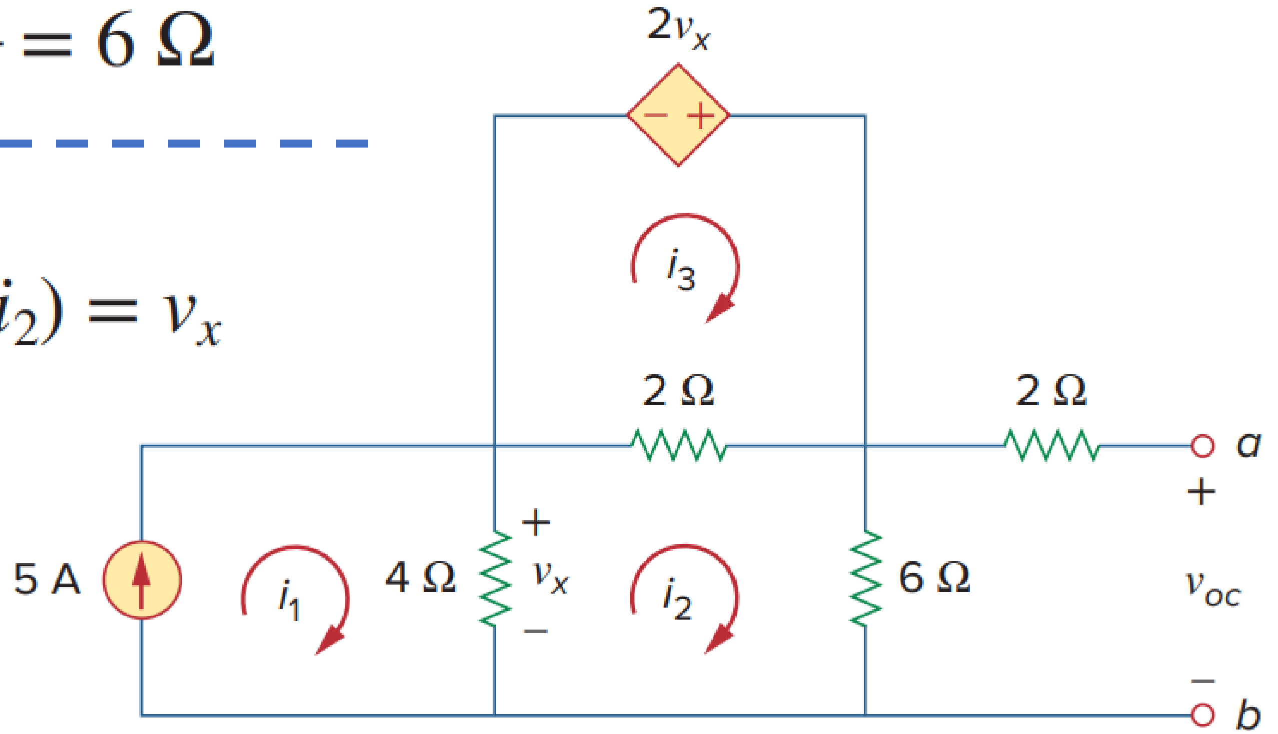
$$i_1 = 5$$

$$-2v_x + 2(i_3 - i_2) = 0 \qquad \text{But } 4(i_1 - i_2) = v_x$$

$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

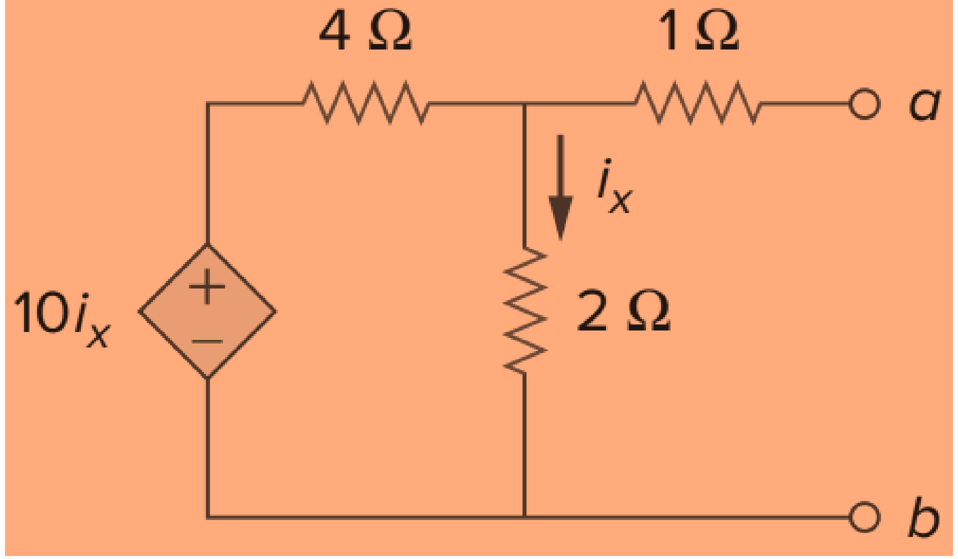
Solving these equations ⟶ $i_2 = 10/3$

$$V_{Th} = v_{oc} = 6i_2 = 20\text{ V}$$

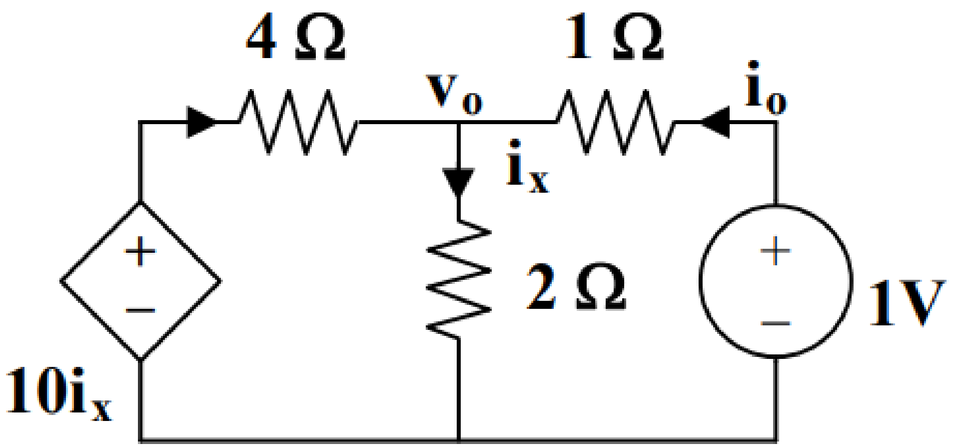


continued...

Obtain the Thevenin equivalent seen at terminals *a-b*



With no independent sources, $V_{Th} = \mathbf{0\ V}$.



$$i_x = [(1 - v_o)/1] + [(10i_x - v_o)/4]$$

$$5v_o = 4 + 6i_x$$

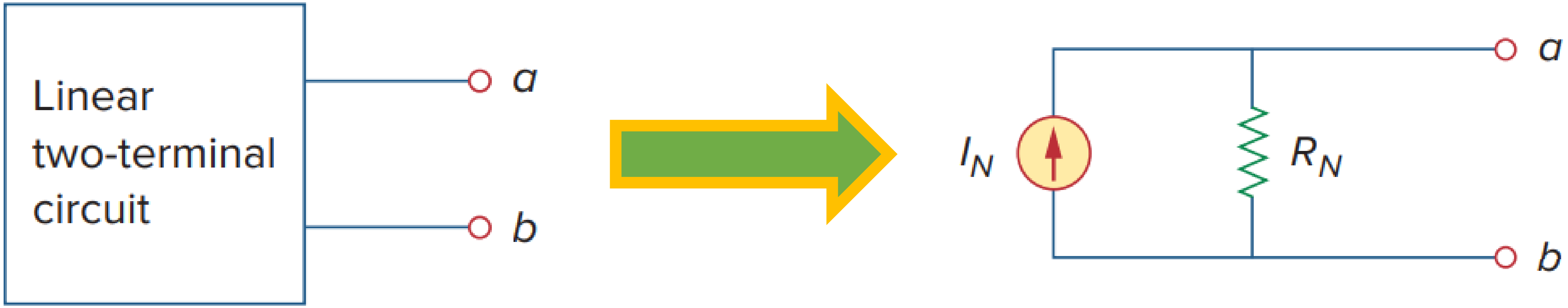
But $i_x = v_o/2$

$$5v_o = 4 + 3v_o \rightarrow v_o = 2$$

$$i_o = (1 - v_o)/1 = -1$$

$$R_{Th} = 1/i_o = \mathbf{-1\ ohm}$$

Norton’s Theorem

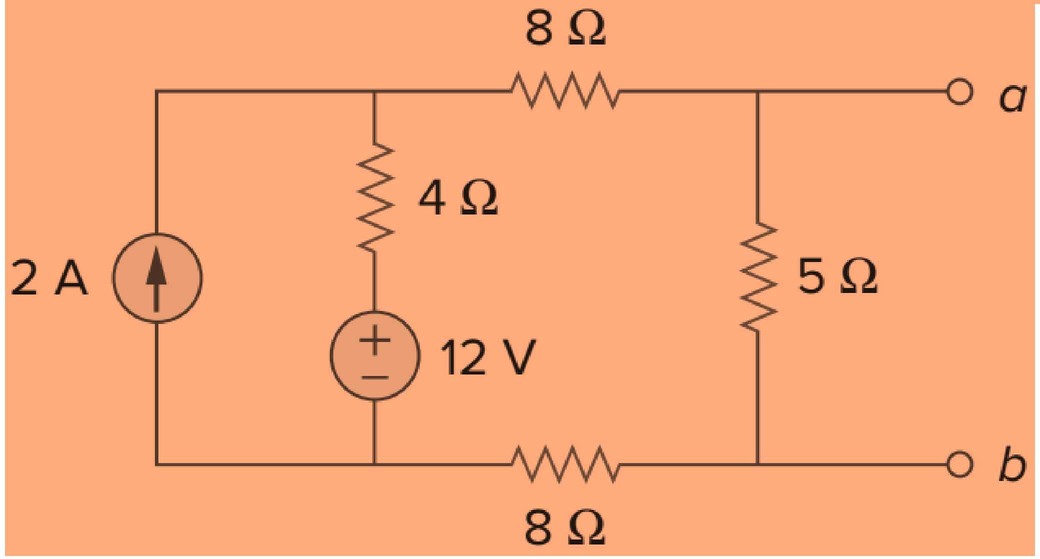


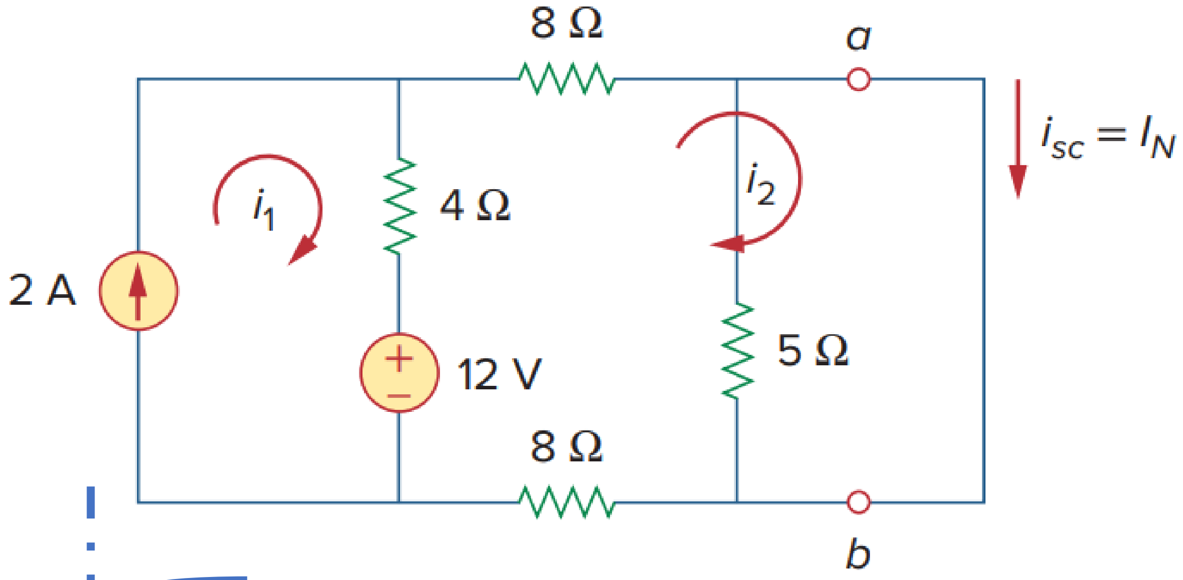
$$R_N = R_{Th}$$

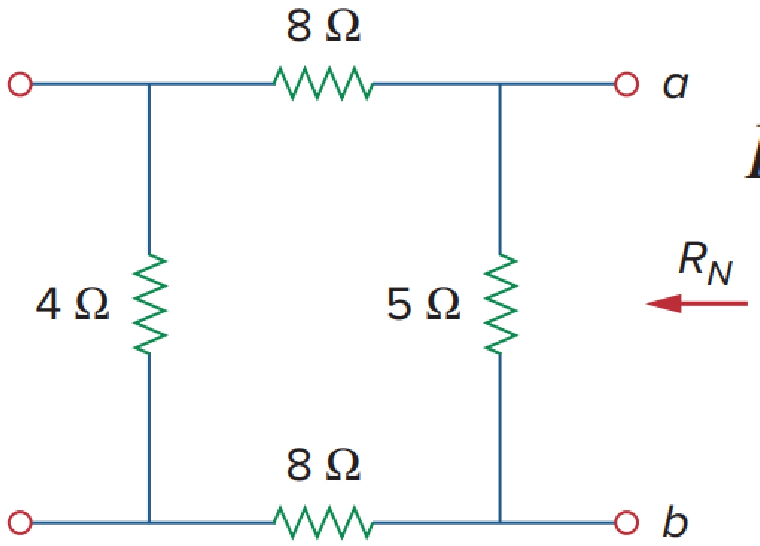
$$I_N = \frac{V_{Th}}{R_{Th}}$$

Example

Determine the Norton equivalent at terminals *a*-*b*







$$R_N = 5 \parallel (8 + 4 + 8)$$

$$= \frac{20 \times 5}{25} = 4\ \Omega$$

$$\begin{cases} i_1 = 2\text{ A} \\ 20i_2 - 4i_1 - 12 = 0 \end{cases}$$

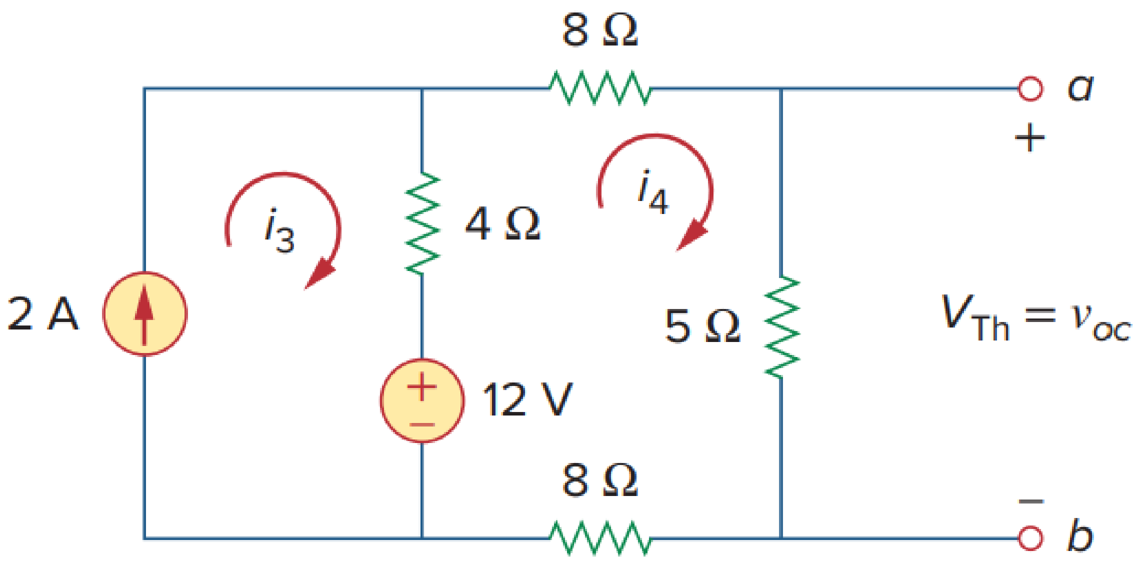
Solving these equations

$$i_2 = 1\text{ A}$$

$$= i_{sc} = I_N$$

continued…

Alternative via Thevenin Voltage:



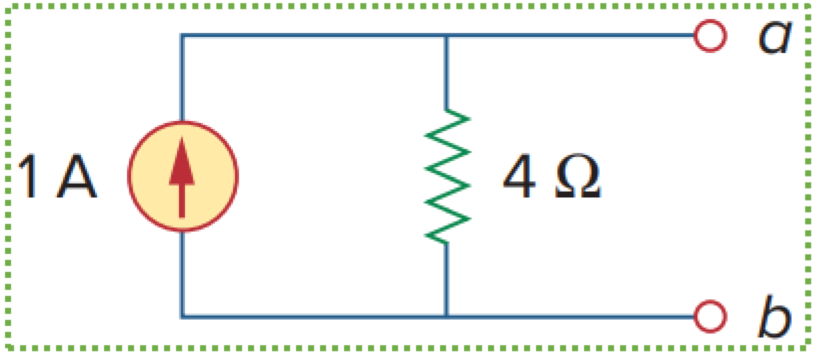
$$i_3 = 2\text{ A}$$
$$25i_4 - 4i_3 - 12 = 0$$

Solving these equations

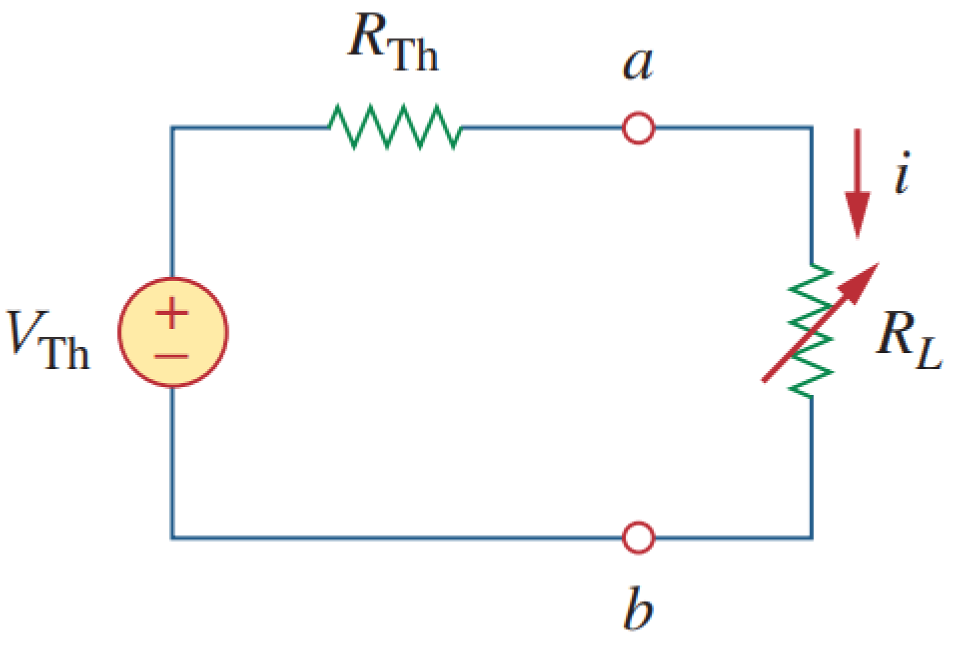
$$i_4 = 0.8\text{ A}$$

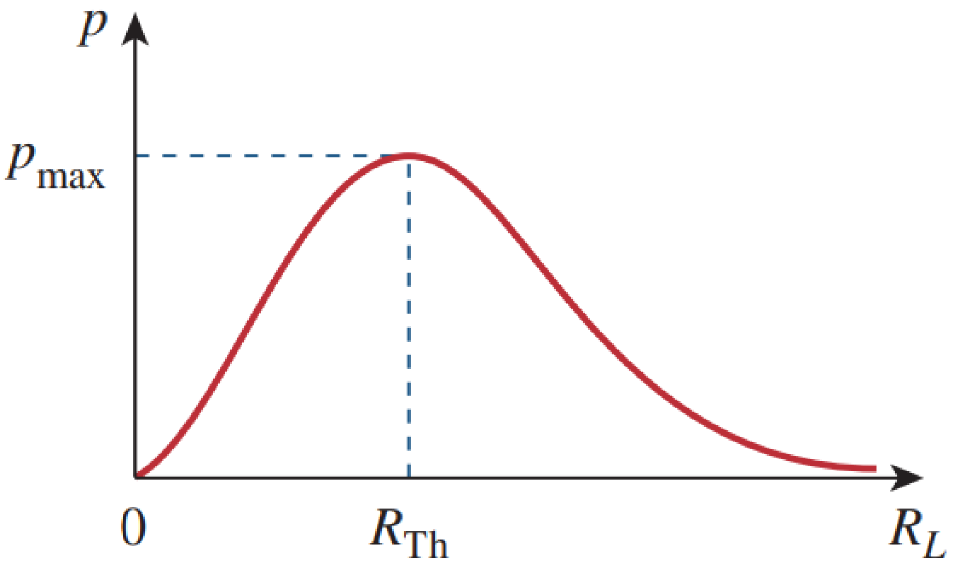
$$v_{oc} = V_{Th} = 5i_4$$
$$= 4\text{ V}$$

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1\text{ A}$$



Maximum Power Transfer





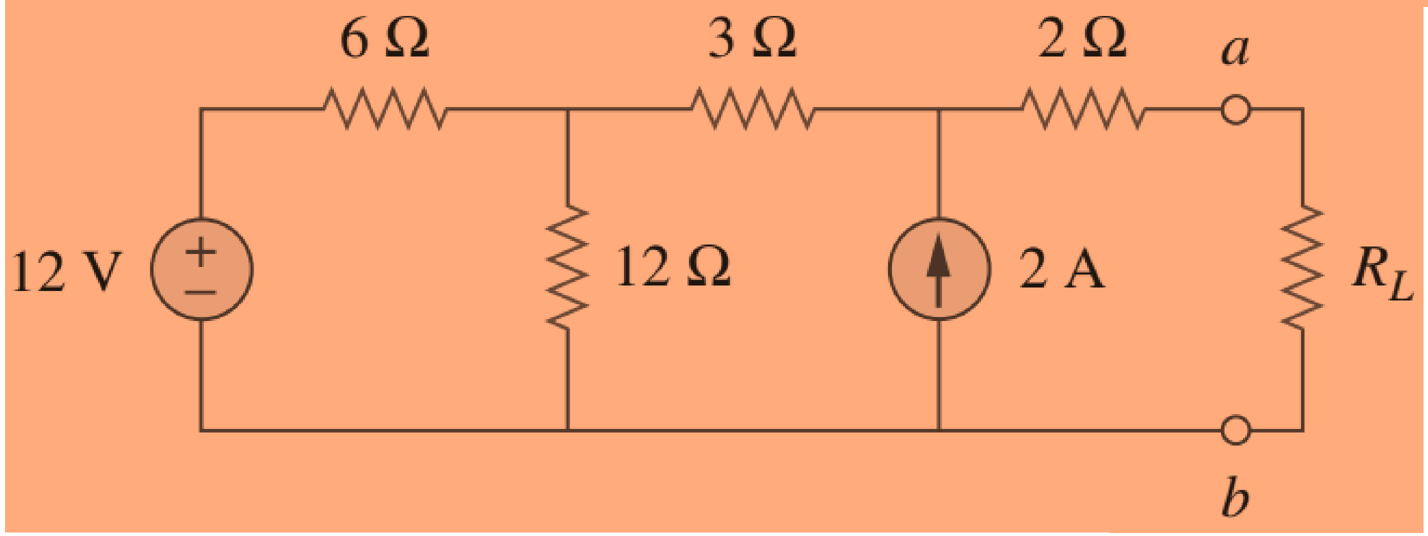
$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

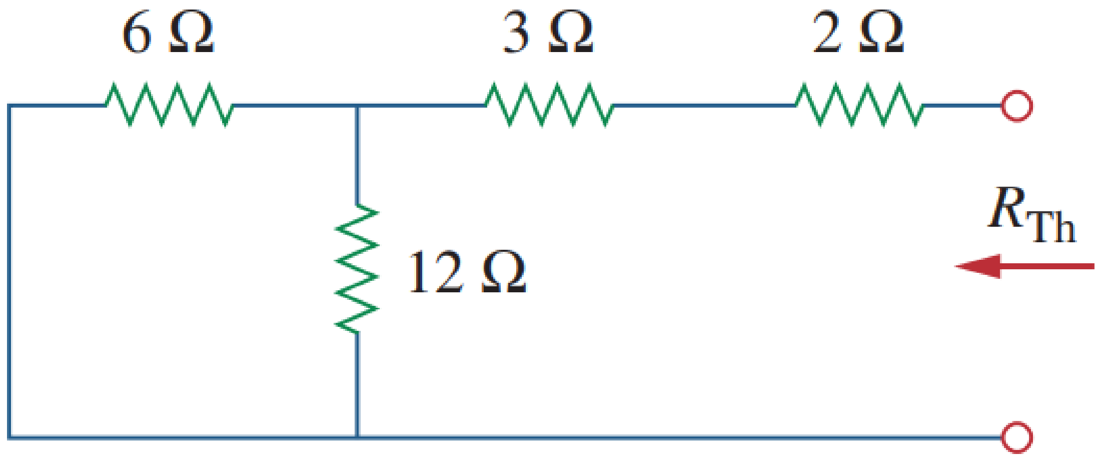
$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

only when $R_L = R_{Th}$.

Example

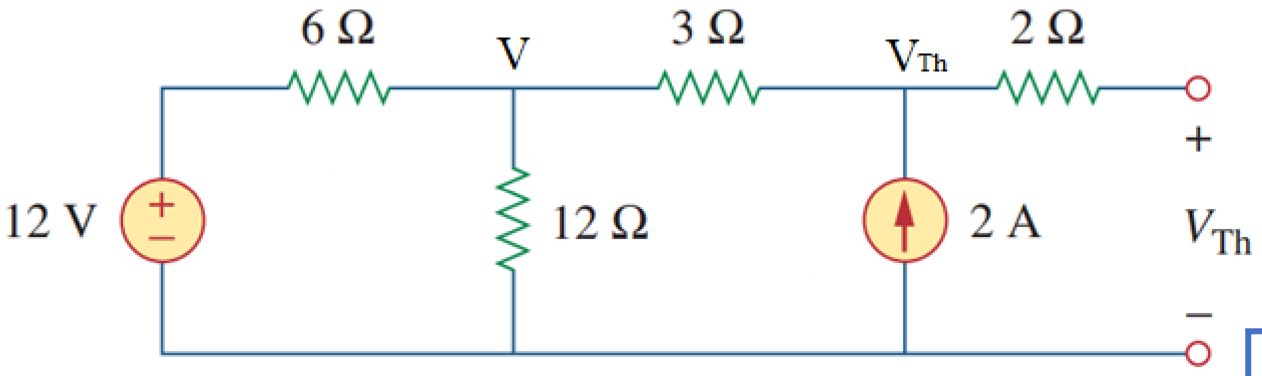
Find the value of R_L for maximum power transfer in the circuit Find the maximum power.





$$R_{Th} = 2 + 3 + 6 \parallel 12$$
$$= 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

continued…



$$\frac{V-12}{6}+\frac{V-0}{12}+\frac{V-V_{Th}}{3}=0$$

$$\Rightarrow 7V-4V_{Th}=24$$

$$\frac{V_{Th}-V}{3}-2=0$$

$$\Rightarrow -V+V_{Th}=6$$

$$V_{\text{Th}}=22\text{ V}$$

For maximum power transfer,

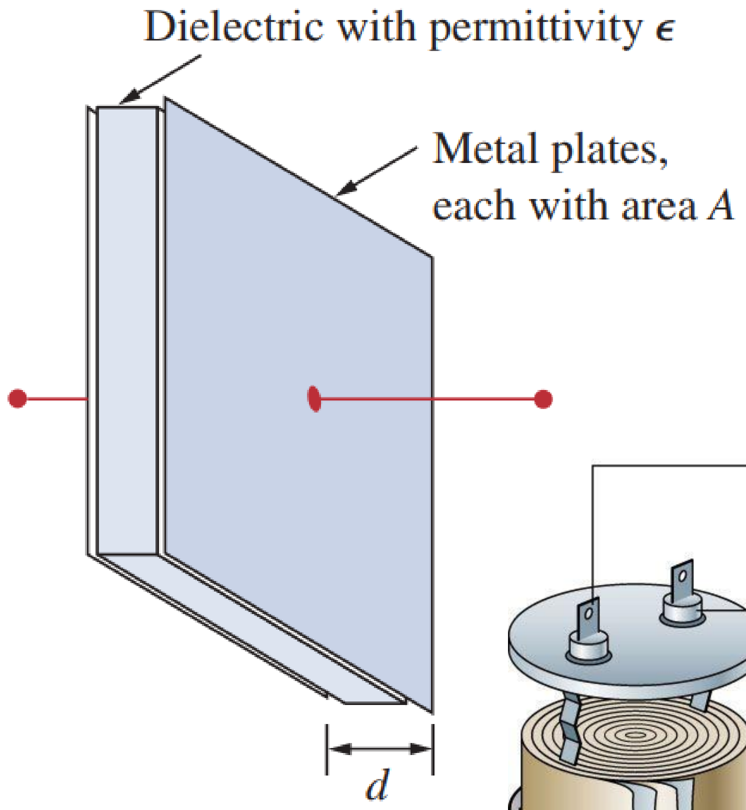
$$R_L=R_{\text{Th}}=9\ \Omega$$

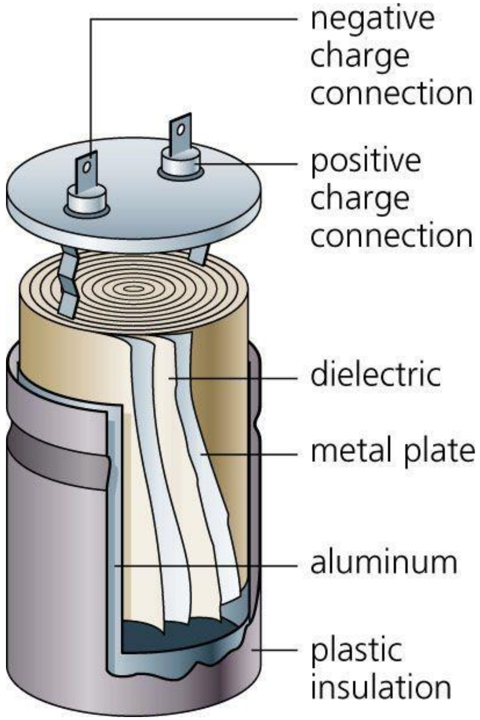
$$p_{\max}=\frac{V_{\text{Th}}^2}{4R_L}=\frac{22^2}{4\times 9}=13.44\text{ W}$$

Electrical Technology (Part 2)

EEE 101

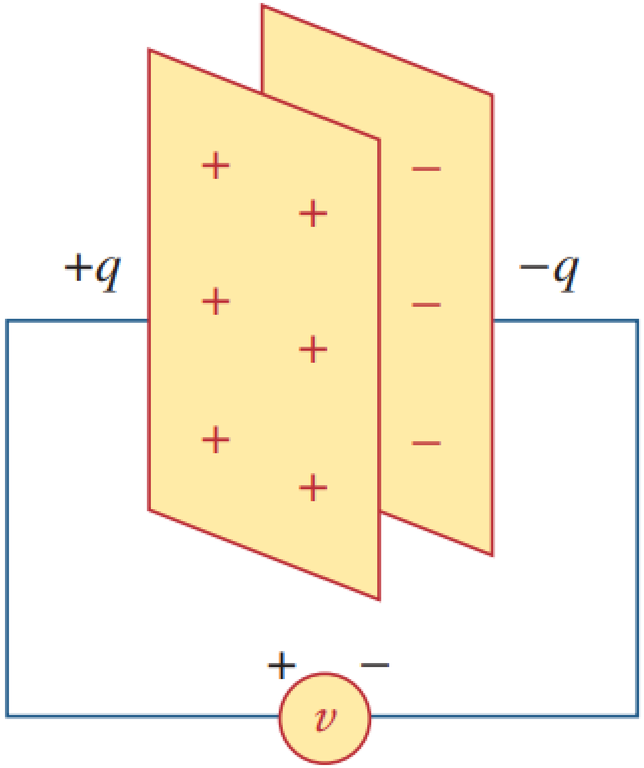
Capacitor


Dielectric with permittivity ϵ
Metal plates,
each with area A
d


negative charge connection
positive charge connection
dielectric
metal plate
aluminum
plastic insulation


nichicon
4700μF 35v
VITAMIN Q
CAPACITOR
22 - 400 D.C.
189-505-5
193-10 8216
103K 500V
151
15
10

continued...

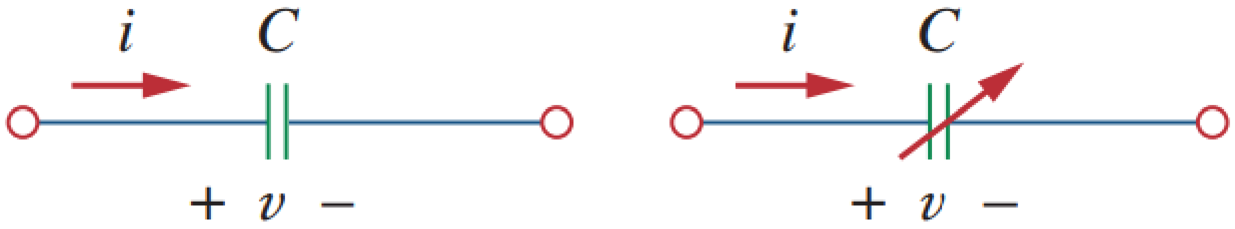


$$q = Cv$$

<u>Unit of Capacitance:</u> Farad (F)

1 Farad = 1 Coulomb / Volt

$$C = \frac{\epsilon A}{d}$$



$$i = \frac{dq}{dt} \rightarrow i = C\frac{dv}{dt}$$

$$w = \frac{1}{2}Cv^2$$

$$w = \frac{q^2}{2C}$$

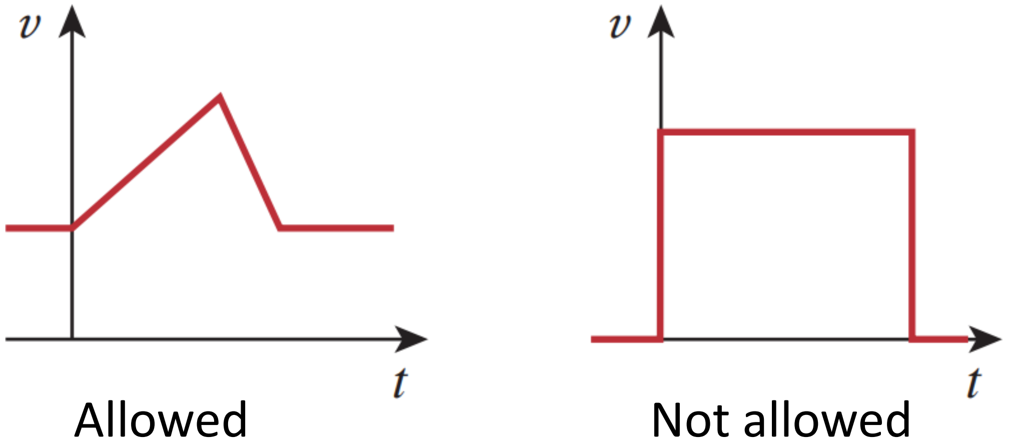
continued…

$$i = C\frac{dv}{dt} \rightarrow v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau \rightarrow v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

- A capacitor is an open-circuit to DC.

If a *battery* (DC voltage) is connected across a capacitor, the capacitor charges.

- The voltage across a capacitor cannot change abruptly.



Example

Calculate the charge stored on a 3-pF capacitor with 20 V across it. Find the energy stored in the capacitor.

$$q = Cv = 3 \times 10^{-12} \times 20 = 60 \text{ pC}$$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ}$$

What is the voltage across a 3-μF capacitor if the charge on one plate is 0.12 mC? How much energy is stored?

$$\text{v} = \frac{\text{q}}{\text{C}} = \frac{120\text{x}10^{-6}}{3\text{x}10^{-6}} = 40 \text{ V}$$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}\text{x}3\text{x}10^{-6}\text{x}1600 = 2.4 \text{ mJ}$$

continued…

The voltage across a 5-μF capacitor is

$$v(t) = 10 \cos 6000t \text{ V}$$

Calculate the current through it.

$$i(t) = C\frac{dv}{dt} = 5 \times 10^{-6}\frac{d}{dt}(10 \cos 6000t)$$
$$= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t$$
$$= -0.3 \sin 6000t \text{ A}$$

continued…

Determine the voltage across a 2-μF capacitor if the current through it is

$$i(t) = 6e^{-3000t} \text{ mA}$$

Assume that the initial capacitor voltage is zero.

$$v = \frac{1}{C}\int_0^t i\,dt + v(0) = \frac{1}{C}\int_0^t i\,dt$$

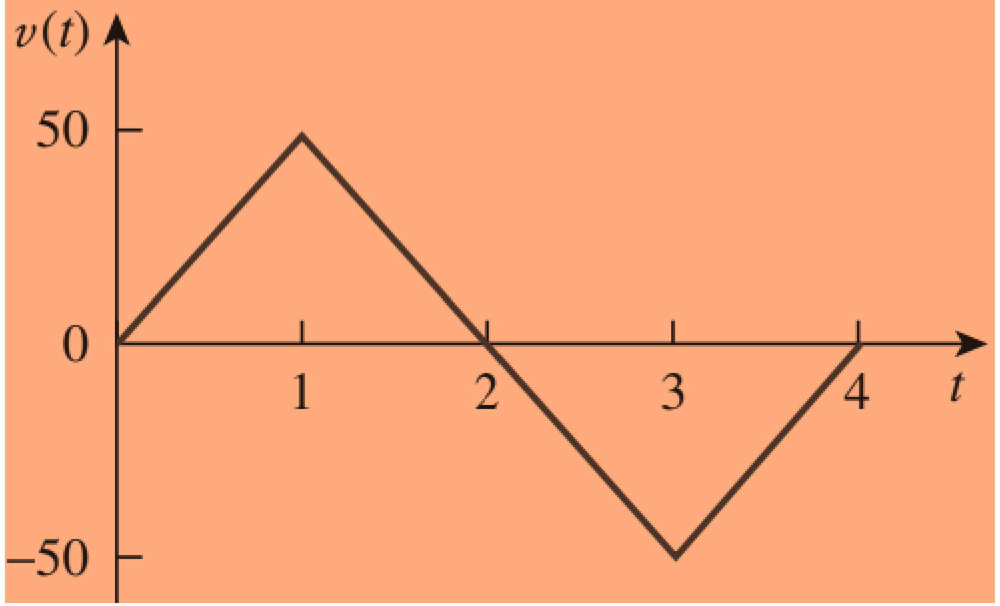
$$= \frac{1}{2 \times 10^{-6}}\int_0^t 6e^{-3000t}\,dt \cdot 10^{-3}$$

$$= \frac{3 \times 10^3}{-3000} e^{-3000t}\Big|_0^t$$

$$= (1 - e^{-3000t}) \text{ V}$$

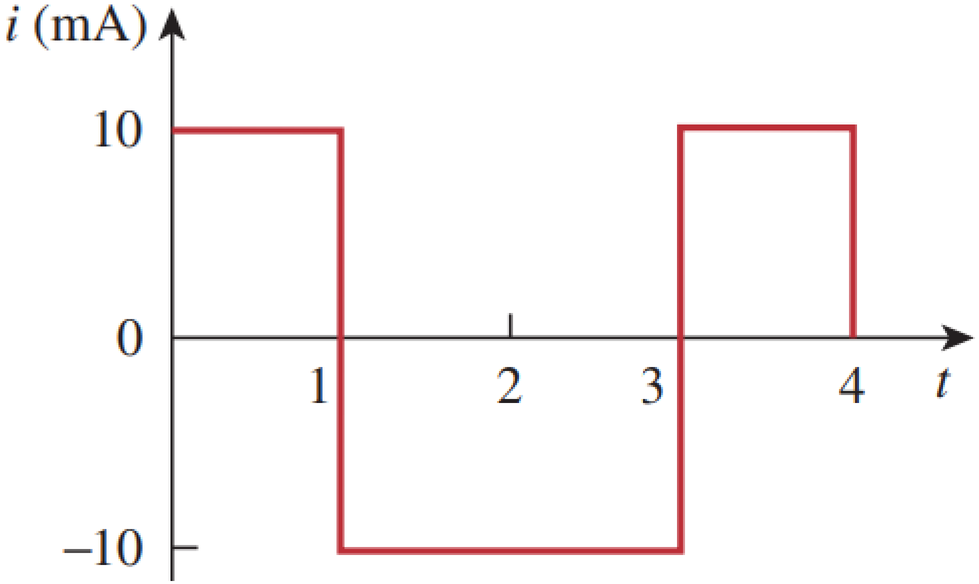
continued…

Determine the current through a 200-μF capacitor whose voltage is



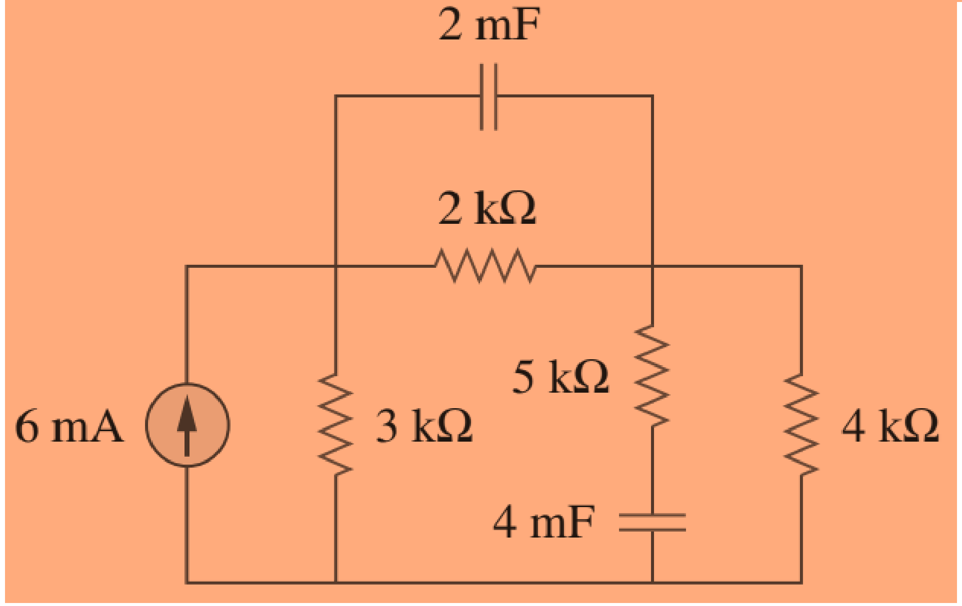
$$i = C\frac{dv}{dt}$$

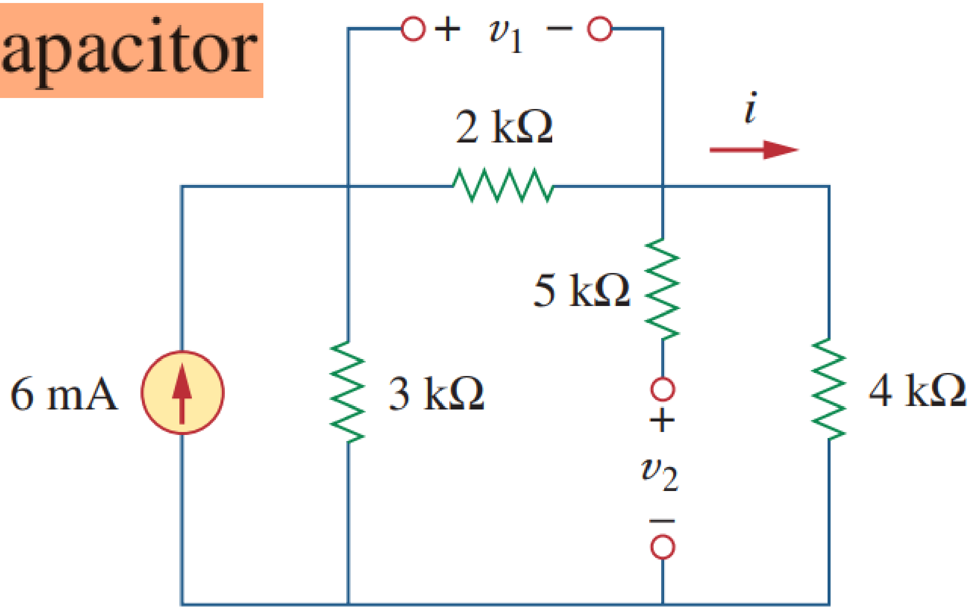
$$= 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$



continued...

Obtain the energy stored in each capacitor



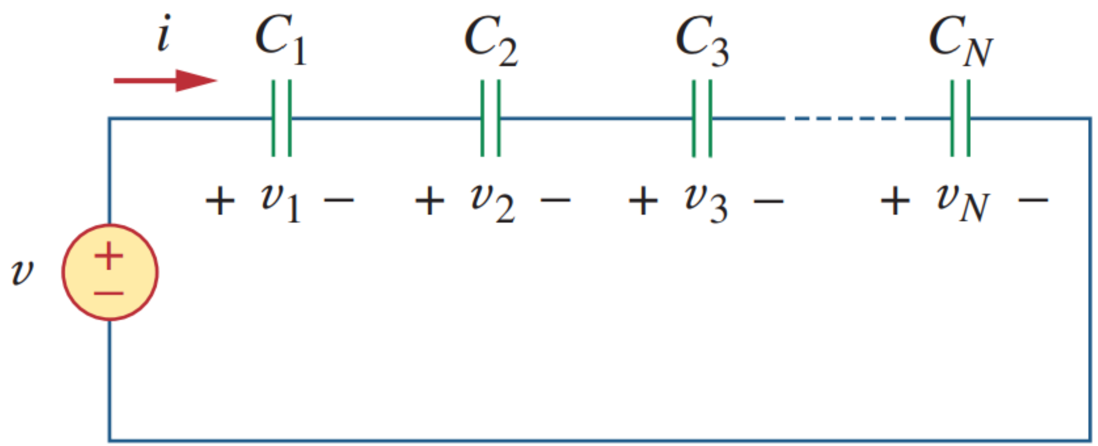


$$i = \frac{3}{3+2+4}(6\text{ mA}) = 2\text{ mA}$$

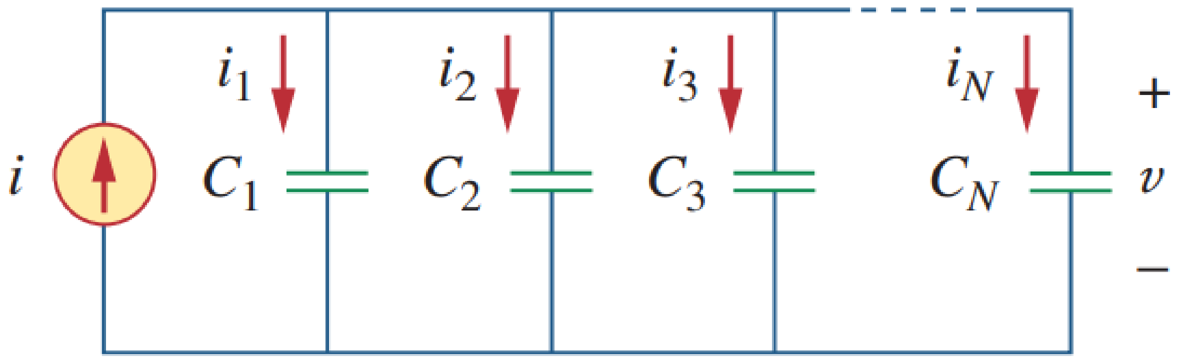
$$v_1 = 2000i = 4\text{ V}$$
$$v_2 = 4000i = 8\text{ V}$$

$$w_1 = \frac{1}{2}C_1v_1^2 = \frac{1}{2}(2\times10^{-3})(4)^2 = 16\text{ mJ}$$
$$w_2 = \frac{1}{2}C_2v_2^2 = \frac{1}{2}(4\times10^{-3})(8)^2 = 128\text{ mJ}$$

Series & Parallel Capacitors



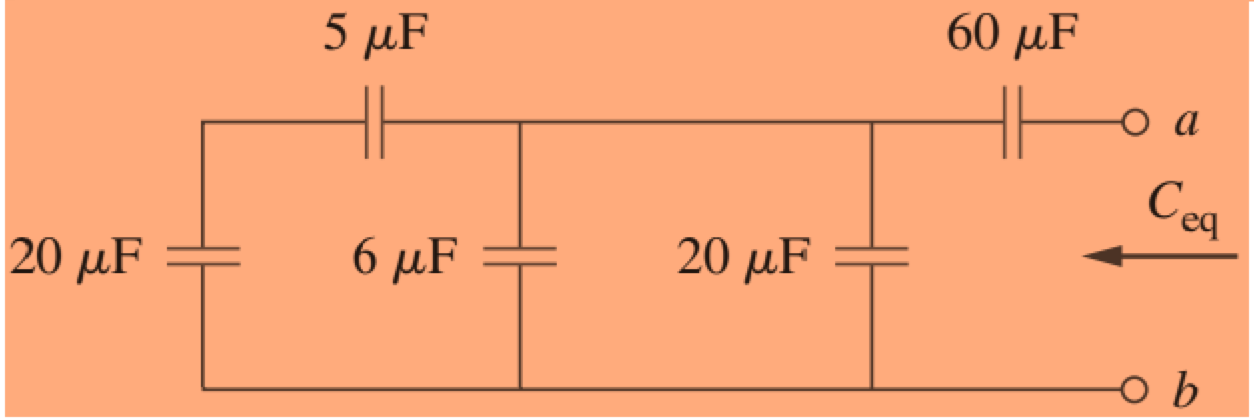
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$



$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$

Example

Find the equivalent capacitance seen between terminals *a* and *b*



$$\frac{20 \times 5}{20 + 5} = 4\ \mu F$$ ← 20-μF and 5-μF capacitors are in series

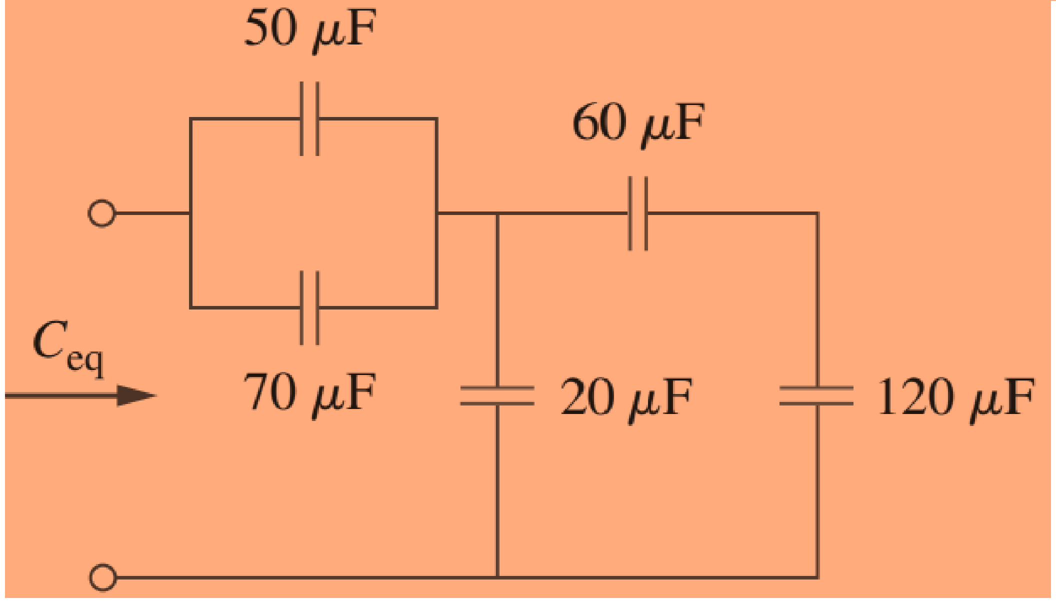
→ is in parallel with the 6-μF and 20-μF capacitors

$$4 + 6 + 20 = 30\ \mu F$$

$$C_{eq} = \frac{30 \times 60}{30 + 60} = 20\ \mu F$$

continued...

Find the equivalent capacitance seen at the terminals



60 and 120μF in series = $\dfrac{60 \times 120}{180}$ = 40μF

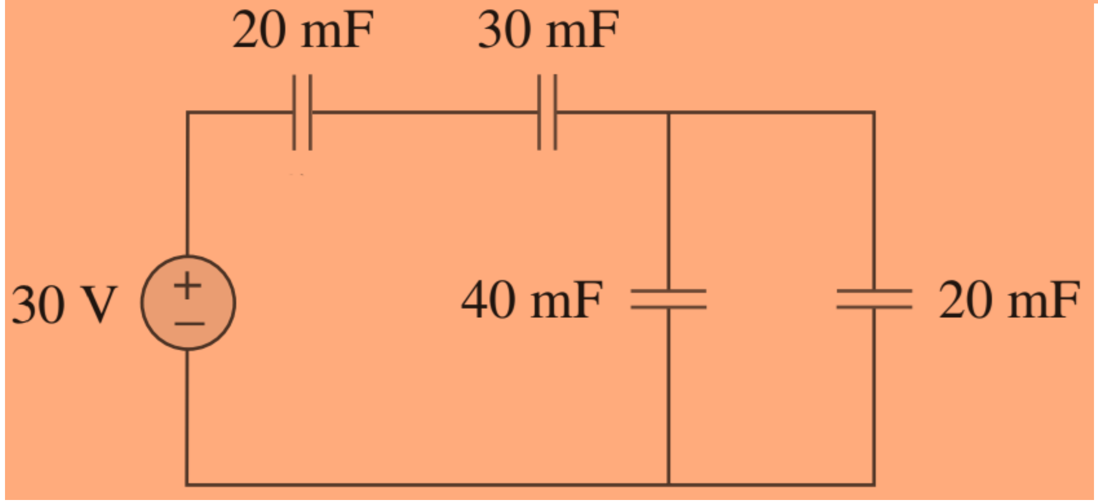
40μF in parallel with 20μF = 40 + 20 = 60μF

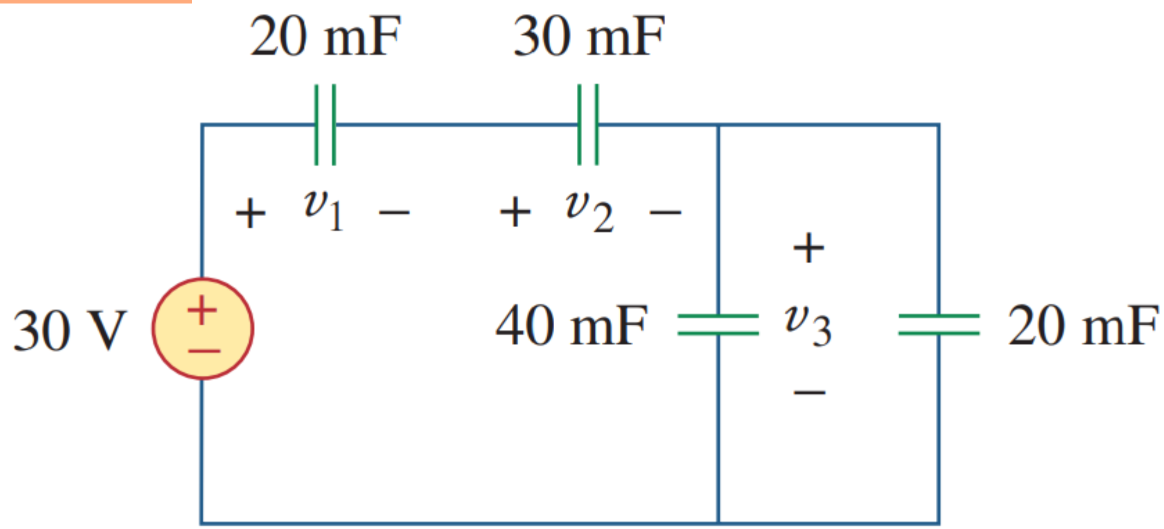
50μF in parallel with 70μF = 50 + 70 = 120μF

60μF in series with 120μF = $\dfrac{60 \times 120}{180}$ = 40 μF

continued...

find the voltage across each capacitor.



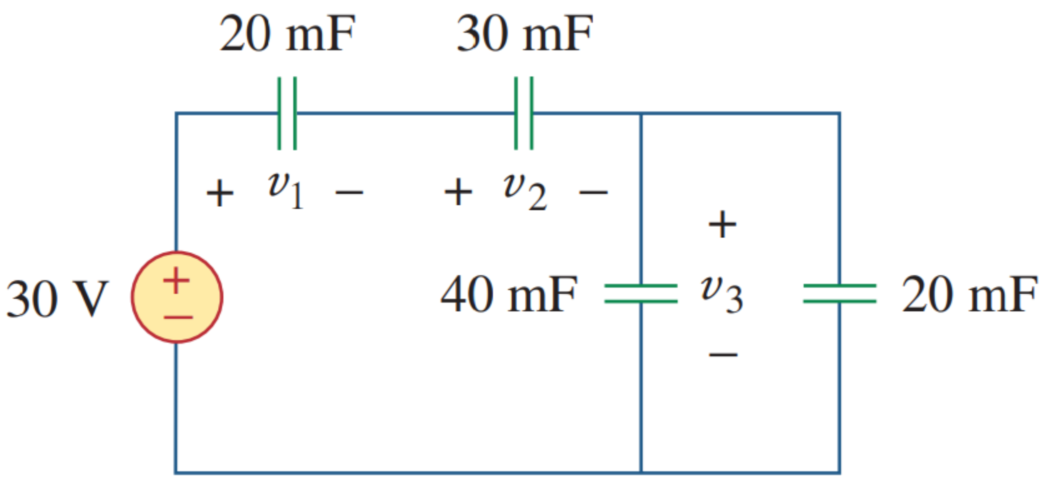


$$40 + 20 = 60 \text{ mF}$$

$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}} \text{ mF} = 10 \text{ mF}$$

The total charge $q = C_{eq} v = 10 \times 10^{-3} \times 30 = 0.3 \text{ C}$

continued…



$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15 \text{ V}$$

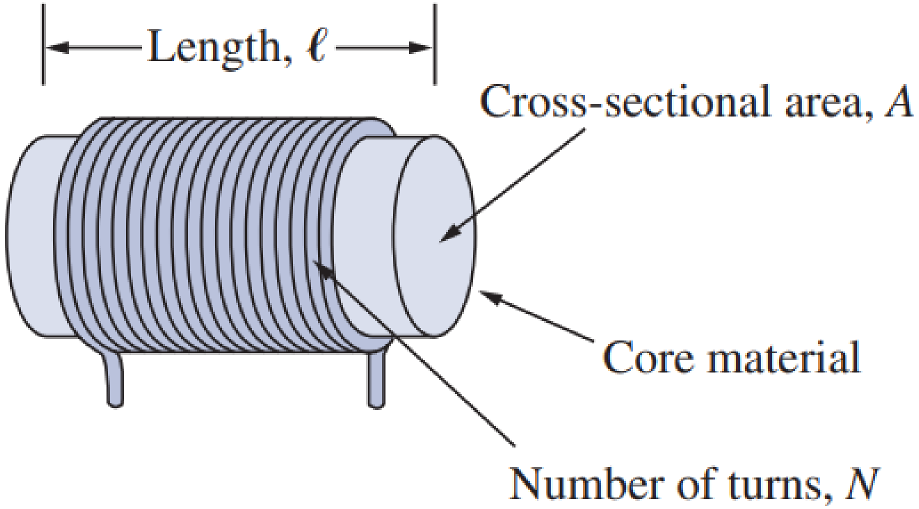
$$v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10 \text{ V}$$

$$v_3 = 30 - v_1 - v_2 = 5 \text{ V}$$

Alternatively,

$$v_3 = \frac{q}{60 \text{ mF}} = \frac{0.3}{60 \times 10^{-3}} = 5 \text{ V}$$

Inductor

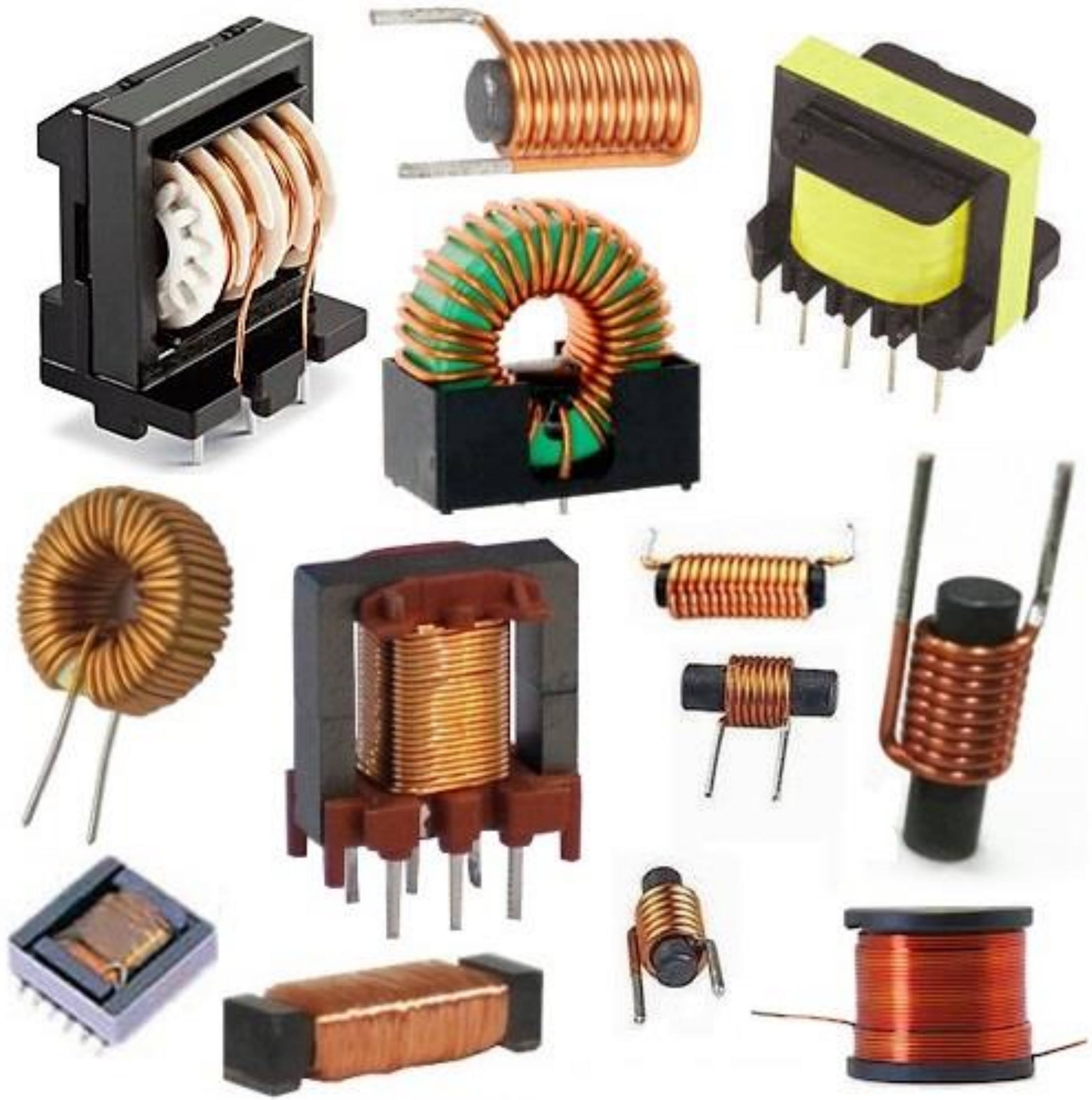


$$v = L\frac{di}{dt}$$

L = Inductance

Unit of inductance: Henry (H)

1 Henry = 1 Volt-second per Ampere

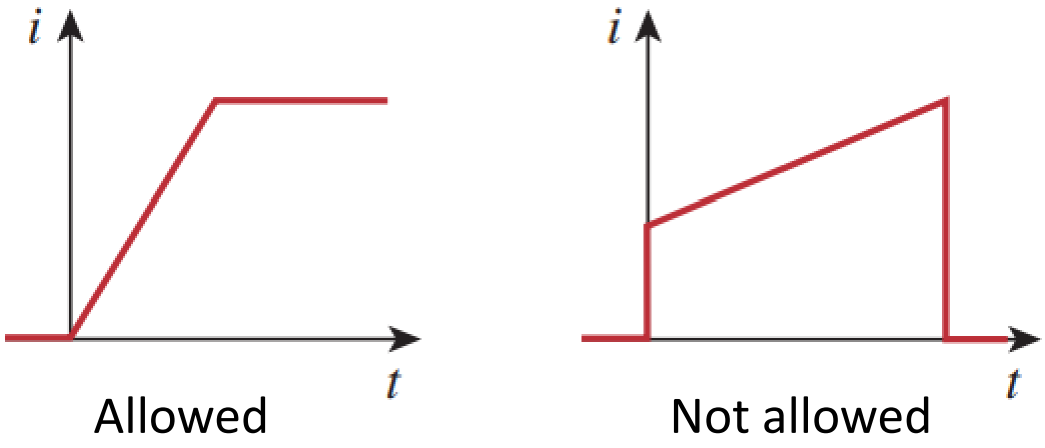


continued…

$$v = L\frac{di}{dt} \rightarrow di = \frac{1}{L} v\, dt \rightarrow i = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\, d\tau \rightarrow i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\, d\tau + i(t_0)$$

$$\boxed{w = \frac{1}{2} L i^2}$$

- An inductor acts like a short-circuit to DC.
- The current through an inductor cannot change instantaneously.



Example

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

$$v = L\frac{di}{dt} = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t}$$
$$= e^{-5t}(1 - 5t) \text{ V}$$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t} \text{ J}$$

Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also, find the energy stored at $t = 5$ s.

$$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$$
$$= \frac{1}{5}\int_0^t 30t^2\,dt + 0 = 6 \times \frac{t^3}{3} = 2t^3 \text{ A}$$

continued…

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(5)(2 \times 5^3)^2 = 156.25 \text{ kJ}$$

The terminal voltage of a 2-H inductor is $v = 10(1 - t)$ V. Find the current flowing through it at $t = 4$ s and the energy stored in it at $t = 4$ s. Assume $i(0) = 2$ A.

$$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0) = \frac{1}{2}\int_0^t 10(1-t)dt + 2 = 5\left(t - \frac{t^2}{2}\right) + 2$$

At t = 4, i = 5(4 - 8) + 2 = **–18A**

$$w = \frac{1}{2}Li^2(4)$$

$$= \frac{1}{2} \cdot 2 \cdot (-18)^2$$

$$= 324 \text{ J}$$

If the question was: Find the energy stored between t=0 and t=4 s.

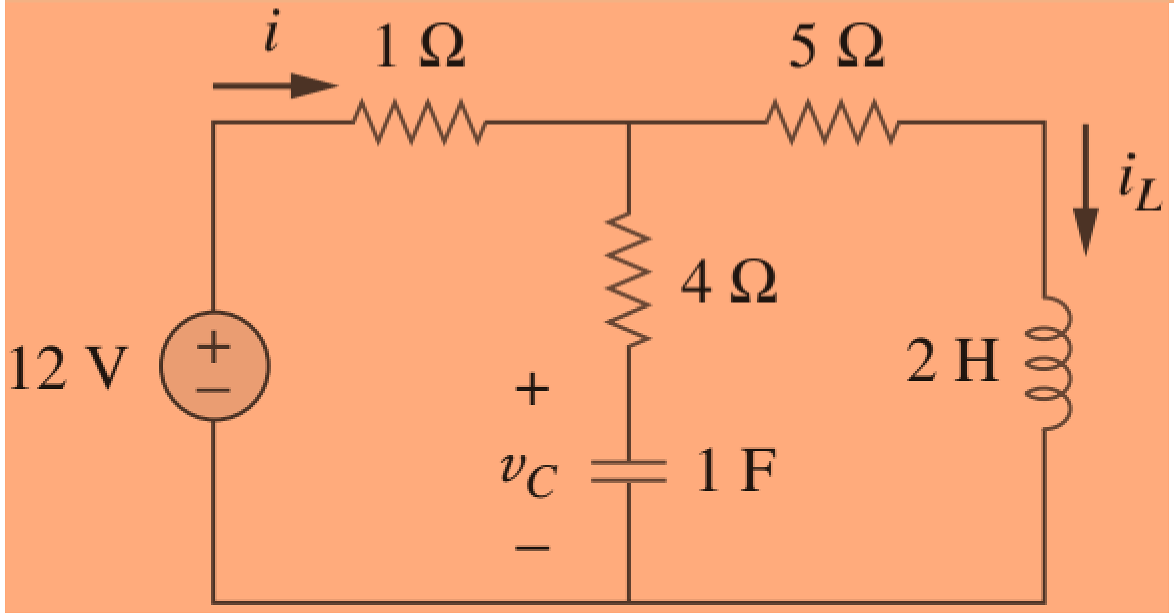
$$w = \frac{1}{2}Li^2(4) - \frac{1}{2}Li^2(0)$$

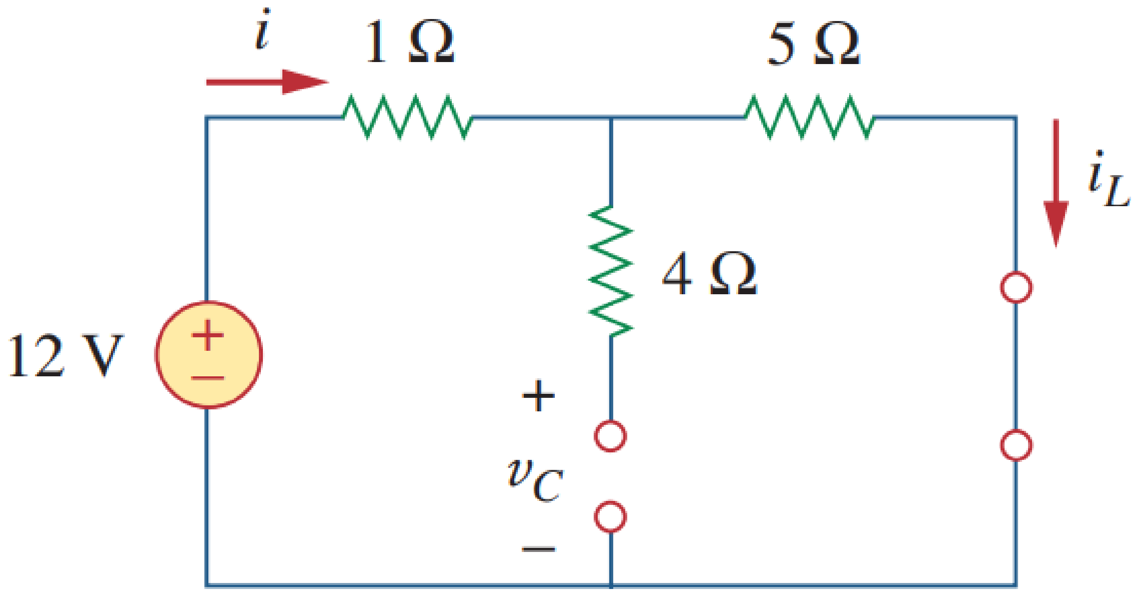
$$= \frac{1}{2} \cdot 2 \cdot (-18)^2 - \frac{1}{2} \cdot 2 \cdot (2)^2$$

$$= 324 - 4 = 320 \text{ J}$$

continued…

Find i, v_C, i_L, and the energy stored in the capacitor and inductor.

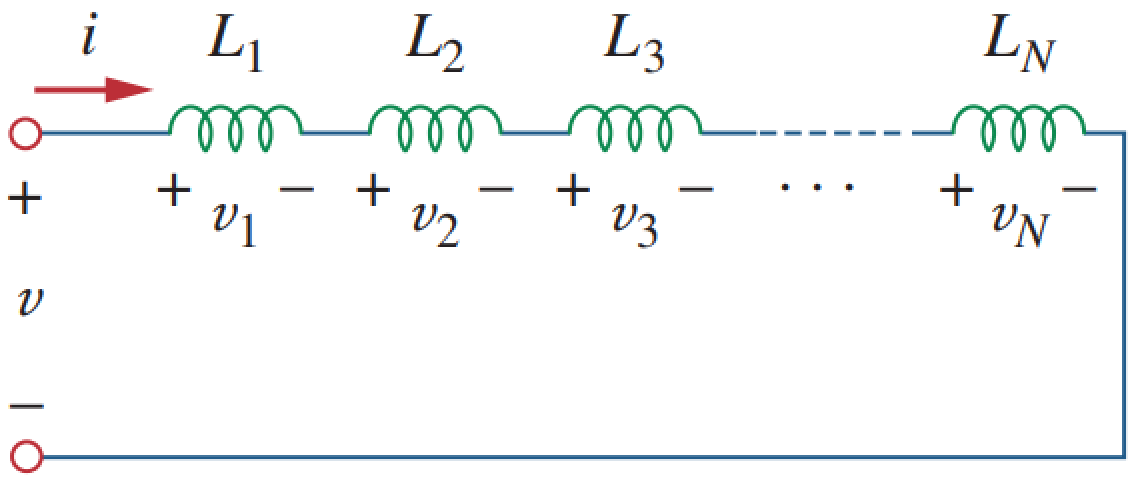




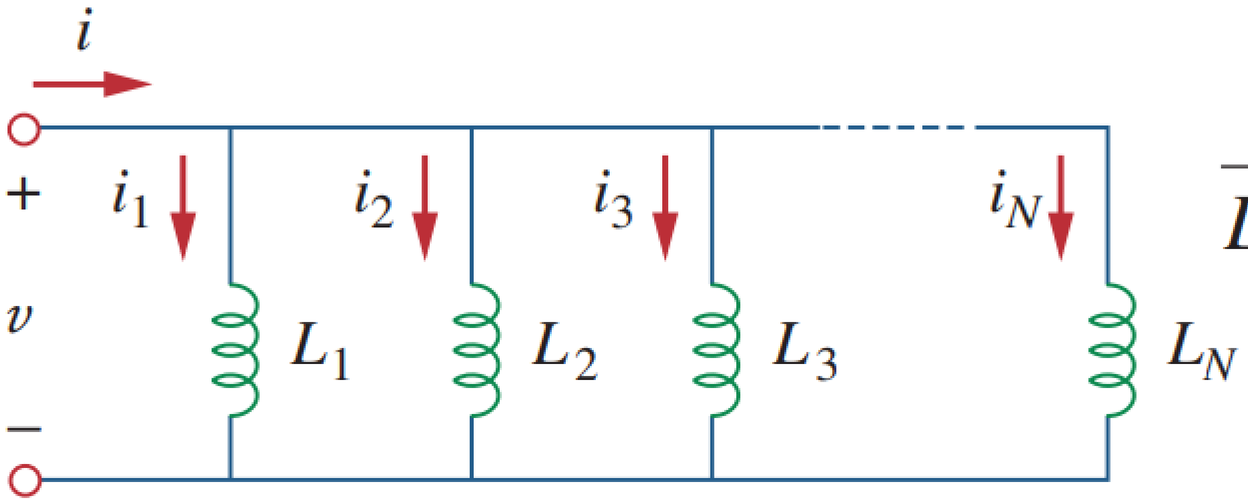
$$i = i_L = \frac{12}{1+5} = 2 \text{ A} \qquad v_C = 5i = 10 \text{ V}$$

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 \text{ J} \qquad w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4 \text{ J}$$

Series and Parallel Inductors



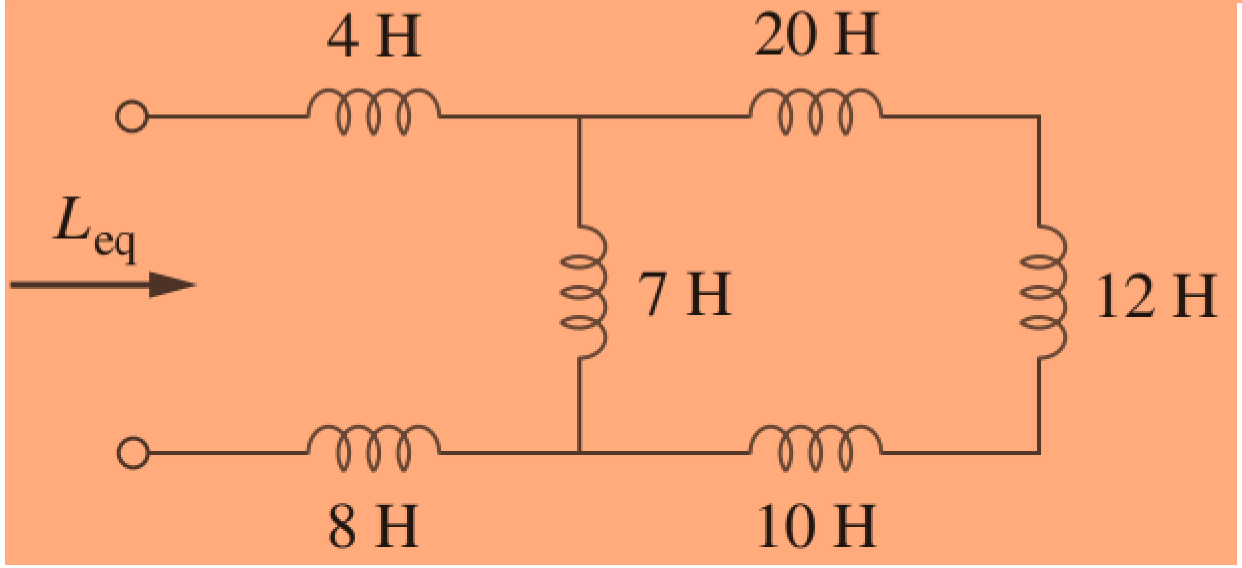
$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

Example

Find the equivalent inductance of the circuit shown



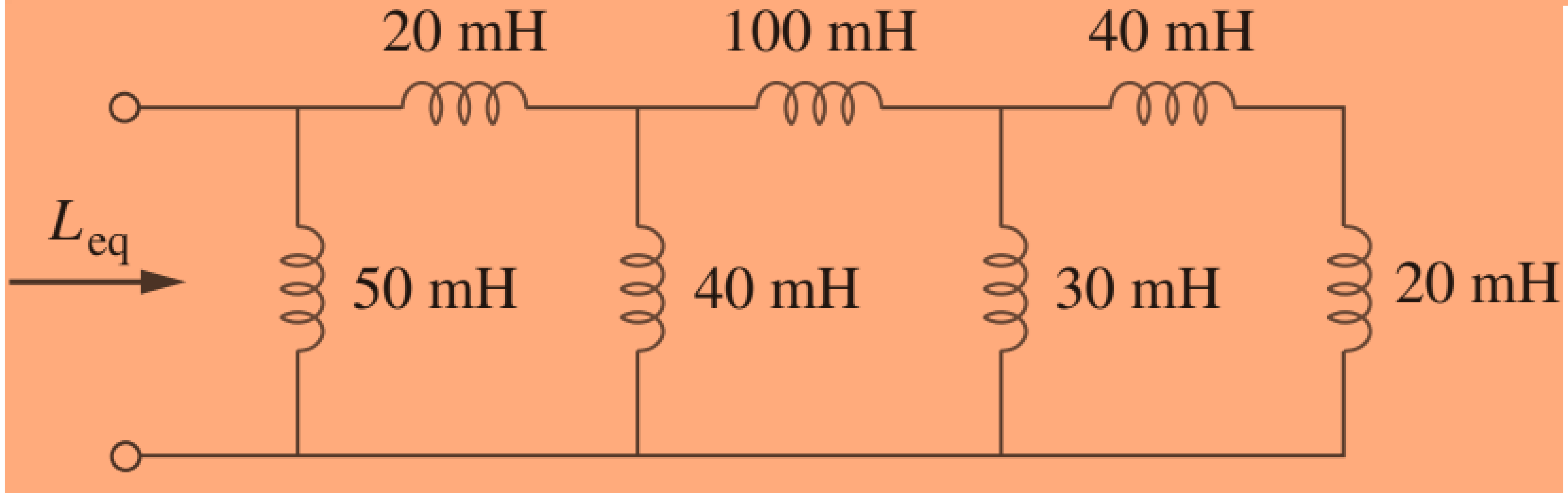
10-H, 12-H, and 20-H inductors are in series → 42-H

$$\frac{7 \times 42}{7 + 42} = 6 \text{ H}$$ → This 6-H inductor is in series with the 4-H and 8-H

$$L_{eq} = 4 + 6 + 8 = 18 \text{ H}$$

continued…

Find the equivalent inductance of the circuit shown



40mH in series with 20mH → 40 + 20 = 60mH

60mH in parallel with 30mH → 30 x 60/(90) = 20mH

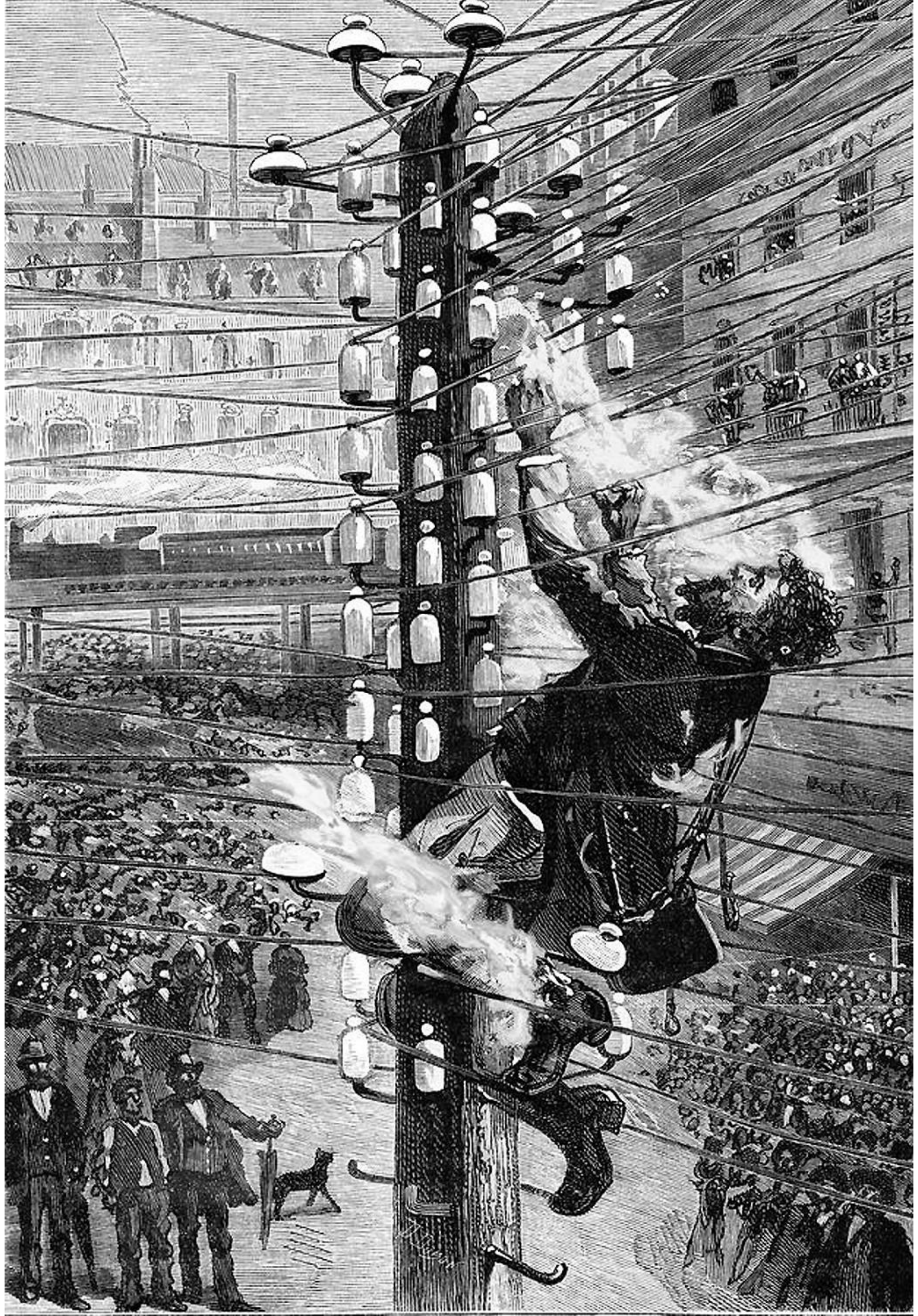
20mH in series with 100mH → 120mH

120mH in parallel with 40mH → 40 x 120/(160) = 30mH

30mH in series with 20mH → 50mH

50mH in parallel with 50mH → 25mH ← L_{eq}

1889 engraving by William Allen Rogers for Harper's Weekly magazine. GRAND STREET, NEW YORK, AT NIGHT



October 11, 1889. The death of John Feeks. Manhattan, New York.

World's Columbian Exposition, 1893, Chicago

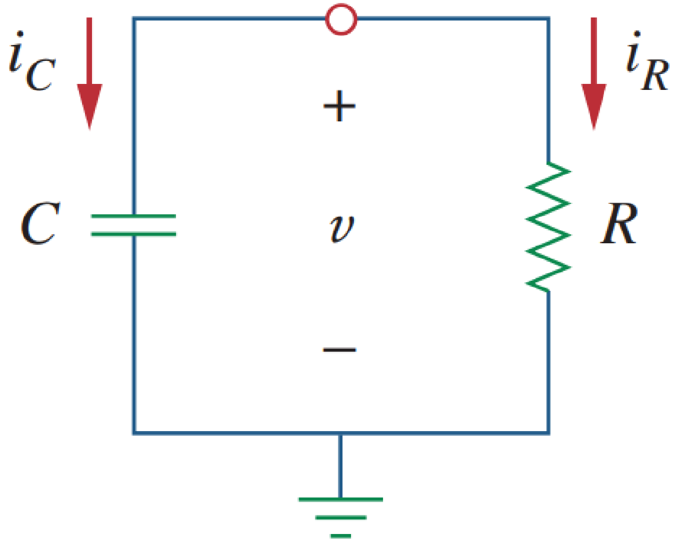
1st-order Circuits

RC and RL circuits: characterized by a 1st-order differential equation.

2 ways of excitation:

- Energy is initially stored in capacitor or inductor; source-free circuit.
- Circuit is excited by independent sources.

Source-Free RC Circuit



at time $t = 0$, the initial voltage is

$$v(0) = V_0$$

$$i_C + i_R = 0 \rightarrow C\frac{dv}{dt} + \frac{v}{R} = 0 \rightarrow \frac{dv}{dt} + \frac{v}{RC} = 0$$

$$\frac{dv}{v} = -\frac{1}{RC}dt \rightarrow \ln v = -\frac{t}{RC} + \ln A$$

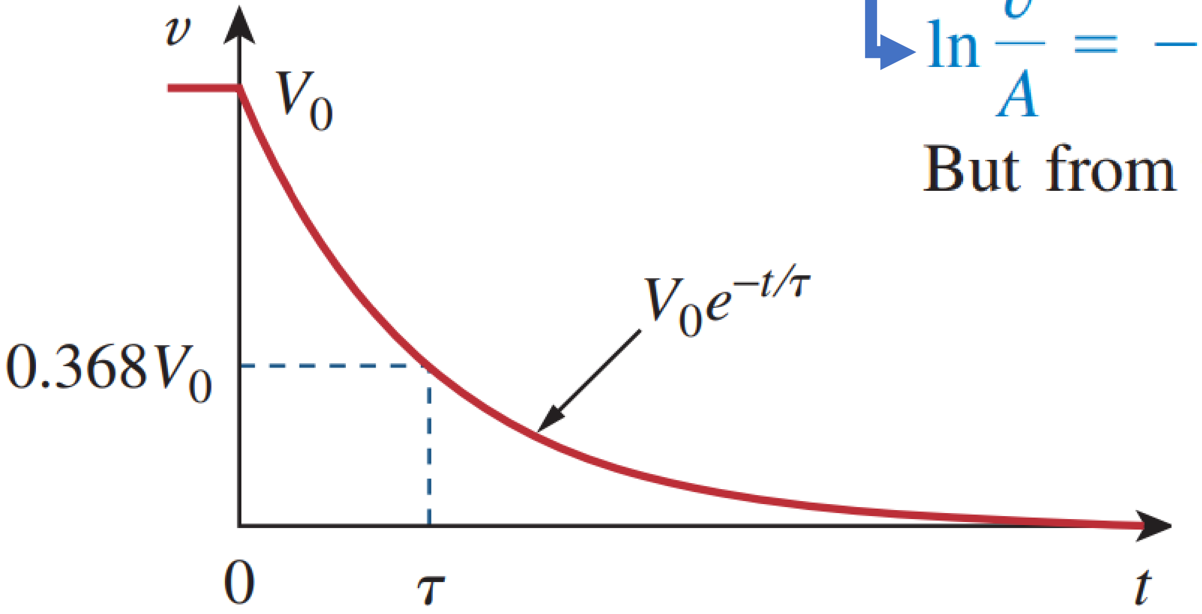
$$\ln\frac{v}{A} = -\frac{t}{RC} \rightarrow v(t) = Ae^{-t/RC}$$

But from the initial conditions, $v(0) = A = V_0$

$$v(t) = V_0 e^{-t/RC}$$

$$\tau = RC$$

$$v(t) = V_0 e^{-t/\tau}$$



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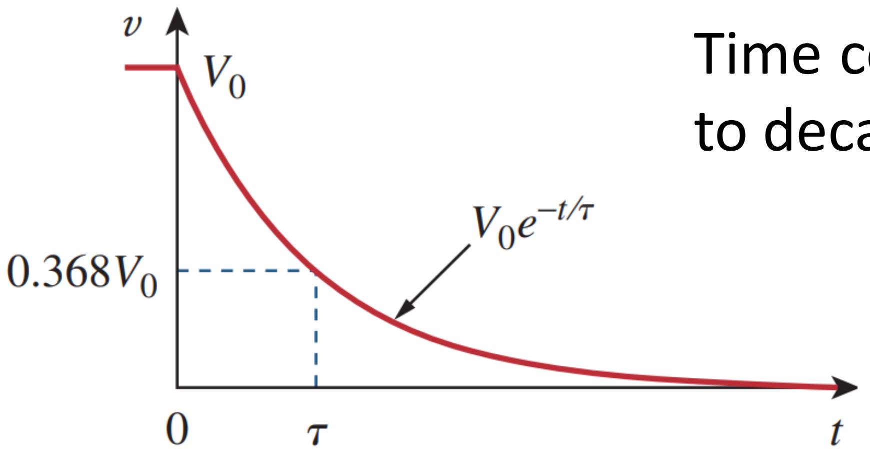
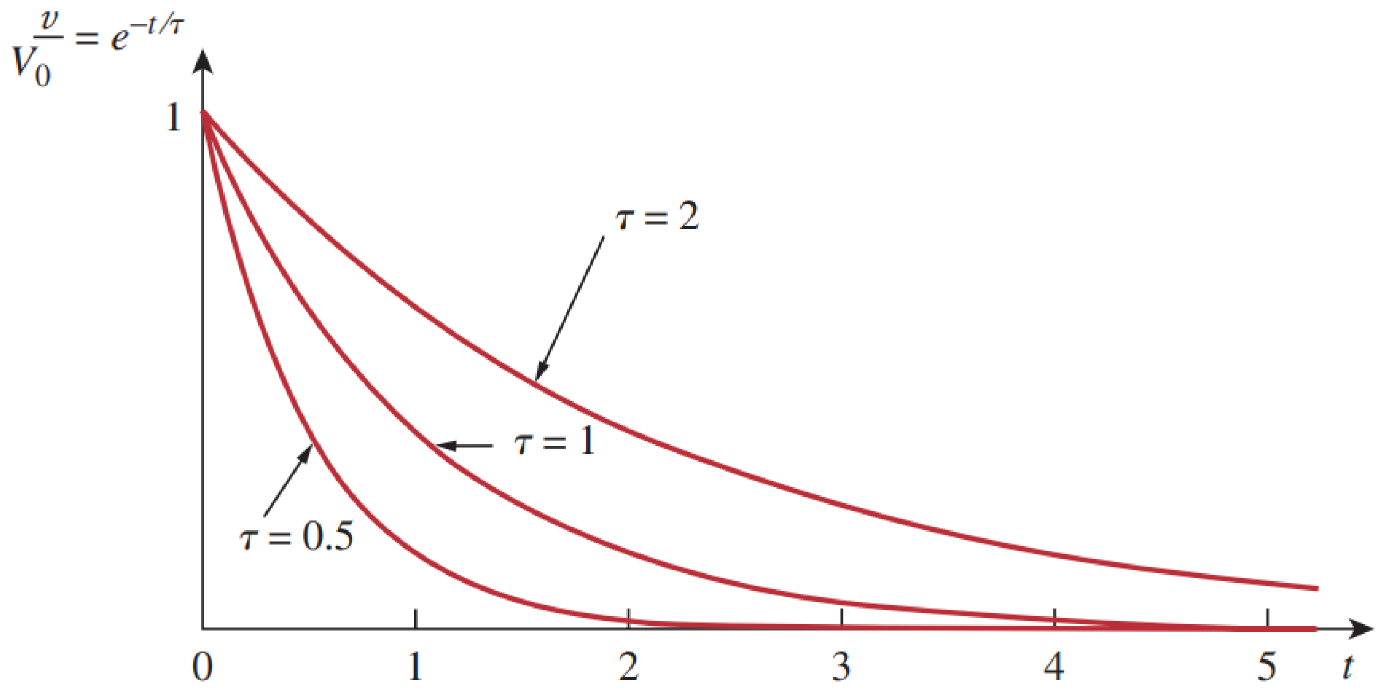


Time constant (τ) of a circuit is the time required for the response to decay to a factor of 1/e or 36.8 percent of its initial value.

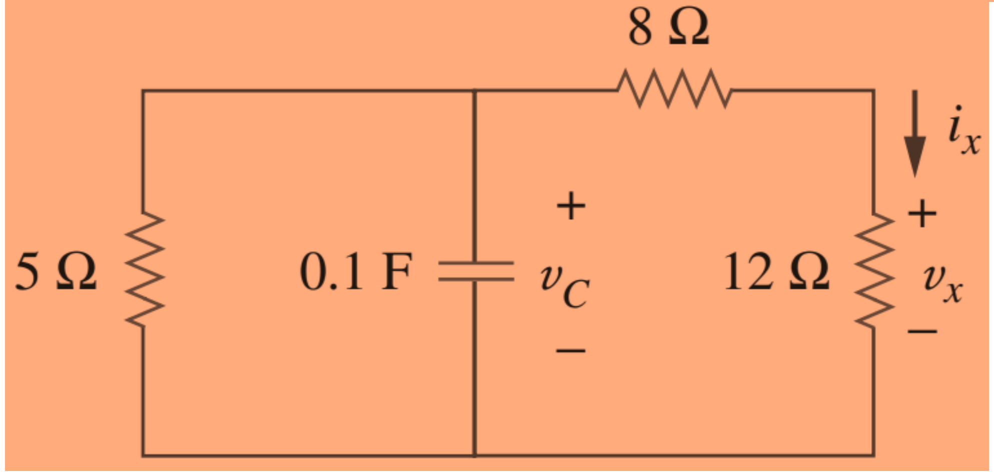
t	$v(t)/V_0$
τ	0.36788
2τ	0.13534
3τ	0.04979
4τ	0.01832
5τ	0.00674



The **natural response** depends on the nature of the circuit alone, with no external sources. In fact, the circuit has a response only because of the energy initially stored in the capacitor.

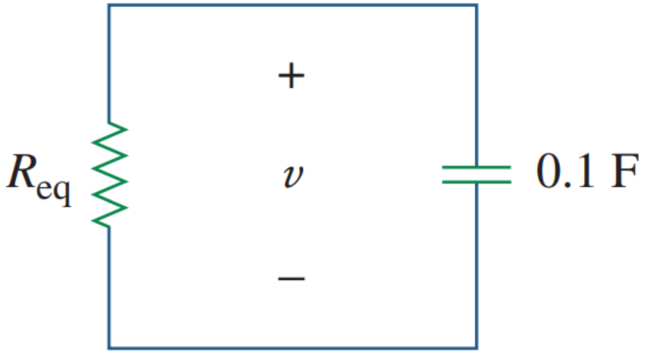
Example

$v_C(0) = 15$ V. Find v_C, v_x, and i_x for $t > 0$.



We have to find Thevenin resistance at the capacitor terminals.

$$R_{eq} = \frac{20 \times 5}{20 + 5} = 4\ \Omega$$



$$\tau = R_{eq}C = 4(0.1) = 0.4\text{ s}$$

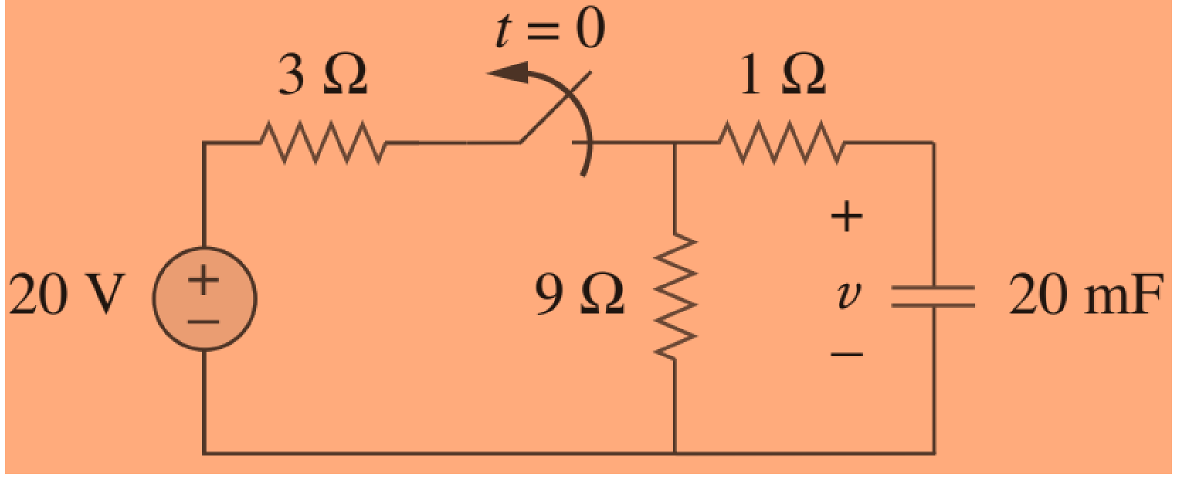
$$v_C = v = v(0)e^{-t/\tau} = 15e^{-t/0.4}\text{ V}$$
$$= 15e^{-2.5t}\text{ V}$$

$$v_x = \frac{12}{12 + 8}v = 0.6(15e^{-2.5t}) = 9e^{-2.5t}\text{ V}$$

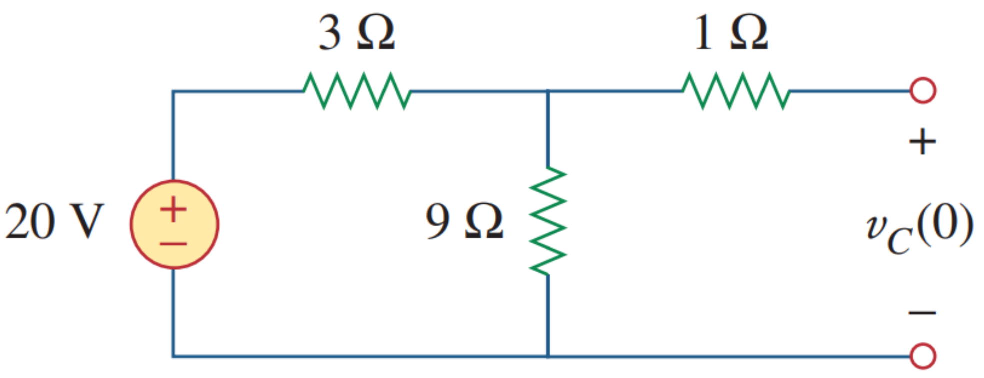
$$i_x = \frac{v_x}{12} = 0.75e^{-2.5t}\text{ A}$$

continued...

The switch in the circuit has been closed for a long time, and it is opened at $t = 0$. Find $v(t)$ for $t \geq 0$. Calculate the initial energy stored in the capacitor.



For $t < 0$, the switch is closed



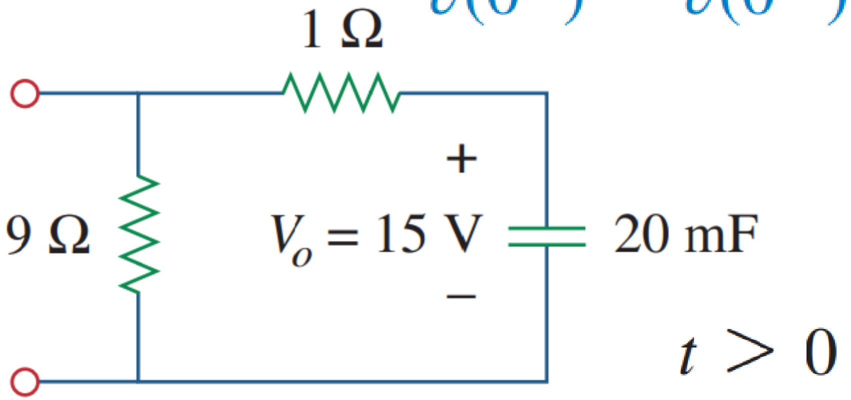
$$v_C(0) = V_0 = 15 \text{ V} \longleftarrow v_C(t) = \frac{9}{9+3}(20) = 15 \text{ V}, \qquad t < 0$$

$$v(0^-) = v(0^+) = V_0$$

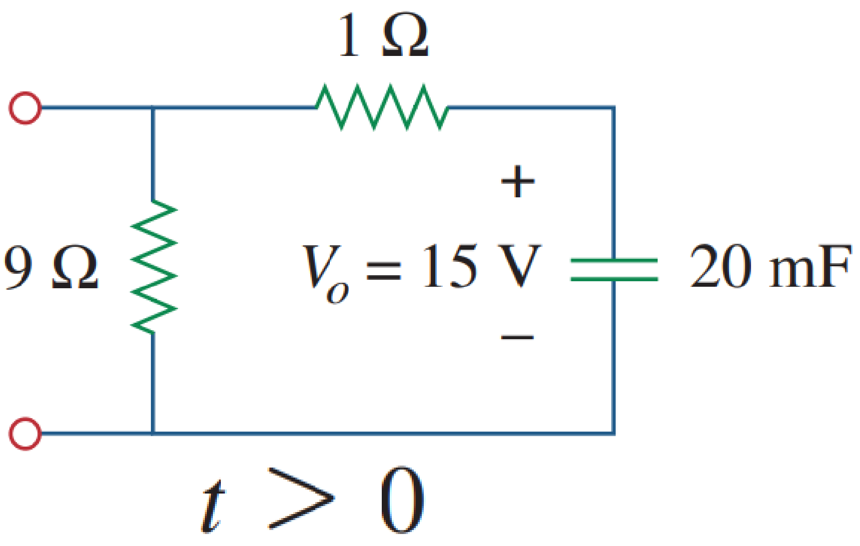
For $t > 0$, the switch is opened

We have to find Thevenin resistance at the capacitor terminals.

$$R_{eq} = 1 + 9 = 10\ \Omega$$



continued...



$$\tau = R_{eq}C = 10 \times 20 \times 10^{-3} = 0.2\ \text{s}$$

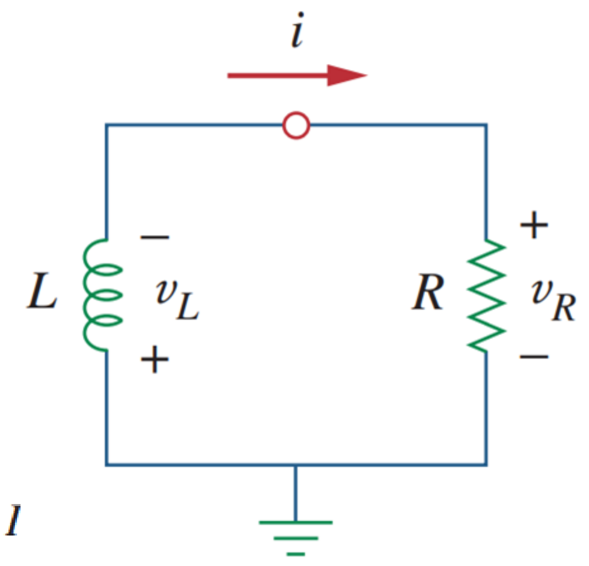
voltage across the capacitor for $t \geq 0$

$$v(t) = v_C(0)e^{-t/\tau} = 15e^{-t/0.2}\ \text{V} = 15e^{-5t}\ \text{V}$$

initial energy stored in the capacitor

$$w_C(0) = \frac{1}{2}Cv_C^2(0) = \frac{1}{2} \times 20 \times 10^{-3} \times 15^2 = 2.25\ \text{J}$$

Source-Free RL Circuit



inductor has an initial current

$$i(0) = I_0$$

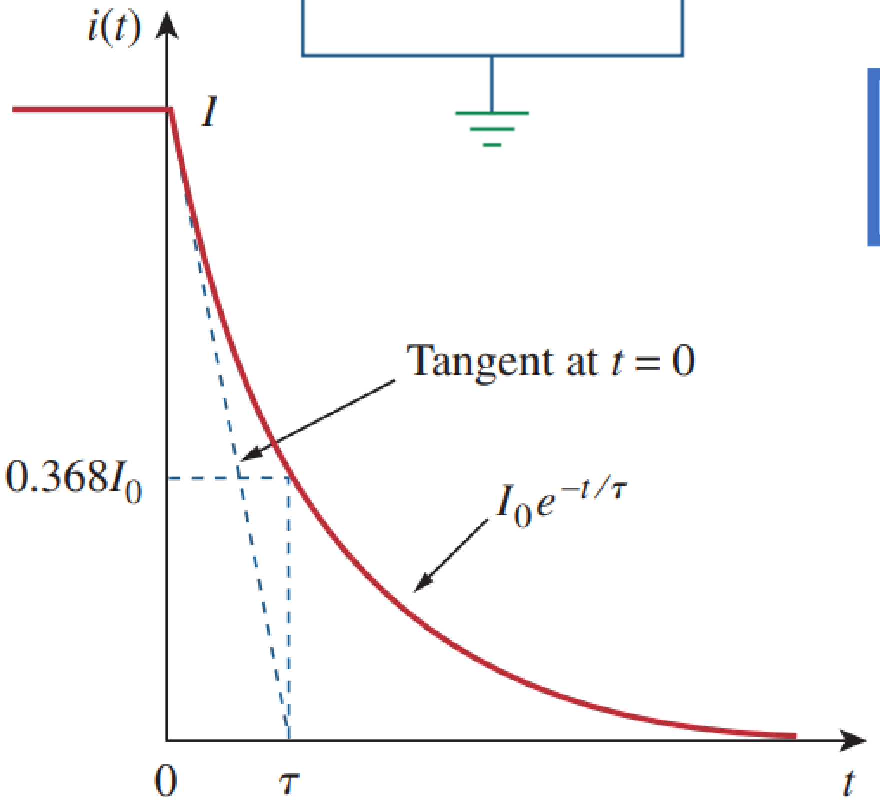
$$v_L + v_R = 0 \rightarrow L\frac{di}{dt} + Ri = 0 \rightarrow \frac{di}{dt} + \frac{R}{L}i = 0$$

$$\int_{I_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{R}{L}dt \rightarrow \ln i(t) - \ln I_0 = -\frac{Rt}{L}$$

$$\ln \frac{i(t)}{I_0} = -\frac{Rt}{L} \rightarrow i(t) = I_0 e^{-Rt/L}$$

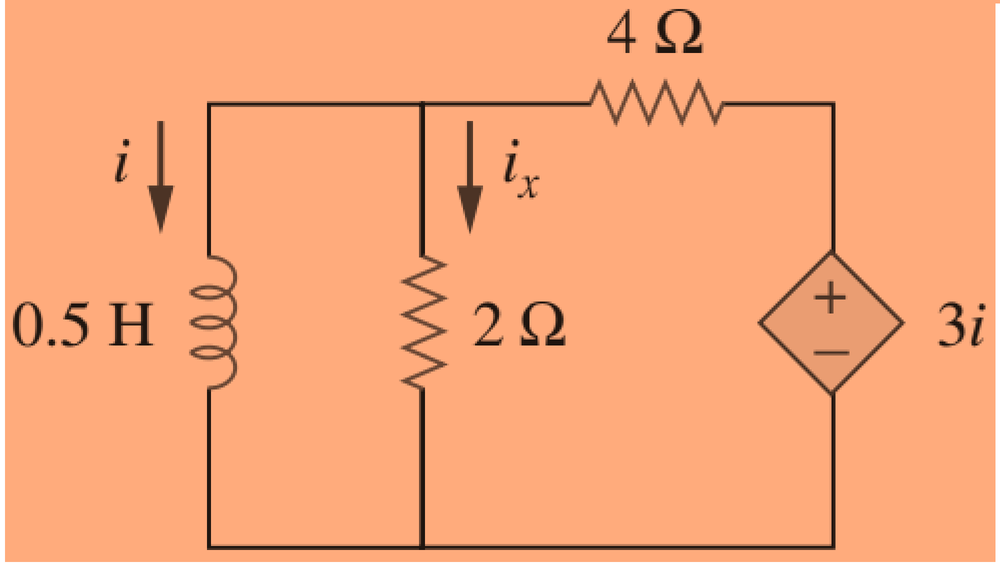
$$\tau = \frac{L}{R}$$

$$i(t) = I_0 e^{-t/\tau}$$



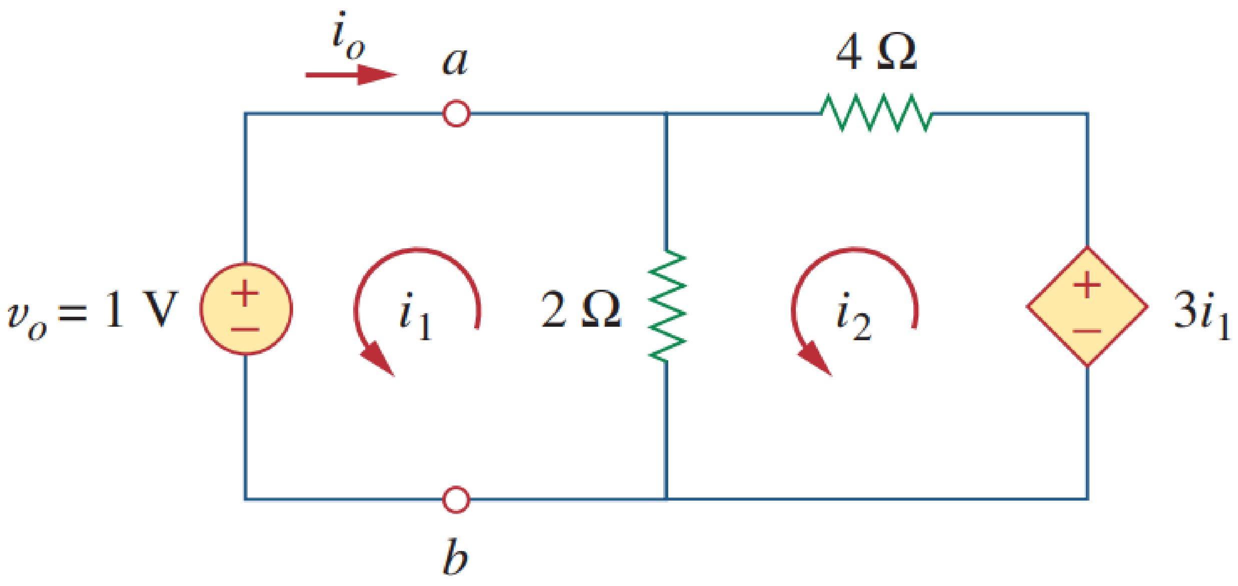
Example

Assuming that $i(0) = 10$ A, calculate $i(t)$ and $i_x(t)$ in the circuit



We have to find Thevenin resistance at the inductor terminals.

Because of the dependent source, we insert a 1V voltage source at the inductor terminals *a*-*b*.



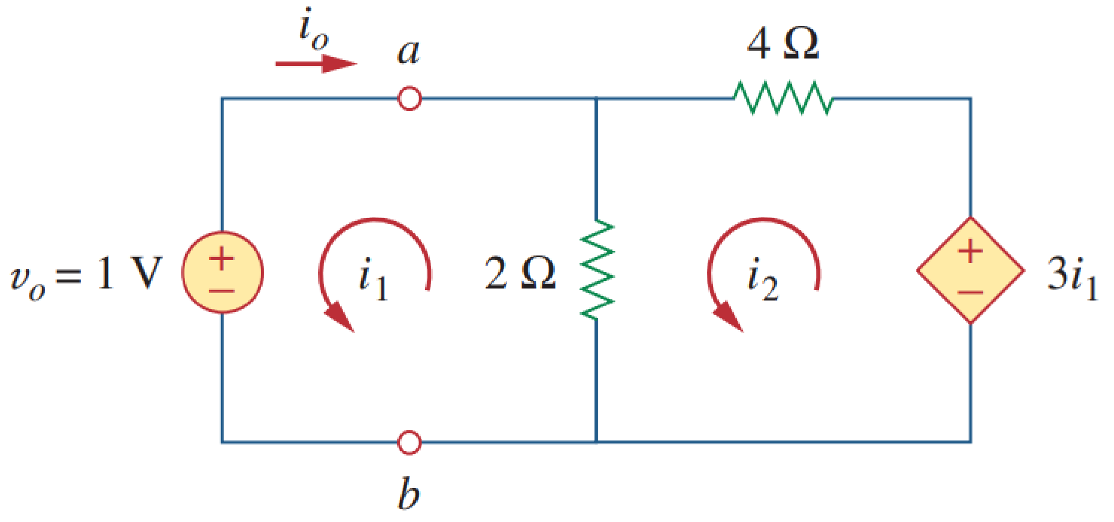
$$2(i_1 - i_2) + 1 = 0$$

$$i_1 - i_2 = -\frac{1}{2}$$

$$6i_2 - 2i_1 - 3i_1 = 0$$

$$i_2 = \frac{5}{6}i_1$$

continued…



$$i_1 = -3 \text{ A}$$

$$i_o = -i_1 = 3 \text{ A}$$

$$R_{eq} = R_{Th} = \frac{v_o}{i_o} = \frac{1}{3}\,\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2}\text{ s}$$

current through the inductor

$$i(t) = i(0)e^{-t/\tau} = 10e^{-(2/3)t}\text{ A} \qquad t > 0$$

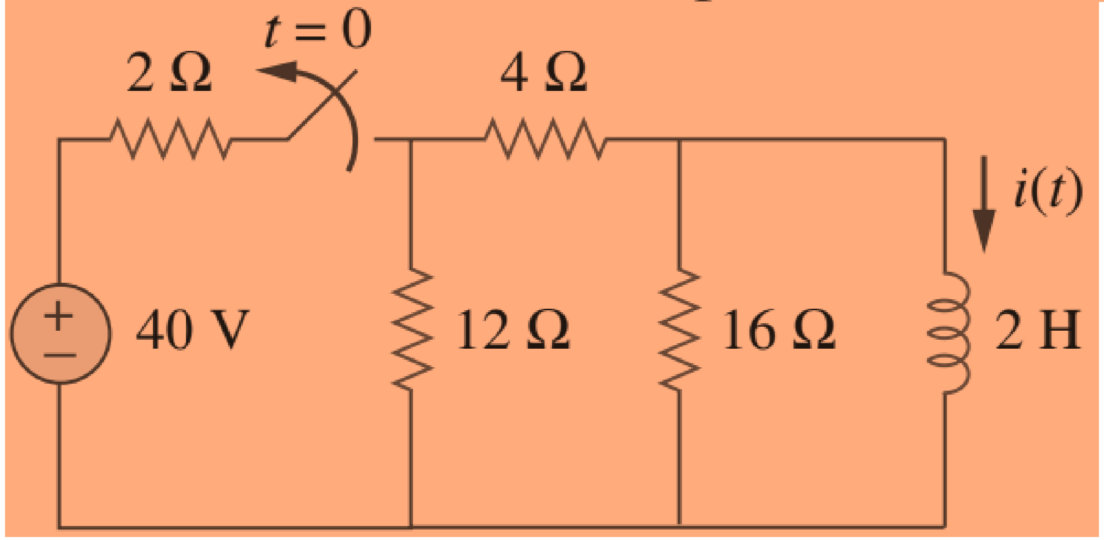
voltage across the inductor

$$v = L\frac{di}{dt} = 0.5(10)\left(-\frac{2}{3}\right)e^{-(2/3)t} = -\frac{10}{3}e^{-(2/3)t}\text{ V} \qquad t > 0$$

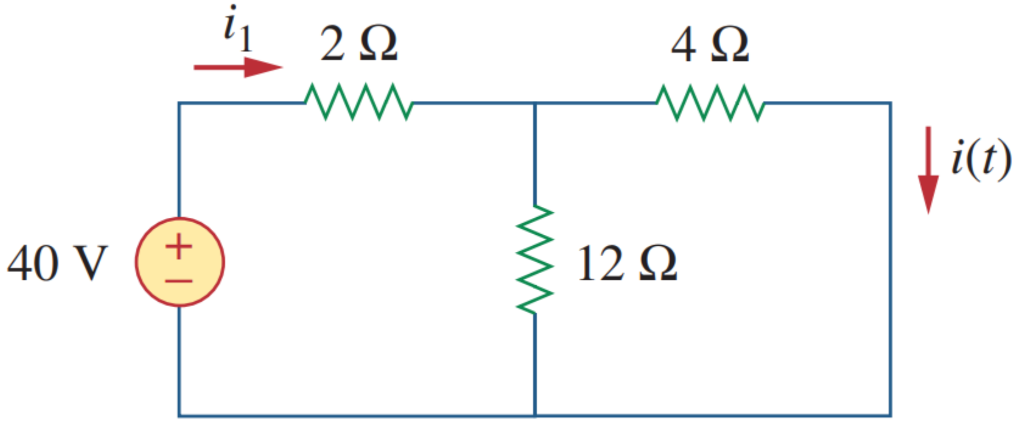
$$i_x(t) = \frac{v}{2} = -1.6667e^{-(2/3)t}\text{ A}, \qquad t > 0$$

continued…

The switch in the circuit has been closed for a long time. At $t = 0$, the switch is opened. Calculate $i(t)$ for $t > 0$.



For $t < 0$, the switch is closed



$$\frac{4 \times 12}{4 + 12} = 3\ \Omega$$

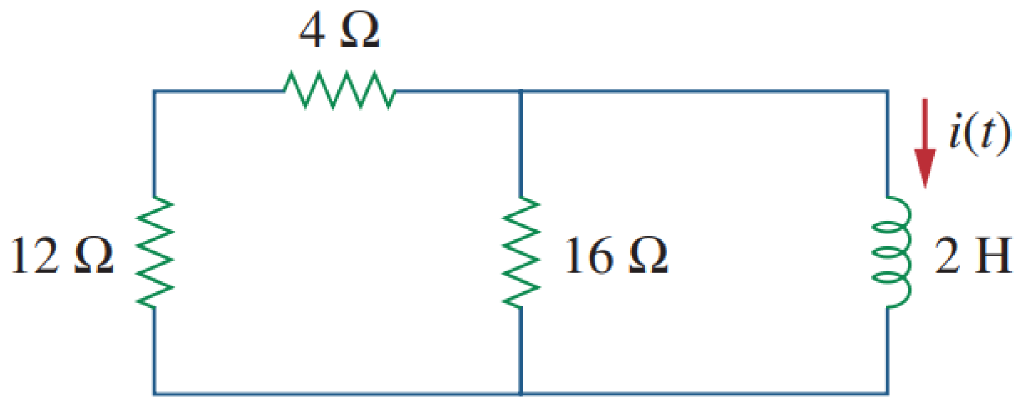
$$i_1 = \frac{40}{2 + 3} = 8\ \text{A}$$

$$i(t) = \frac{12}{12 + 4} i_1 = 6\ \text{A}, \qquad t < 0$$

$$i(0) = i(0^-) = 6\ \text{A}$$

continued…

For $t > 0$, the switch is opened



We have to find Thevenin resistance at the inductor terminals.

$$R_{eq} = (12 + 4) \parallel 16 = 8\ \Omega$$

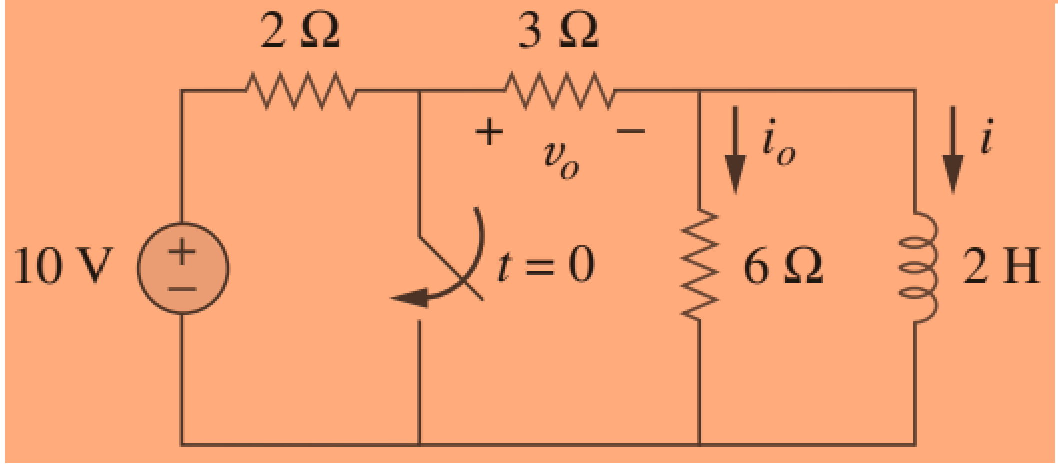
$$\tau = \frac{L}{R_{eq}} = \frac{2}{8} = \frac{1}{4}\ \text{s}$$

current through the inductor

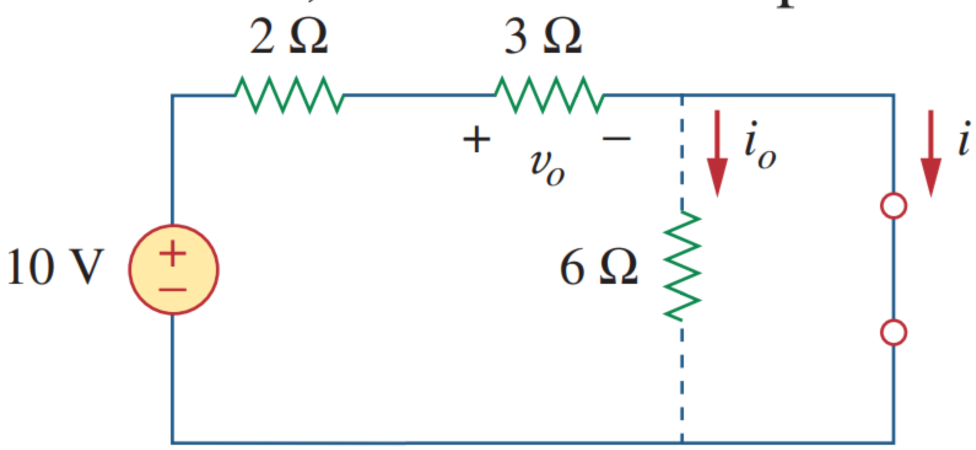
$$i(t) = i(0)e^{-t/\tau} = 6e^{-4t}\ \text{A} \qquad t > 0$$

continued...

In the circuit shown , find i_o, v_o, and i for all time, assuming that the switch was open for a long time.



For $t < 0$, the switch is open



$$i_o = 0$$

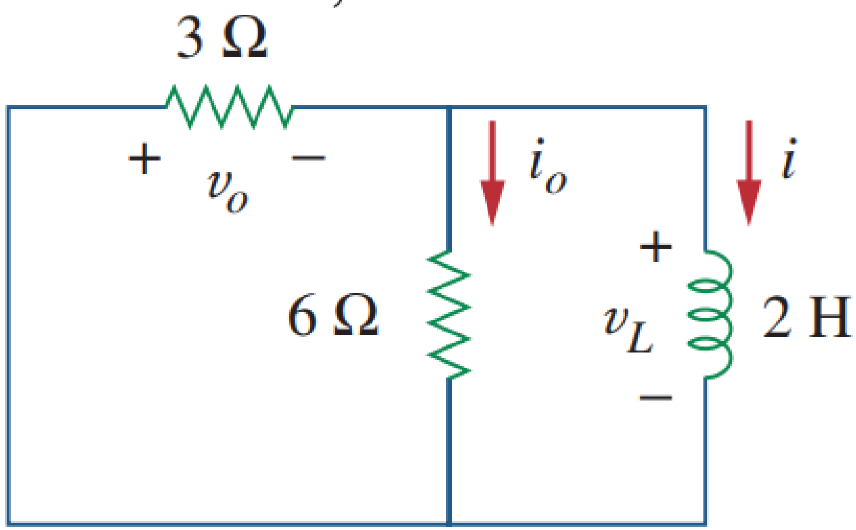
$$i(t) = \frac{10}{2 + 3} = 2 \text{ A}, \qquad t < 0$$

$$v_o(t) = 3i(t) = 6 \text{ V}, \qquad t < 0$$

$$i(0) = 2$$

continued...

For $t > 0$, the switch is closed



We have to find Thevenin resistance at the inductor terminals.

$$R_{\text{Th}} = 3 \,\|\, 6 = 2\ \Omega$$

$$\tau = \frac{L}{R_{\text{Th}}} = 1\ \text{s}$$

$$i(t) = i(0)e^{-t/\tau} = 2e^{-t}\ \text{A}, \qquad t > 0$$

$$v_o(t) = -v_L = -L\frac{di}{dt} = -2(-2e^{-t}) = 4e^{-t}\ \text{V}, \qquad t > 0$$

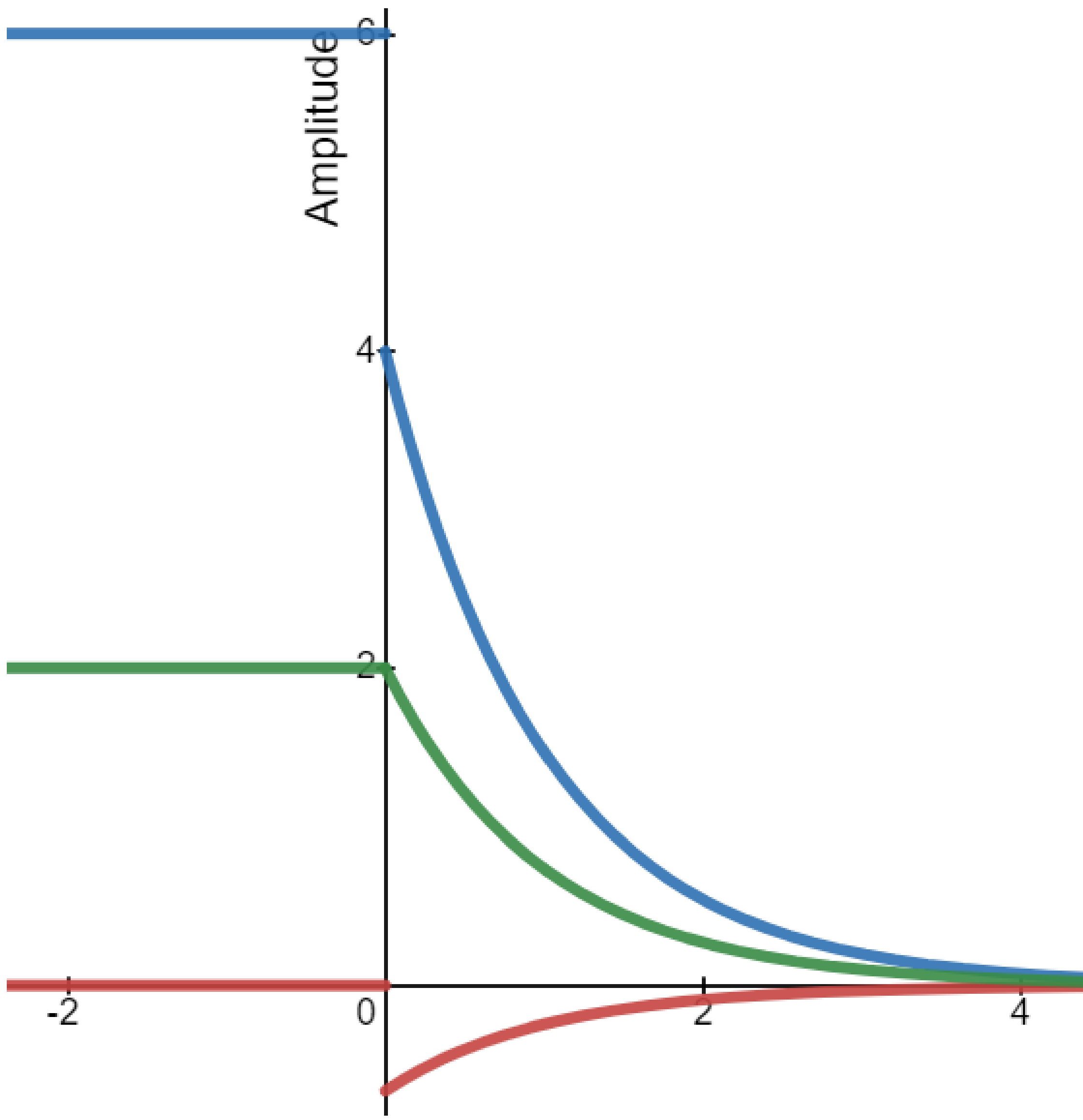
$$i_o(t) = \frac{v_L}{6} = -\frac{2}{3}e^{-t}\ \text{A}, \qquad t > 0$$

continued...

$$v_o(t) = \begin{cases} 6\text{ V}, & t < 0 \\ 4e^{-t}\text{ V}, & t > 0 \end{cases}$$

$$i(t) = \begin{cases} 2\text{ A}, & t < 0 \\ 2e^{-t}\text{ A}, & t \geq 0 \end{cases}$$

$$i_o(t) = \begin{cases} 0\text{ A}, & t < 0 \\ -\frac{2}{3}e^{-t}\text{ A}, & t > 0 \end{cases}$$

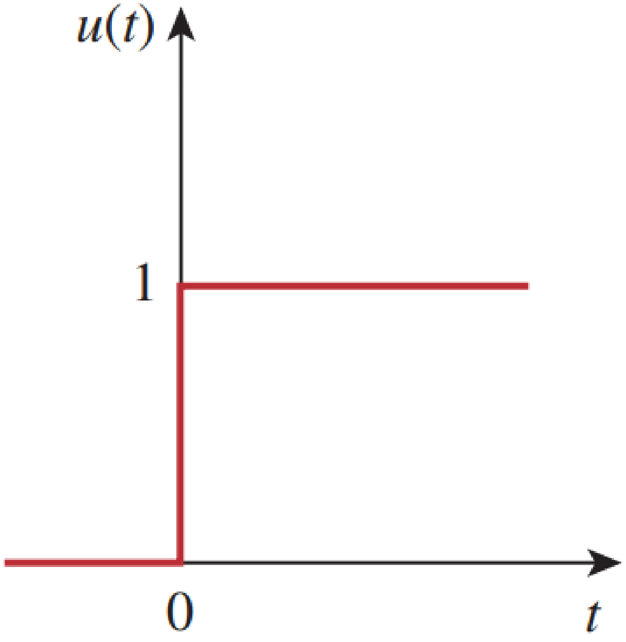


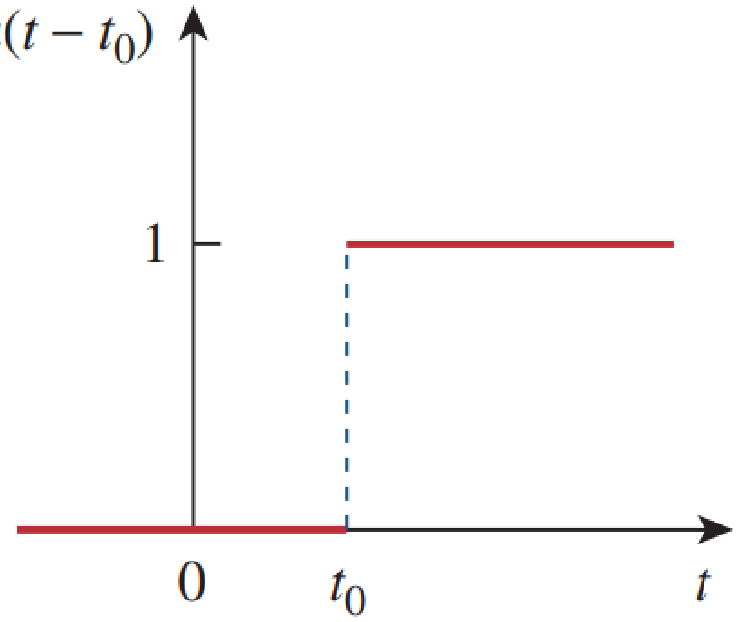
Singularity Functions

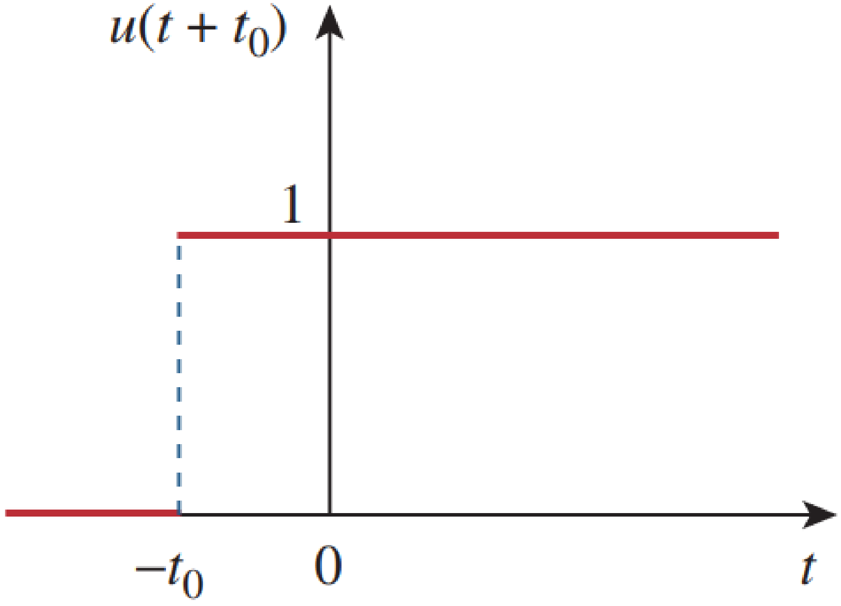
(also called *switching functions*): either are discontinuous or have discontinuous derivatives.

Unit Step	Unit Impulse	Unit Ramp
u(t)	δ(t)	r(t)
$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$	$\delta(t) = \begin{cases} 0, & t < 0 \\ \infty, & t = 0 \\ 0, & t > 0 \end{cases}$	$r(t) = \begin{cases} 0, & t \leq 0 \\ t, & t \geq 0 \end{cases}$
$u(t) = \int_{-\infty}^{t} \delta(t)dt$ $u(t) = \frac{d}{dt} r(t)$	$\delta(t) = \frac{d}{dt} u(t)$	$r(t) = tu(t)$

Step Function







$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

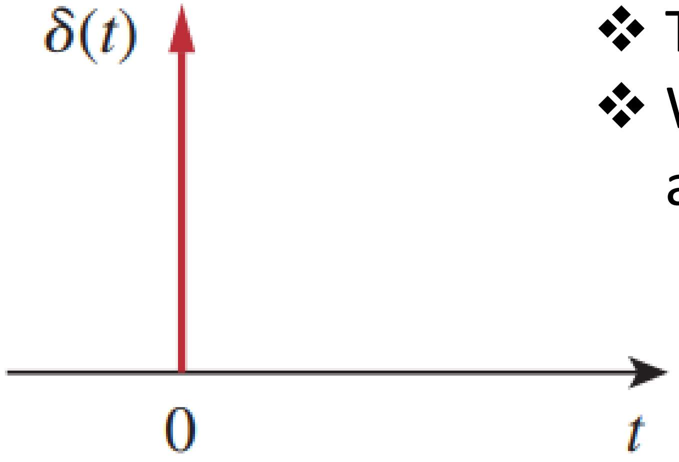
$$u(t - t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$

$$u(t + t_0) = \begin{cases} 0, & t < -t_0 \\ 1, & t > -t_0 \end{cases}$$

Impulse Function

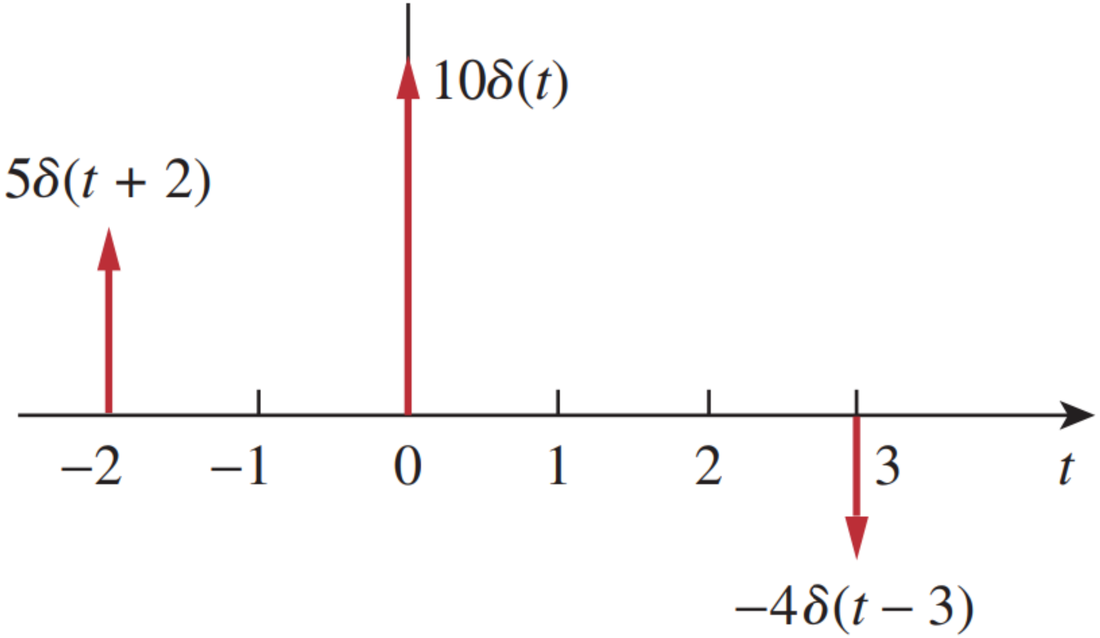
(also known as **Dirac delta** function): NOT physically realizable.

$$\int_{0^-}^{0^+} \delta(t)dt = 1$$

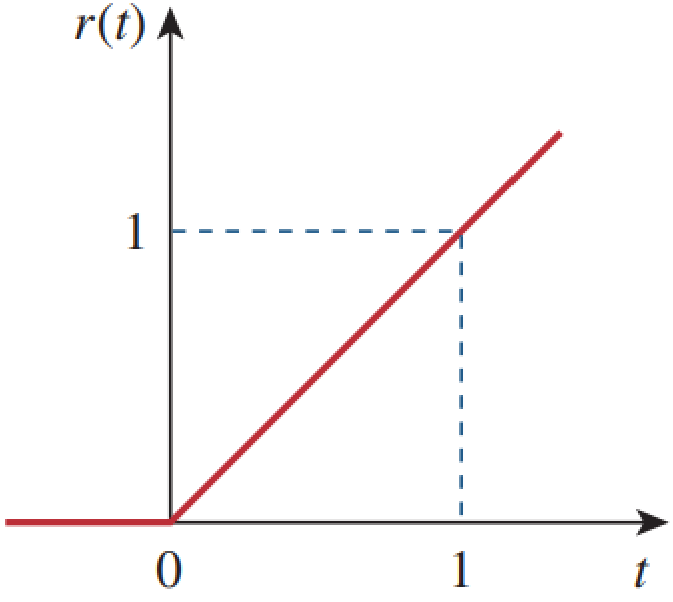


$$\delta(t) = \begin{cases} 0, & t < 0 \\ \infty, & t = 0 \\ 0, & t > 0 \end{cases}$$

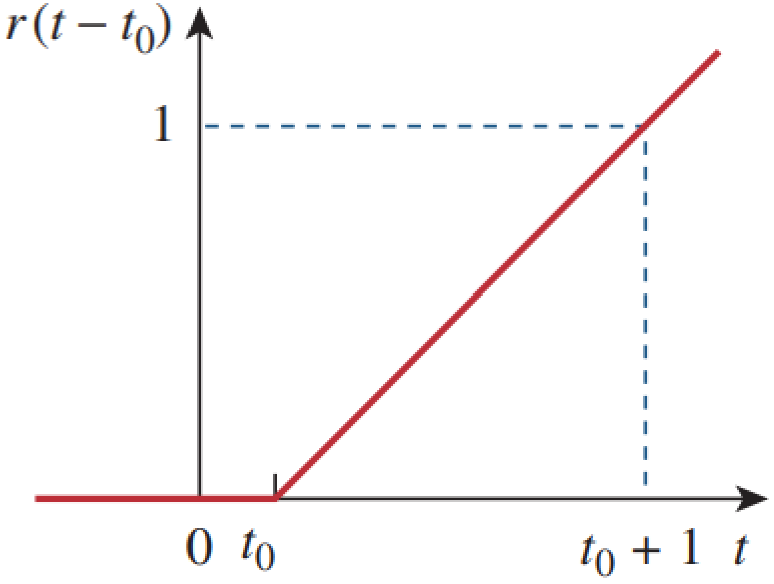
- The unit area is known as the **strength** of the impulse function.
- When an impulse function has a strength other than unity, the area of the impulse is equal to its strength.



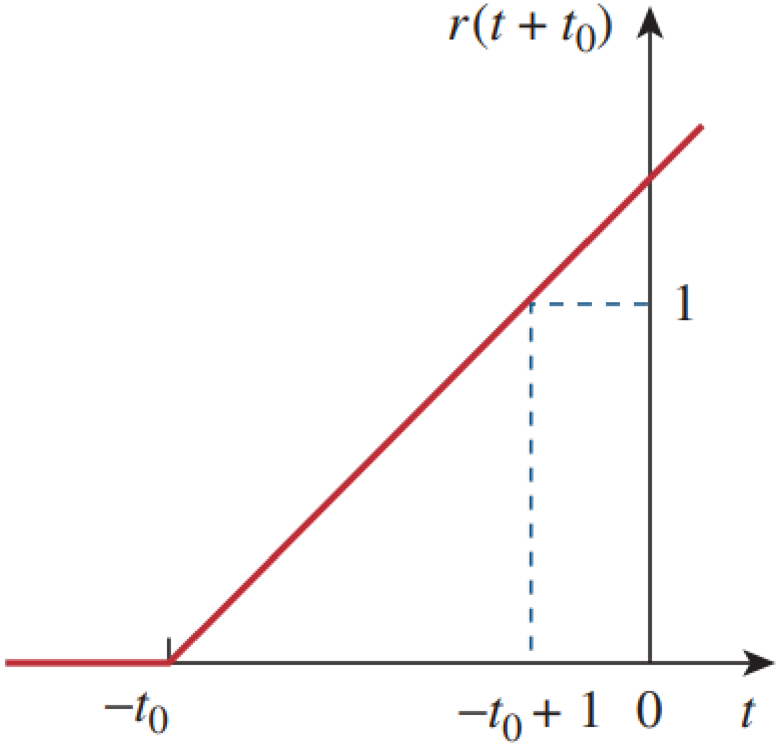
Ramp Function



$$r(t) = \begin{cases} 0, & t \leq 0 \\ t, & t \geq 0 \end{cases}$$



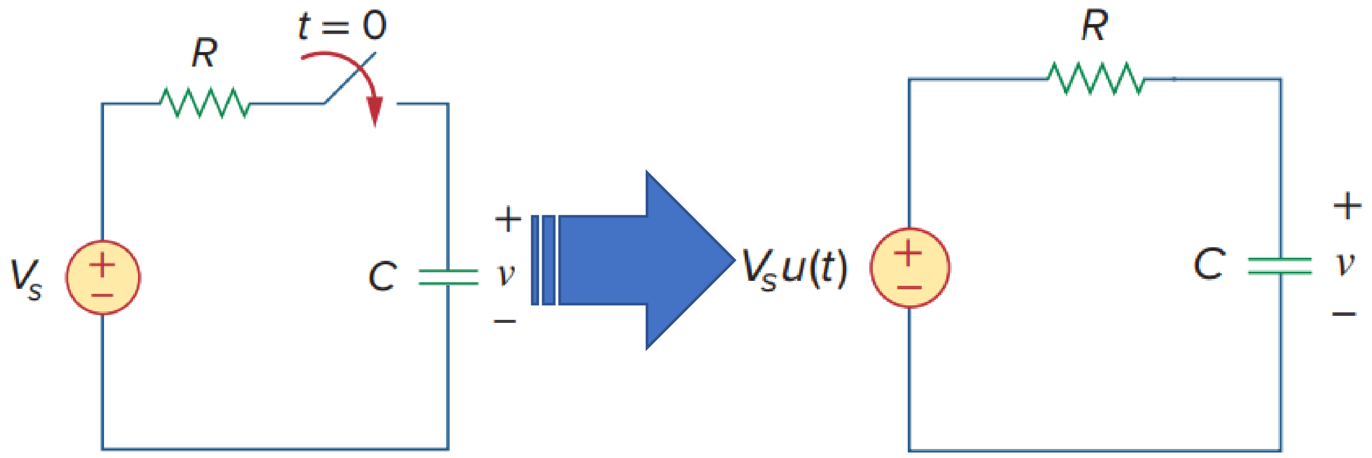
$$r(t - t_0) = \begin{cases} 0, & t \leq t_0 \\ t - t_0, & t \geq t_0 \end{cases}$$



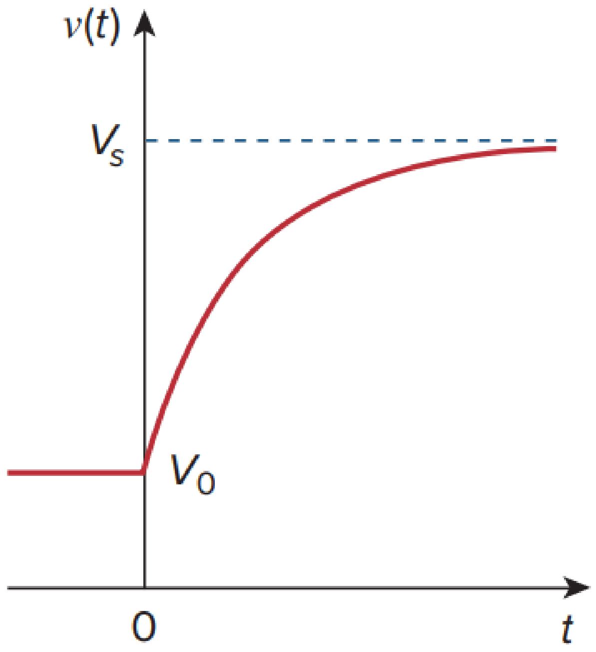
$$r(t + t_0) = \begin{cases} 0, & t \leq -t_0 \\ t + t_0, & t \geq -t_0 \end{cases}$$

Step Response of RC Circuit

The step response is the response of the circuit due to a sudden application of a dc voltage or current source.

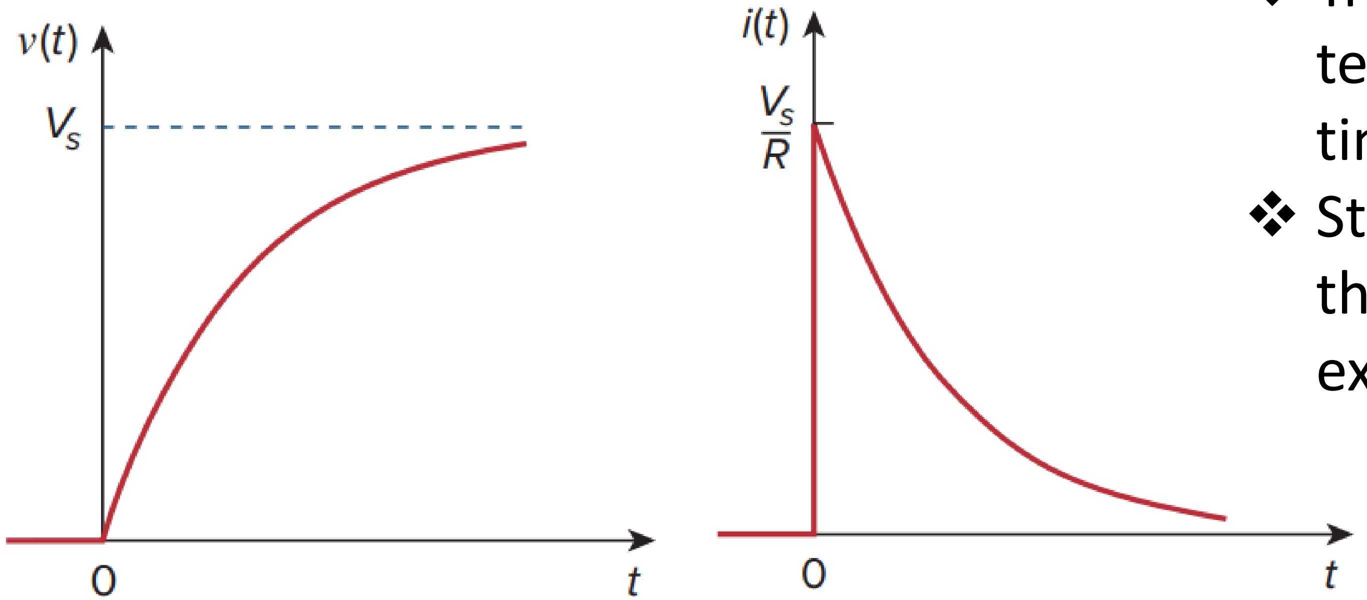


$$v(0^-) = v(0^+) = V_0$$



Response of an RC circuit with initially charged capacitor

continued…



Step response of an RC circuit with initially uncharged capacitor: (left) voltage response, (right) current response

- Transient response is the circuit's temporary response that will die out with time.
- Steady-state response is the behavior of the circuit a long time after an external excitation is applied.

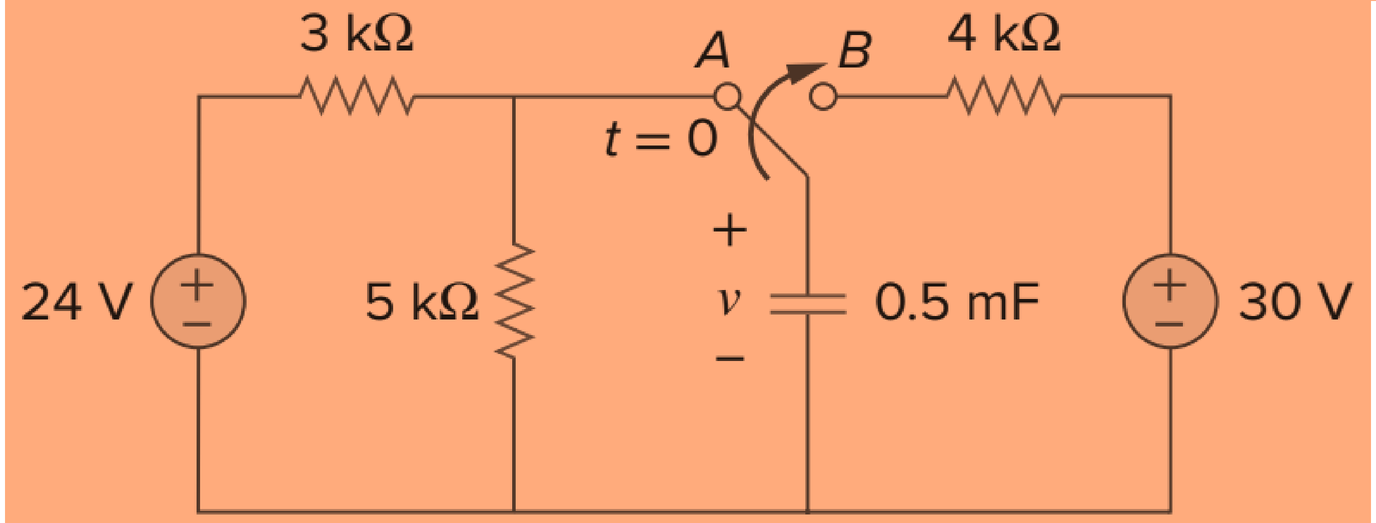
$$\underbrace{\text{Complete response}}\ = \underbrace{\text{natural response}}_{\text{stored energy}} + \underbrace{\text{forced response}}_{\text{independent source}}$$

$$\text{Complete response} = \underbrace{\text{transient response}}_{\text{temporary part}} + \underbrace{\text{steady-state response}}_{\text{permanent part}}$$

If the switch changes position at time $t = t_0$ → $v(t) = v(\infty) + [v(t_0) - v(\infty)]e^{-(t - t_0)/\tau}$

Example

The switch has been in position A for a long time. At $t = 0$, the switch moves to B. Determine $v(t)$ for $t > 0$ and calculate its value at $t = 1$ and 4 s.



$$v(0^-) = \frac{5}{5+3}(24) = 15 \text{ V}$$

$$v(0) = v(0^-) = v(0^+) = 15 \text{ V}$$

For $t > 0$,

$$R_{\text{Th}} = 4 \text{ k}\Omega$$

$$\tau = R_{\text{Th}} C = 4 \times 10^3 \times 0.5 \times 10^{-3} = 2 \text{ s}$$

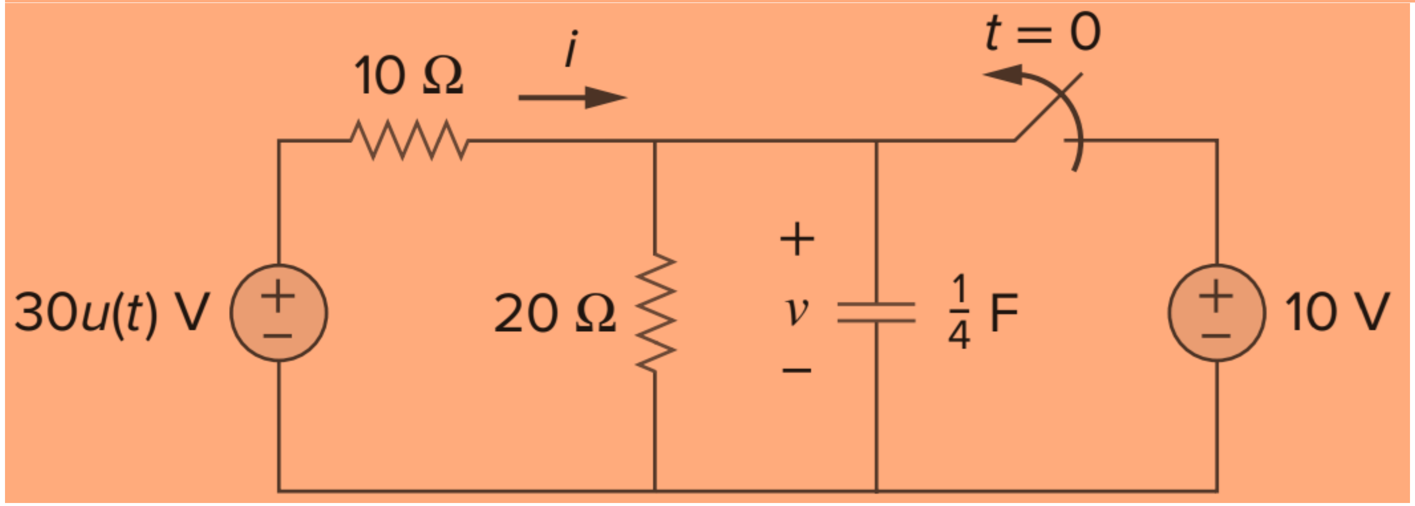
$$v(\infty) = 30 \text{ V}$$

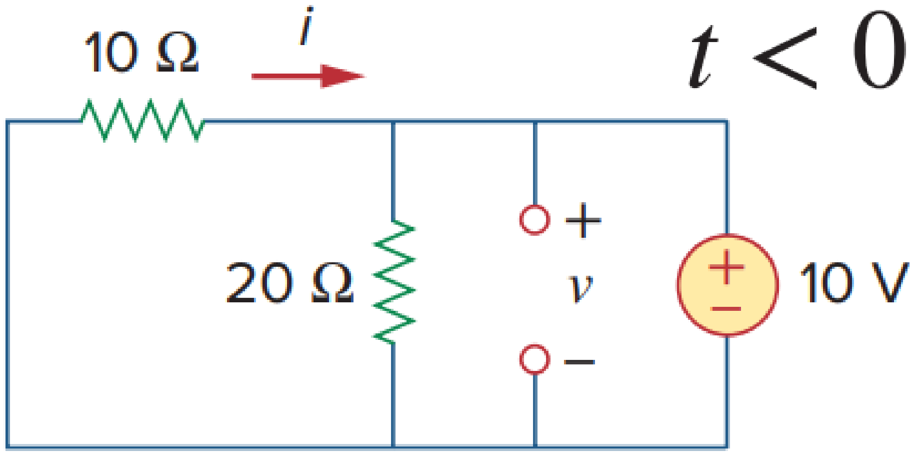
$$\begin{aligned} v(t) &= v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \\ &= 30 + (15 - 30)e^{-t/2} \\ &= (30 - 15e^{-0.5t}) \text{ V} \end{aligned}$$

continued...

$$\text{At } t = 1,\ v(1) = 30 - 15e^{-0.5} = 20.9 \text{ V}$$
$$\text{At } t = 4,\ v(4) = 30 - 15e^{-2} = 27.97 \text{ V}$$

the switch has been closed for a long time and is opened at $t = 0$. Find i and v for all time.





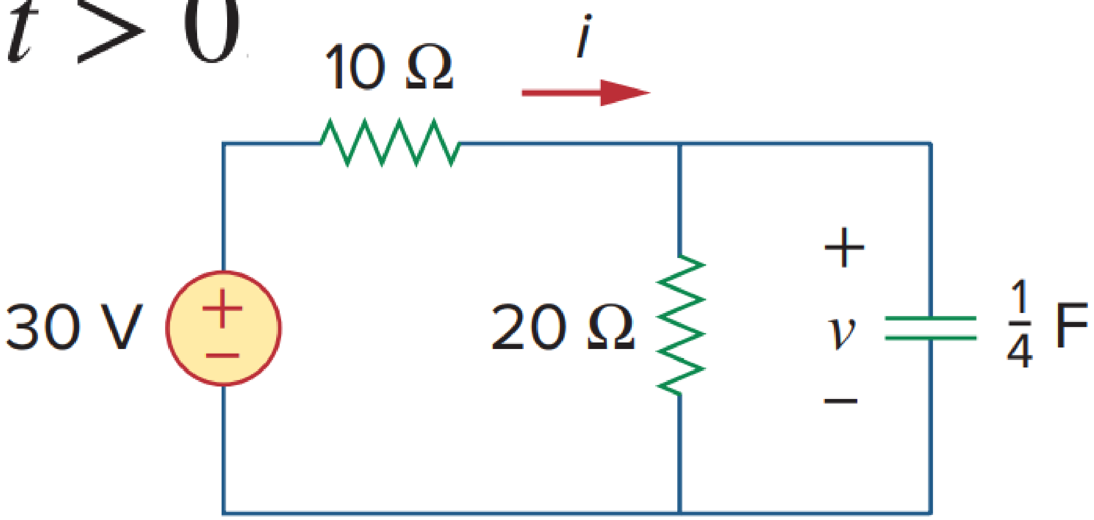
$t < 0$

$$30u(t) = \begin{cases} 0, & t < 0 \\ 30, & t > 0 \end{cases}$$

$$v(0) = v(0^-) = 10 \text{ V}$$
$$i = -\frac{v}{10} = -1 \text{ A} \qquad t < 0$$

continued…

$t > 0$



$$v(\infty) = \frac{20}{20+10}(30) = 20\ \text{V}$$

We have to find Thevenin resistance at the capacitor terminals.

$$R_{\text{Th}} = 10 \| 20 = \frac{10 \times 20}{30} = \frac{20}{3}\ \Omega$$

$$\tau = R_{\text{Th}}\, C = \frac{20}{3} \cdot \frac{1}{4} = \frac{5}{3}\ \text{s}$$

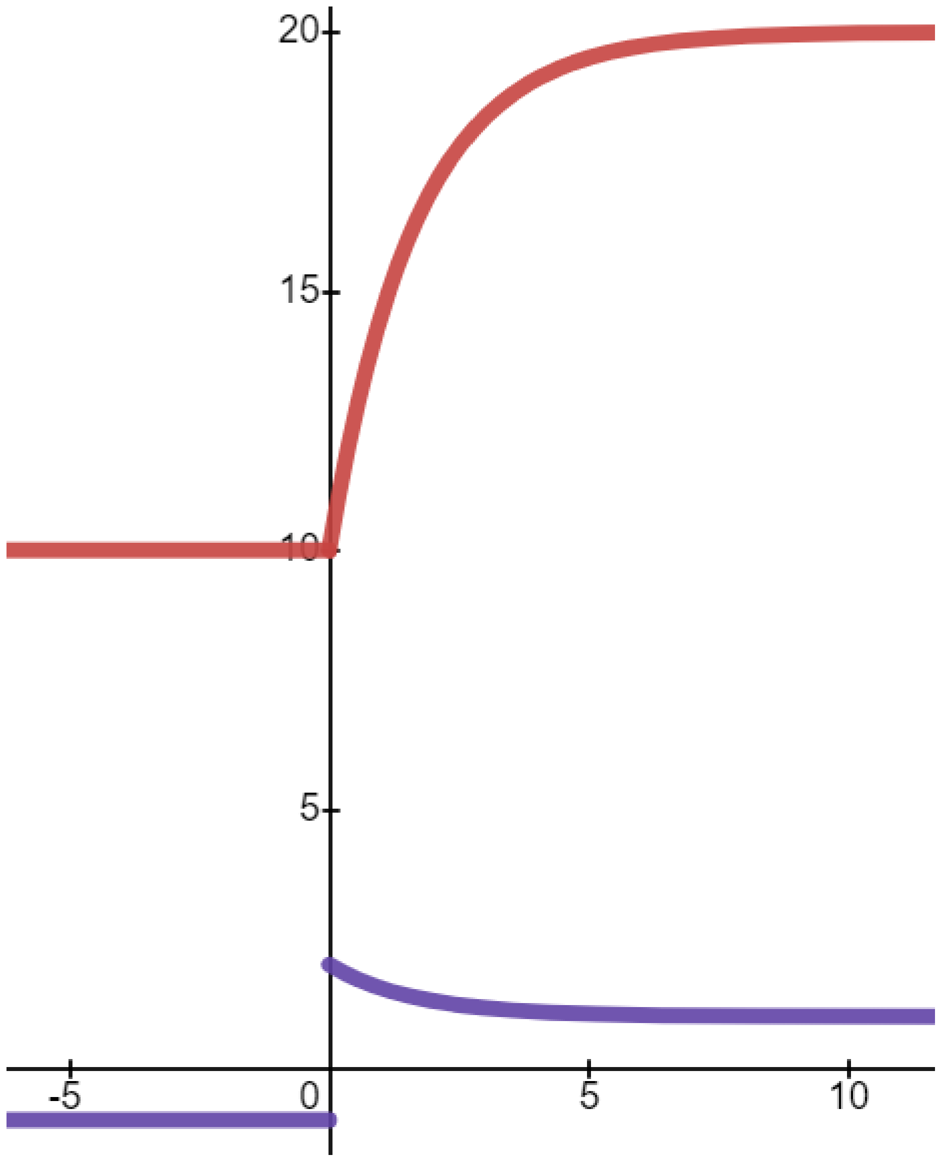
$$\begin{aligned} v(t) &= v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \\ &= 20 + (10 - 20)e^{-(3/5)t} \\ &= (20 - 10e^{-0.6t})\ \text{V} \end{aligned}$$

$$\begin{aligned} i &= \frac{v}{20} + C\frac{dv}{dt} \\ &= 1 - 0.5e^{-0.6t} \\ &\quad + 0.25(-0.6)(-10)e^{-0.6t} \\ &= (1 + e^{-0.6t})\ \text{A} \end{aligned}$$

continued...

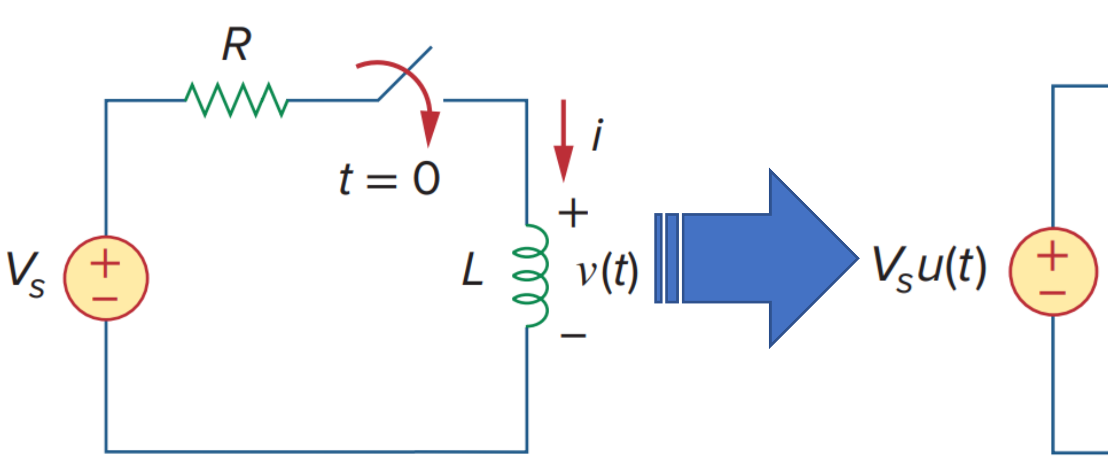
$$v = \begin{cases} 10 \text{ V}, & t < 0 \\ (20 - 10e^{-0.6t}) \text{ V}, & t \geq 0 \end{cases}$$

$$i = \begin{cases} -1 \text{ A}, & t < 0 \\ (1 + e^{-0.6t}) \text{ A}, & t > 0 \end{cases}$$

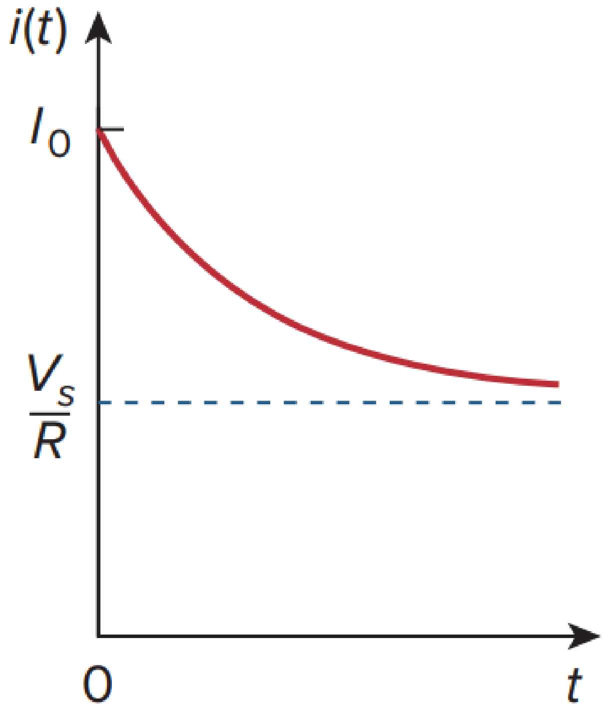


Step Response of RL Circuit

The step response is the response of the circuit due to a sudden application of a dc voltage or current source.

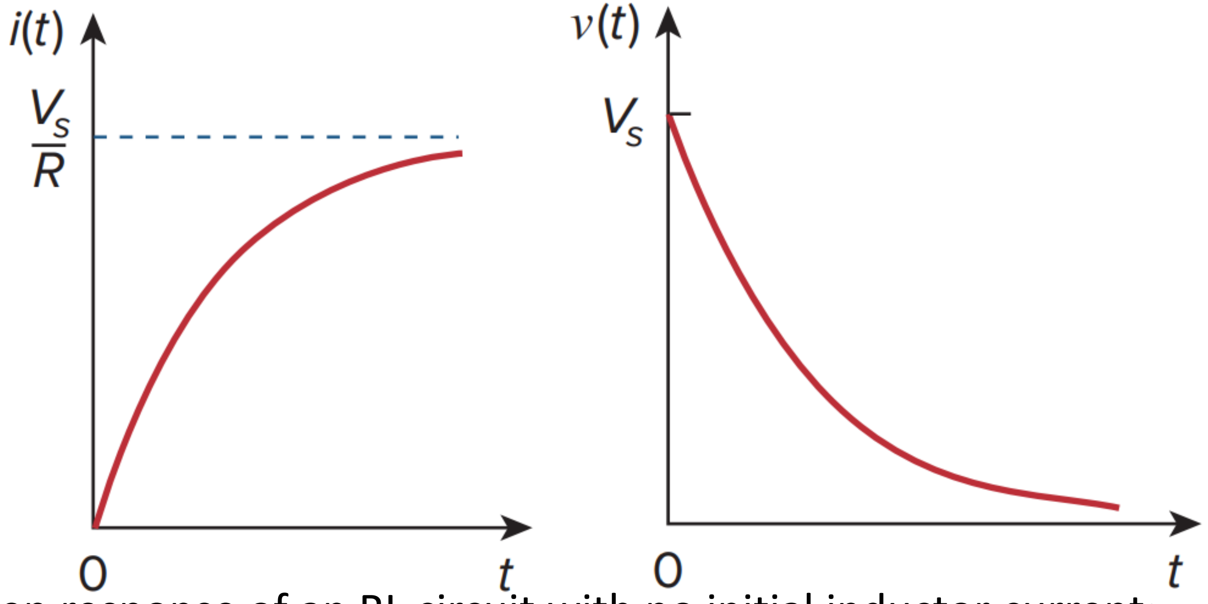


$$i(0^+) = i(0^-) = I_0$$



Total response of an RL circuit with initial inductor current I_0

continued...



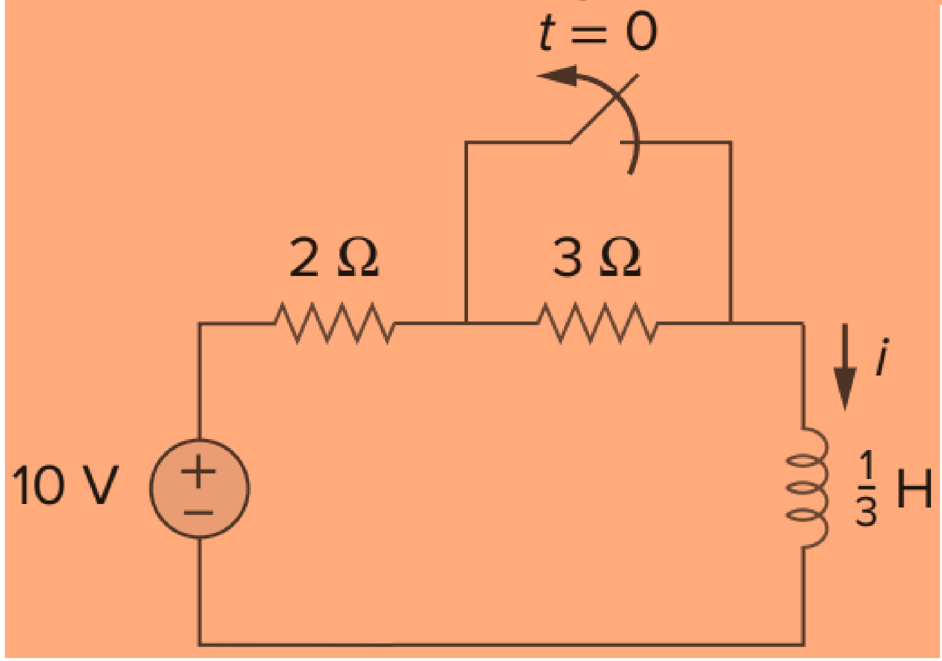
Step response of an RL circuit with no initial inductor current: (left) current response, (right) voltage response

If the switch changes position at time $t = t_0$

$$i(t) = i(\infty) + [i(t_0) - i(\infty)]e^{-(t-t_0)/\tau}$$

Example

Find $i(t)$ in the circuit for $t > 0$. Assume that the switch has been closed for a long time.



$$i(0^-) = \frac{10}{2} = 5\text{ A}$$

$$i(0) = i(0^+) = i(0^-) = 5\text{ A}$$

$$i(\infty) = \frac{10}{2+3} = 2\text{ A}$$

We have to find Thevenin resistance at the inductor terminals.

$$R_{\text{Th}} = 2 + 3 = 5\ \Omega$$

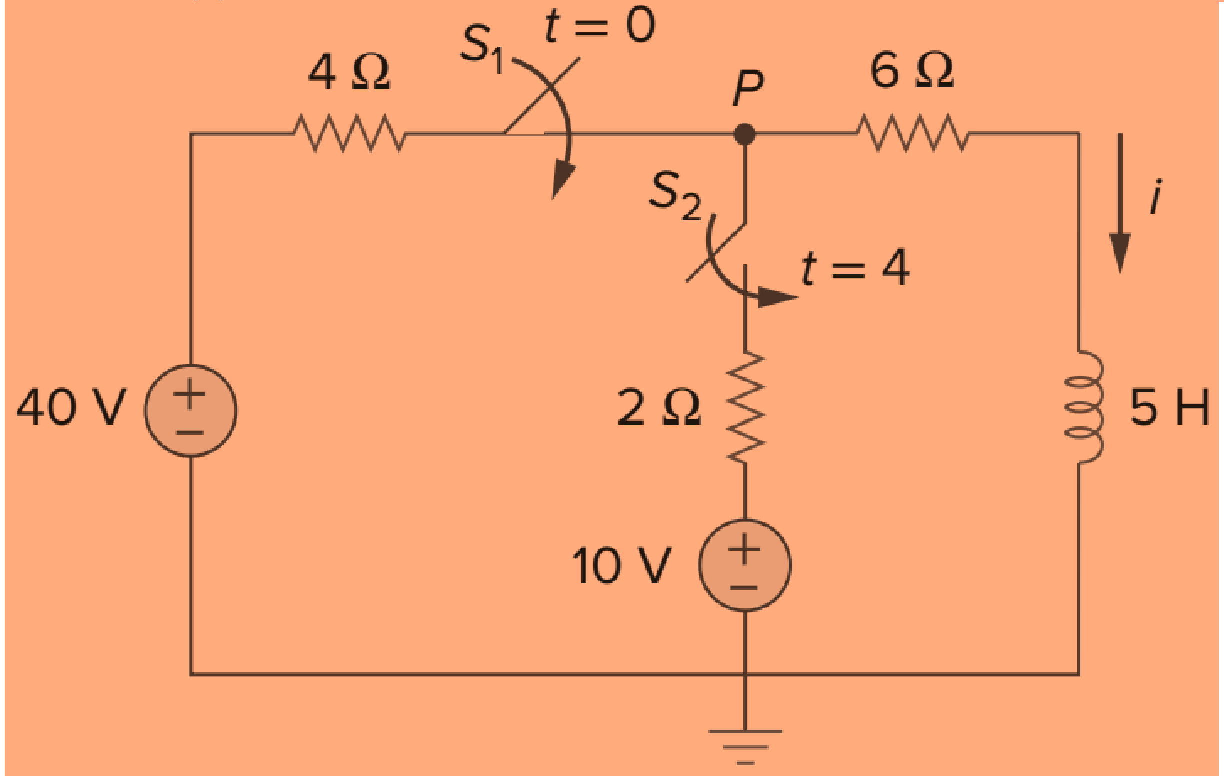
$$\tau = \frac{L}{R_{\text{Th}}} = \frac{\frac{1}{3}}{5} = \frac{1}{15}\text{ s}$$

$t > 0$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$
$$= 2 + (5-2)e^{-15t}$$
$$= 2 + 3e^{-15t}\text{ A}$$

continued…

At $t = 0$, switch 1 is closed, and switch 2 is closed 4 s later. Find $i(t)$ for $t > 0$. Calculate i for $t = 2$ s and $t = 5$ s.



For $t < 0$, switches S_1 and S_2 are open

$$i(0^-) = i(0) = i(0^+) = 0$$

For $0 \le t \le 4$, S_1 is closed

$$i(\infty) = \frac{40}{4 + 6} = 4 \text{ A}$$

$$R_{Th} = 4 + 6 = 10\ \Omega$$

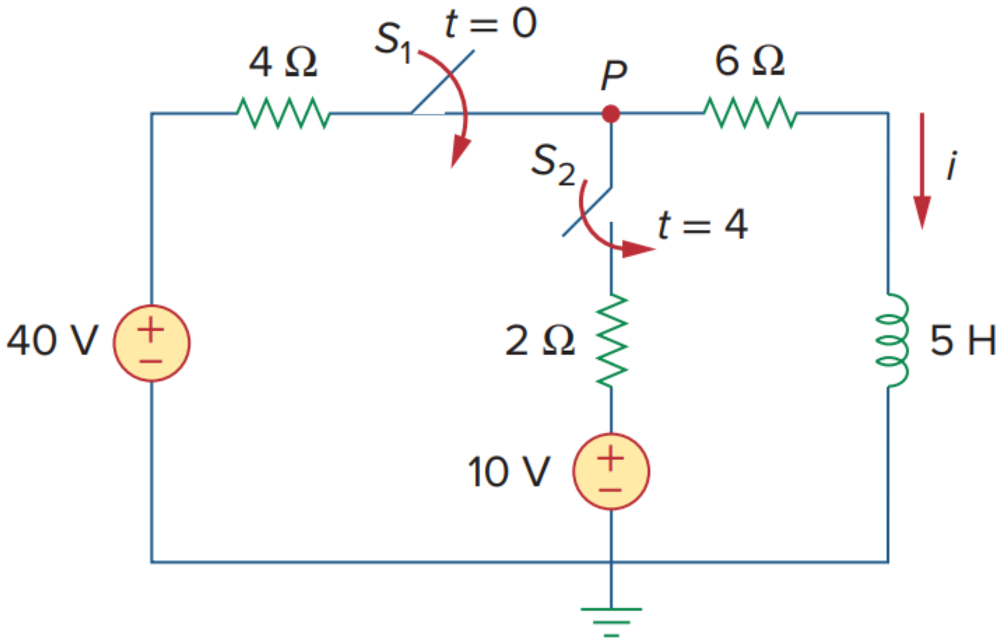
$$\tau = \frac{L}{R_{Th}} = \frac{5}{10} = \frac{1}{2} \text{ s}$$

$0 \le t \le 4$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} = 4 + (0 - 4)e^{-2t}$$

$$= 4(1 - e^{-2t}) \text{ A}$$

continued...



For $t \geq 4$, S_2 is closed

$$i(4) = i(4^-)$$
$$= 4(1 - e^{-8}) \simeq 4 \text{ A}$$

at node P

$$\frac{40 - v}{4} + \frac{10 - v}{2} = \frac{v}{6} \rightarrow v = \frac{180}{11} \text{ V}$$

$$i(\infty) = \frac{v}{6} = \frac{30}{11} = 2.727 \text{ A}$$

$$R_{\text{Th}} = 4 \| 2 + 6$$
$$= \frac{4 \times 2}{6} + 6 = \frac{22}{3} \, \Omega$$

$$\tau = \frac{L}{R_{\text{Th}}} = \frac{5}{\frac{22}{3}} = \frac{15}{22} \text{ s}$$

continued...

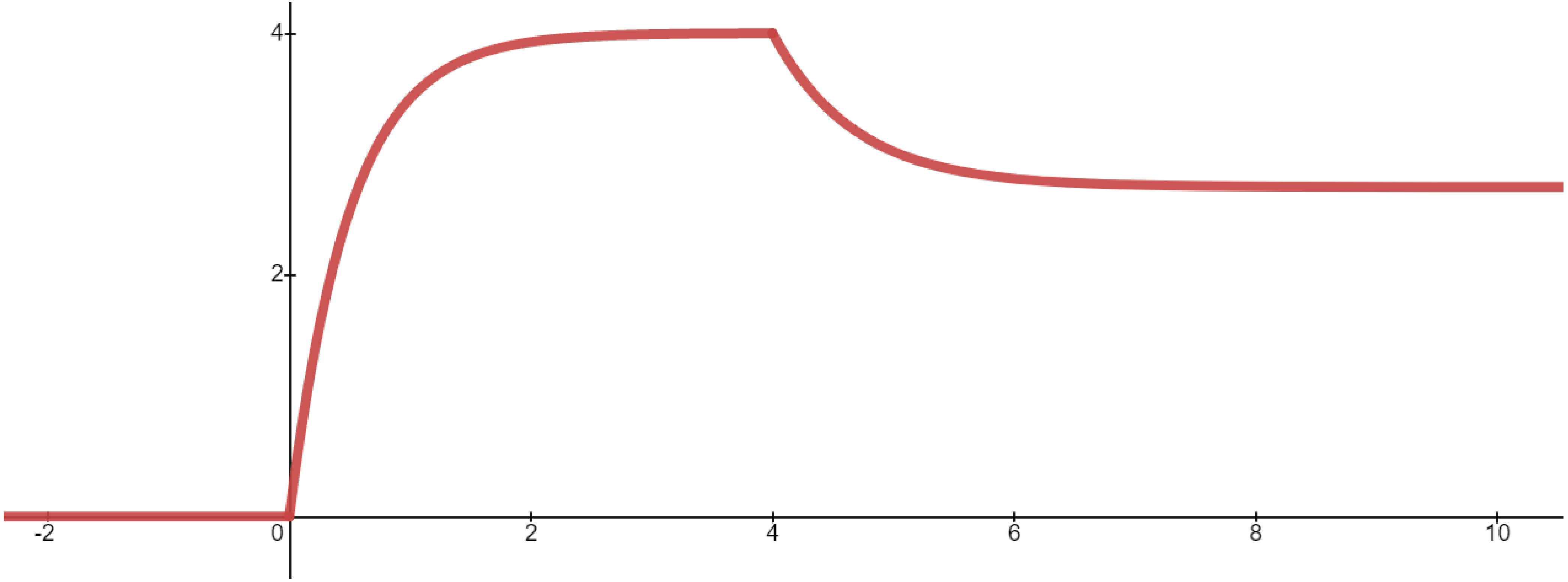
$$i(t) = i(\infty) + [i(4) - i(\infty)]e^{-(t-4)/\tau} \qquad t \geq 4$$
$$= 2.727 + (4 - 2.727)e^{-(t-4)/\tau}$$
$$= 2.727 + 1.273e^{-1.4667(t-4)}$$

$$i(t) = \begin{cases} 0, & t \leq 0 \\ 4(1 - e^{-2t}), & 0 \leq t \leq 4 \\ 2.727 + 1.273e^{-1.4667(t-4)}, & t \geq 4 \end{cases}$$

$$i(2) = 4(1 - e^{-4}) = 3.93 \text{ A}$$
$$i(5) = 2.727 + 1.273e^{-1.4667} = 3.02 \text{ A}$$

continued…

$$i(t) = \begin{cases} 0, & t \leq 0 \\ 4(1 - e^{-2t}), & 0 \leq t \leq 4 \\ 2.727 + 1.273e^{-1.4667(t-4)}, & t \geq 4 \end{cases}$$



Electrical Technology (Part 3)

EEE 101

Basic: Magnetic Field

Flux Density

$$B = \frac{\Phi}{A}$$

B = Wb/m^2 = teslas (T)
Φ = webers (Wb)
A = m^2

Relative Permeability

$$\mu_r = \frac{\mu}{\mu_o}$$

$$\mu_o = 4\pi \times 10^{-7}\,\text{Wb/Am}$$

Magnetomotive Force

$$\mathscr{F} = NI$$

$\mathscr{F}$ = ampere-turns (At)
N = turns (t)
I = amperes (A)

Reluctance

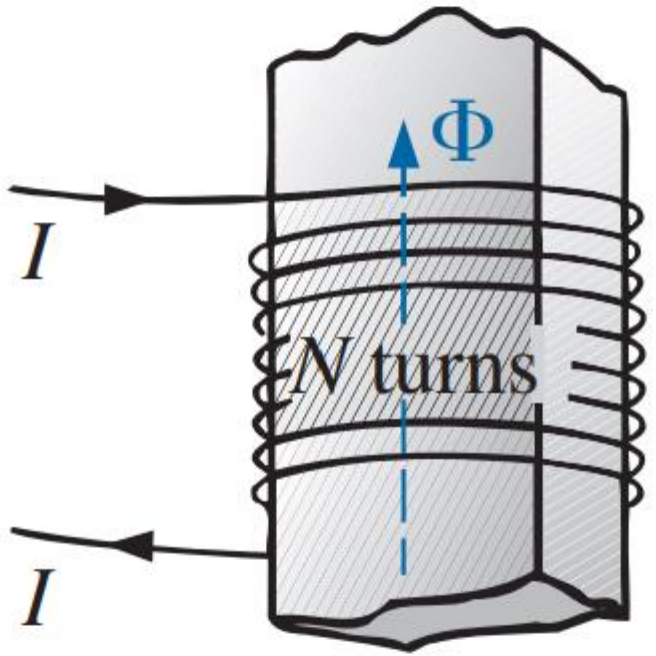
$$\mathscr{R} = \frac{l}{\mu A}$$

(rels, or At/Wb)

Ohm's Law for Magnetic Circuits

$$\text{Effect} = \frac{\text{cause}}{\text{opposition}}$$

- Desired **effect** is the flux Φ.
- The **cause** is the magnetomotive force (mmf) $\mathcal{F}$, which is the external force required to set up the flux lines within the magnetic material.
- The **opposition** to the setting up of the flux is the reluctance $\Re$.



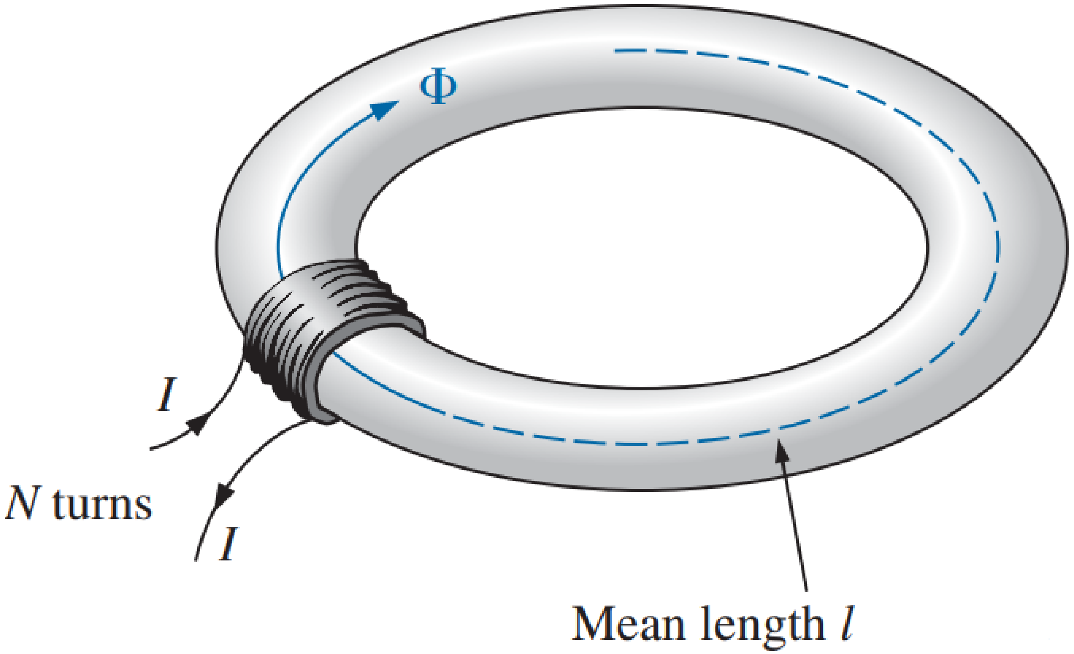
$$\Phi = \frac{\mathcal{F}}{\mathcal{R}}$$

Φ is established in the core through the alteration of the atomic structure of the core due to $\mathcal{F}$, and is **not** a measure of the flow of some charged particles through the core.

Magnetizing Force

mmf per unit length is called the magnetizing force (*H*).

$$H = \frac{\mathscr{F}}{l} \qquad \text{(At/m)}$$

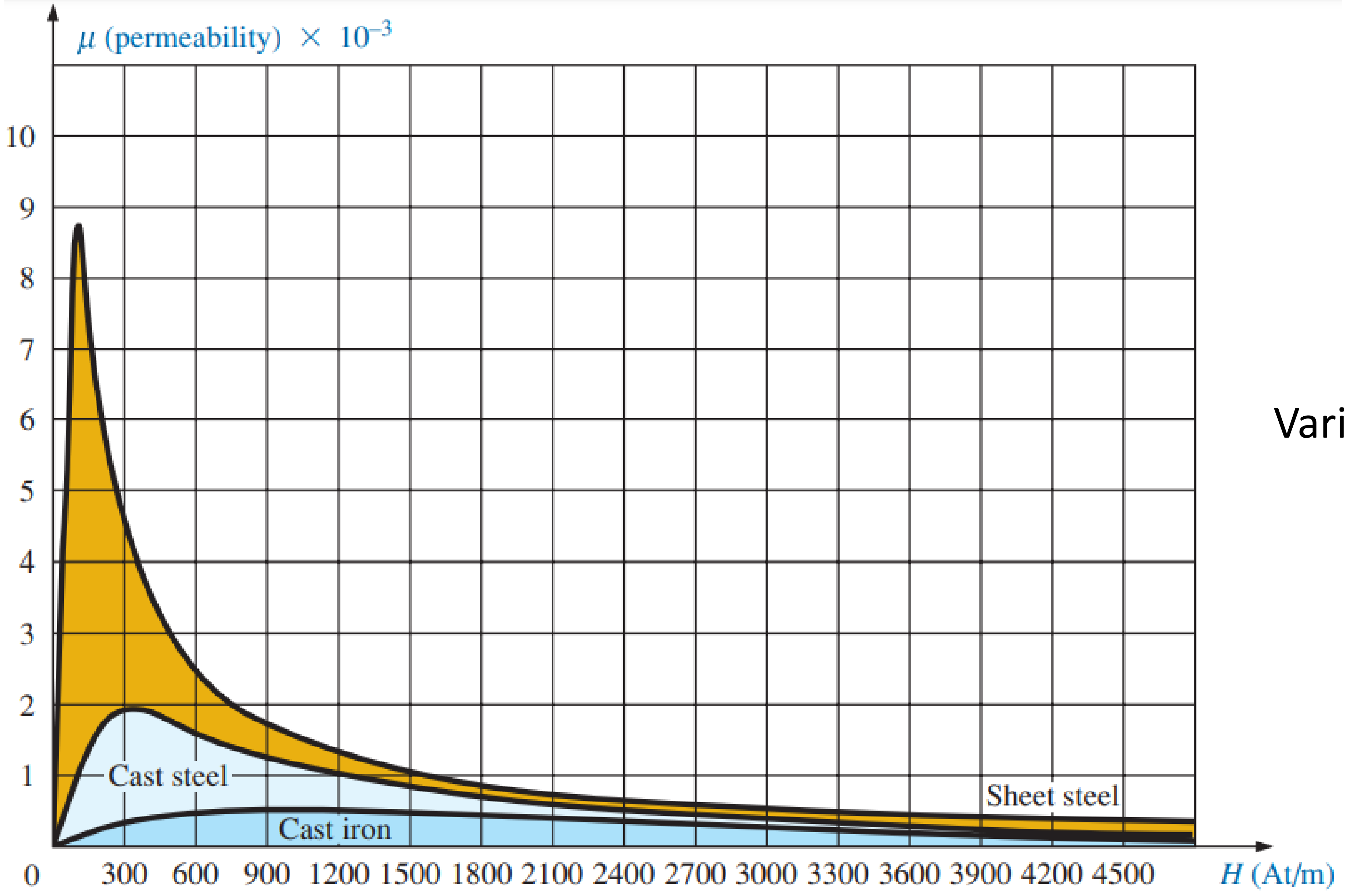


H is independent of the type of core material.

$$B = \mu H$$

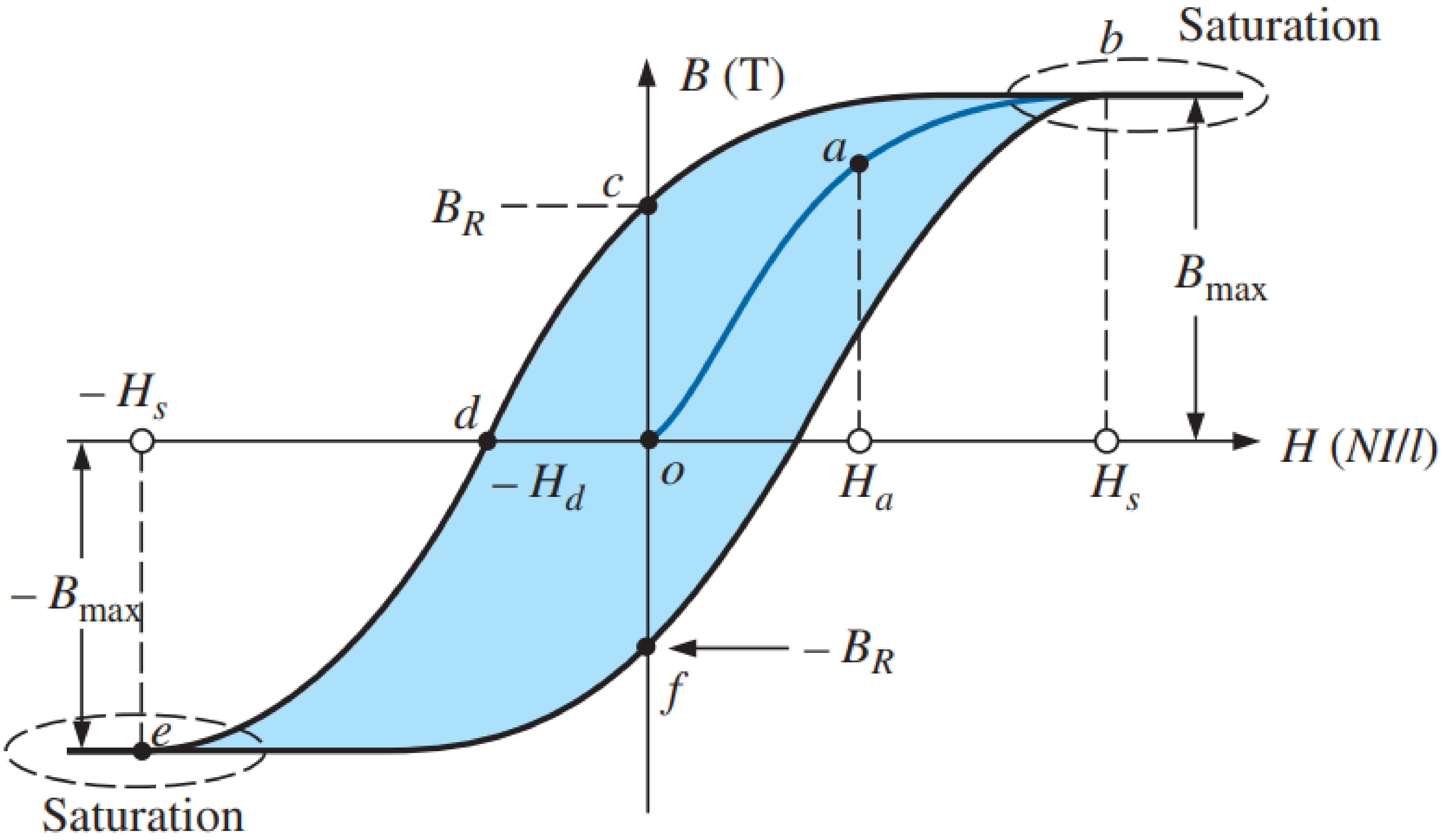
For a particular *H*, the greater the μ, the greater is the induced flux density B.

continued…



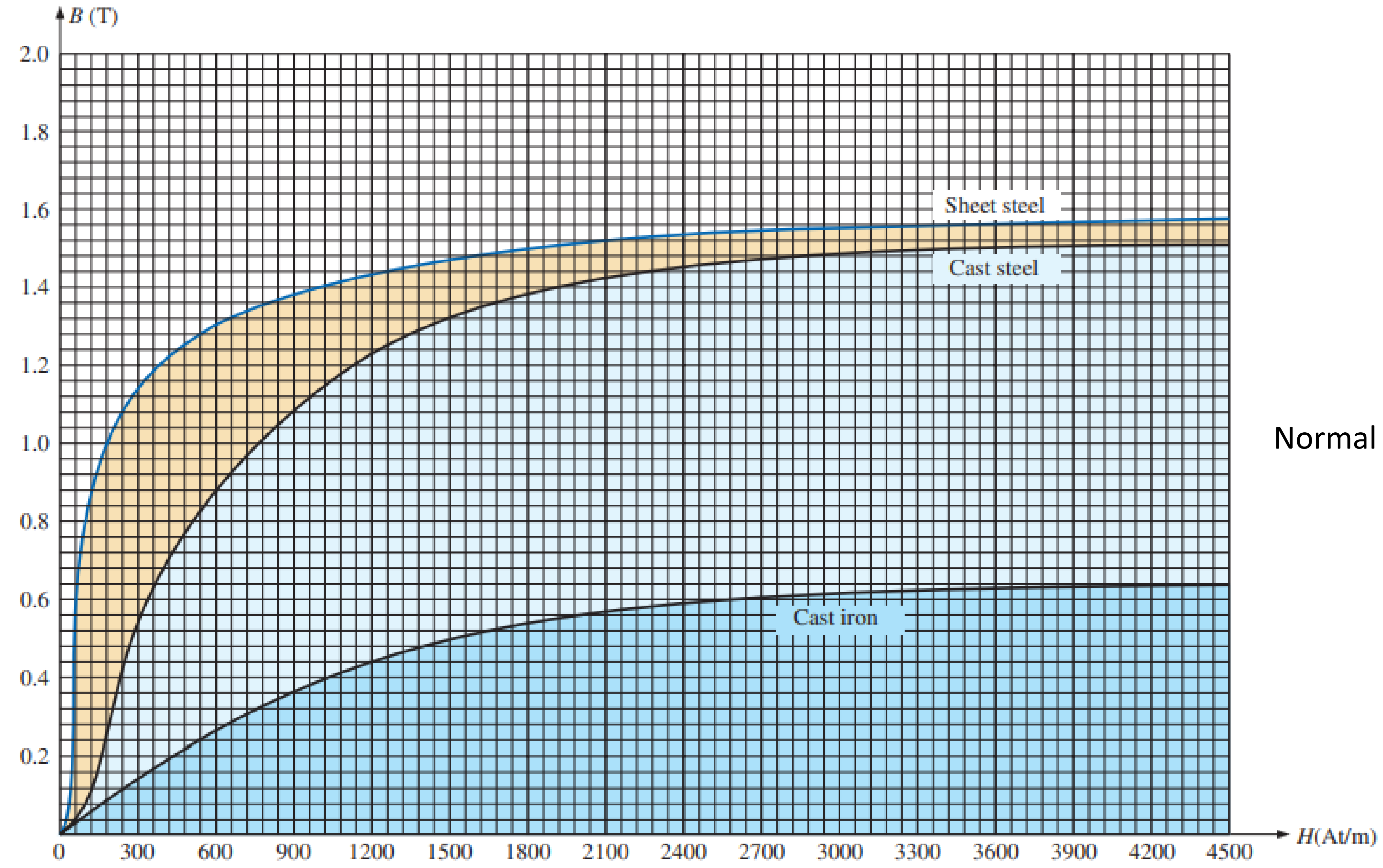
Variation of μ with the *H*

Hysteresis



Saturation
b
B (T)
a
c
B_R
B_max
– H_s
d
H (NI/l)
– H_d
o
H_a
H_s
– B_max
– B_R
f
e
Saturation

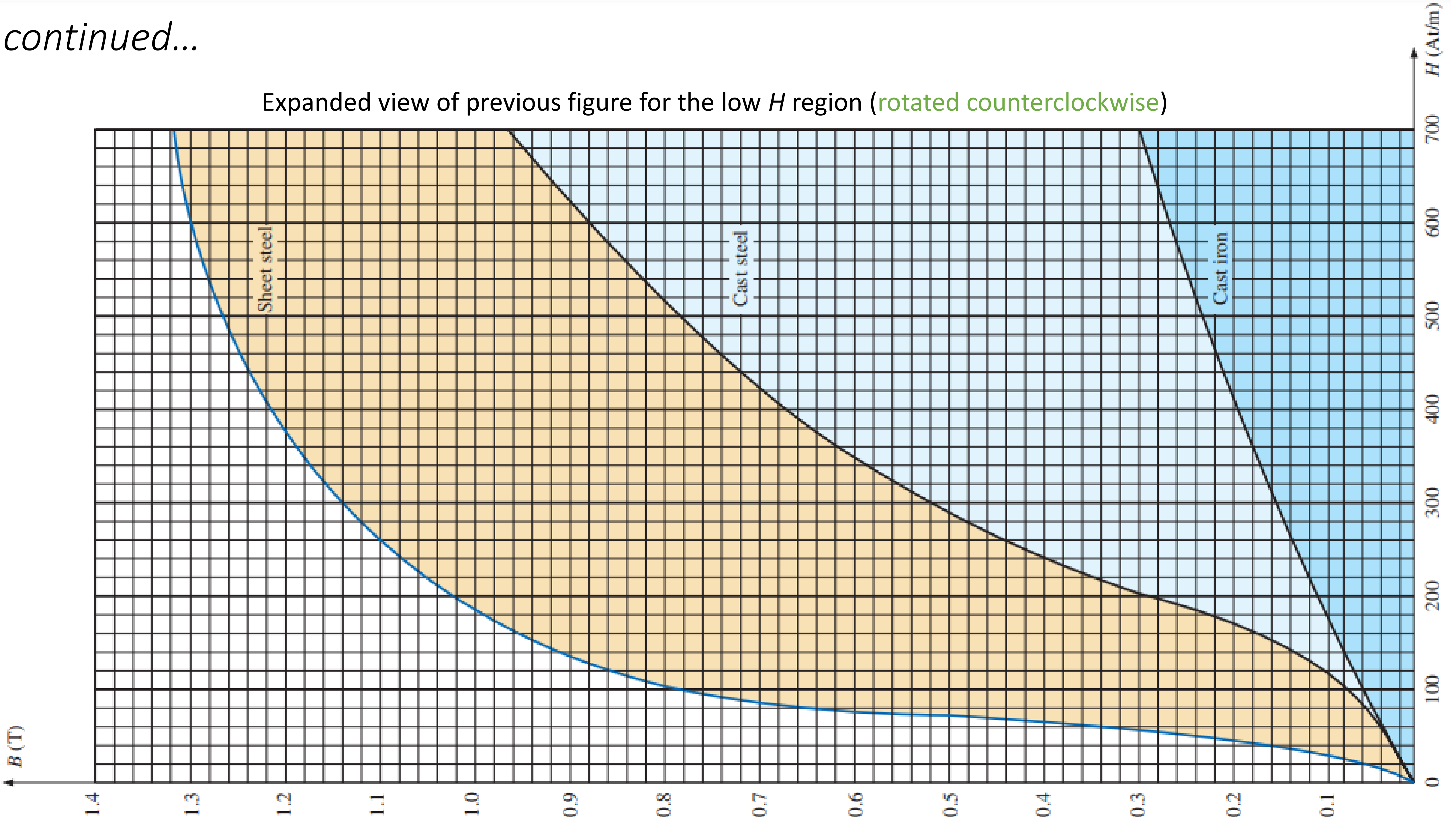
continued...



Normal magnetization curve

continued…

Expanded view of previous figure for the low *H* region (rotated counterclockwise)



Ampère’s Circuital Law

- Algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero.

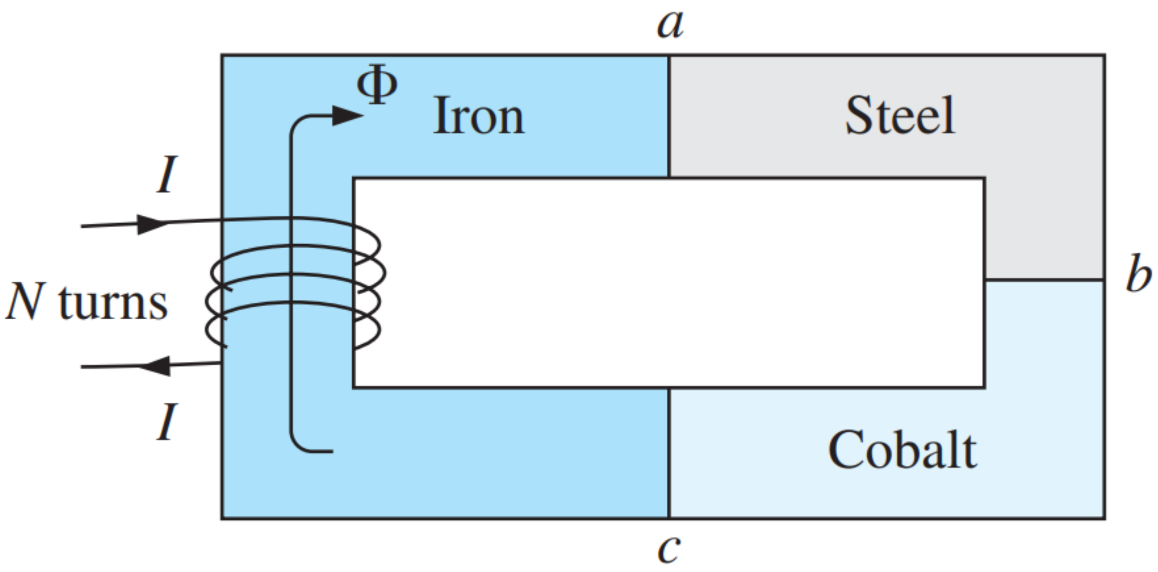
or

- Sum of the rises in mmf equals the sum of the drops in mmf around a closed loop.

$$\Sigma_{\circlearrowright} \mathcal{F} = 0$$

	Electric Circuits	Magnetic Circuits
Cause	E	$\mathcal{F}$
Effect	I	Φ
Opposition	R	$\mathcal{R}$

continued…



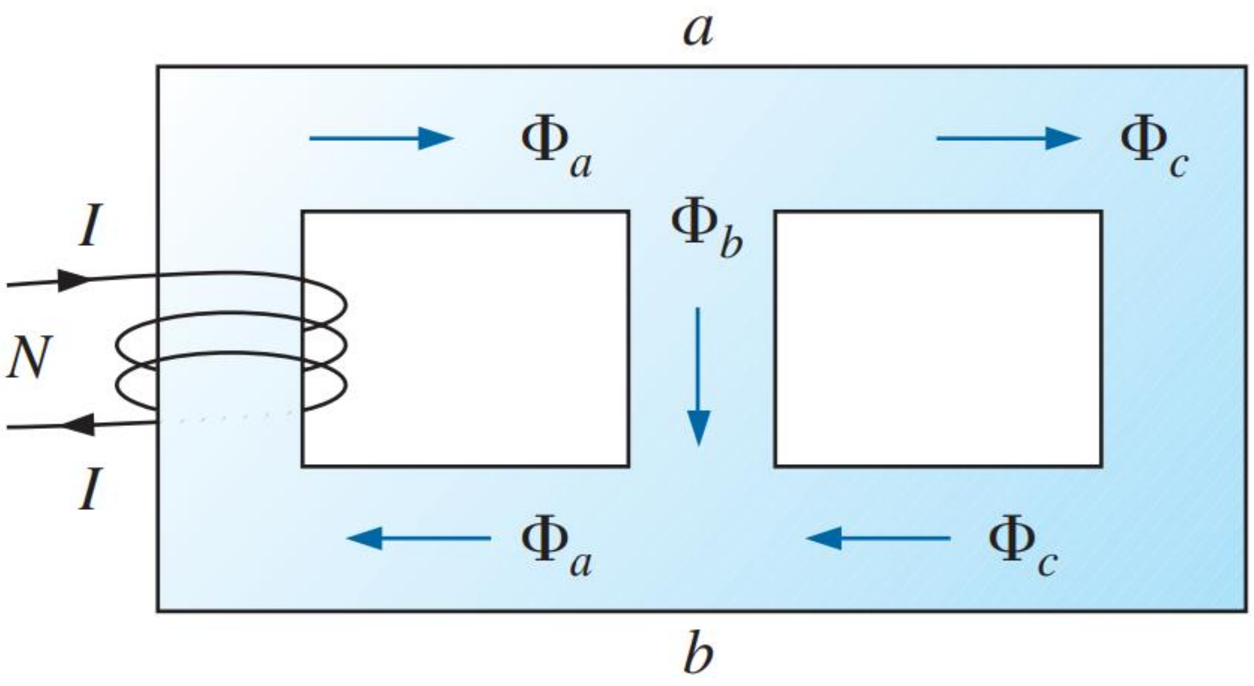
$$\Sigma_{\circlearrowright} \mathscr{F} = 0$$

$$\underbrace{+NI}_{\text{Rise}} - \underbrace{H_{ab}l_{ab}}_{\text{Drop}} - \underbrace{H_{bc}l_{bc}}_{\text{Drop}} - \underbrace{H_{ca}l_{ca}}_{\text{Drop}} = 0$$

$$\underbrace{NI}_{\substack{\text{Impressed}\\ \text{mmf}}} = \underbrace{H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca}}_{\text{mmf drops}}$$

Flux: remember KCL?

$$\sum \text{fluxes entering a junction} = \sum \text{fluxes leaving a junction}$$



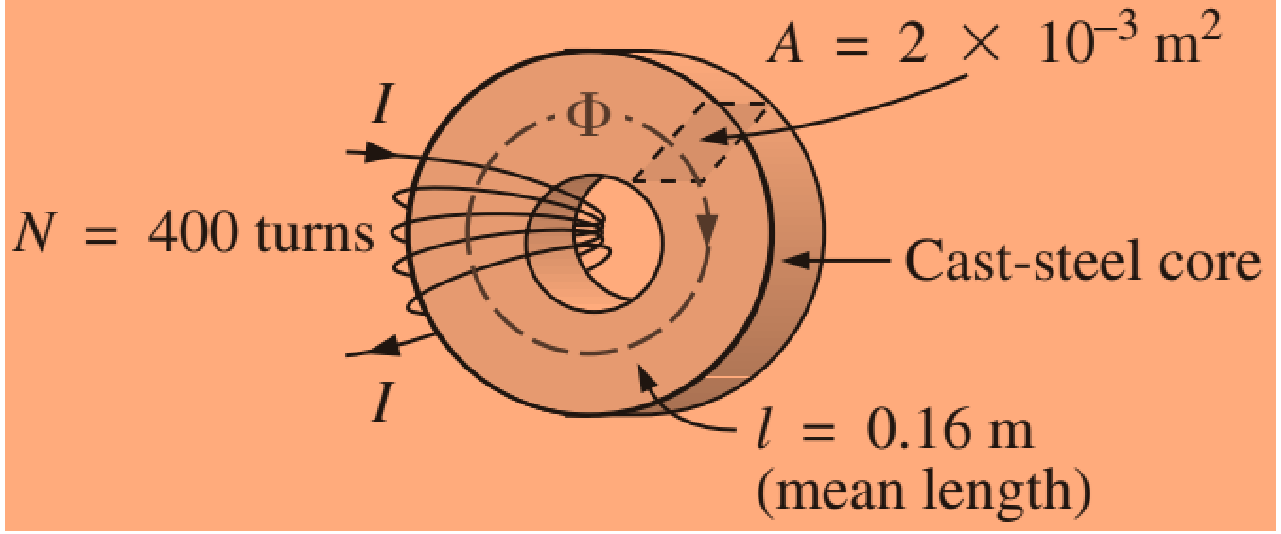
$$\Phi_a = \Phi_b + \Phi_c \qquad \text{(at junction } a\text{)}$$

$$\Phi_b + \Phi_c = \Phi_a \qquad \text{(at junction } b\text{)}$$

Example

a. Find the value of I required to develop a magnetic flux of $\Phi = 4 \times 10^{-4}$ Wb.

b. Determine μ and μ_r for the material under these conditions.



a. $$B = \frac{\Phi}{A} = \frac{4 \times 10^{-4}\ \text{Wb}}{2 \times 10^{-3}\ \text{m}^2} = 2 \times 10^{-1}\ \text{T} = 0.2\ \text{T}$$

Using the B-H curves

$$H\ (\text{cast steel}) = 170\ \text{At/m}$$

Applying Ampère's circuital law

$$NI = Hl \longrightarrow I = \frac{Hl}{N} = \frac{(170\ \text{At/m})(0.16\ \text{m})}{400\ \text{t}} = \mathbf{68\ mA}$$

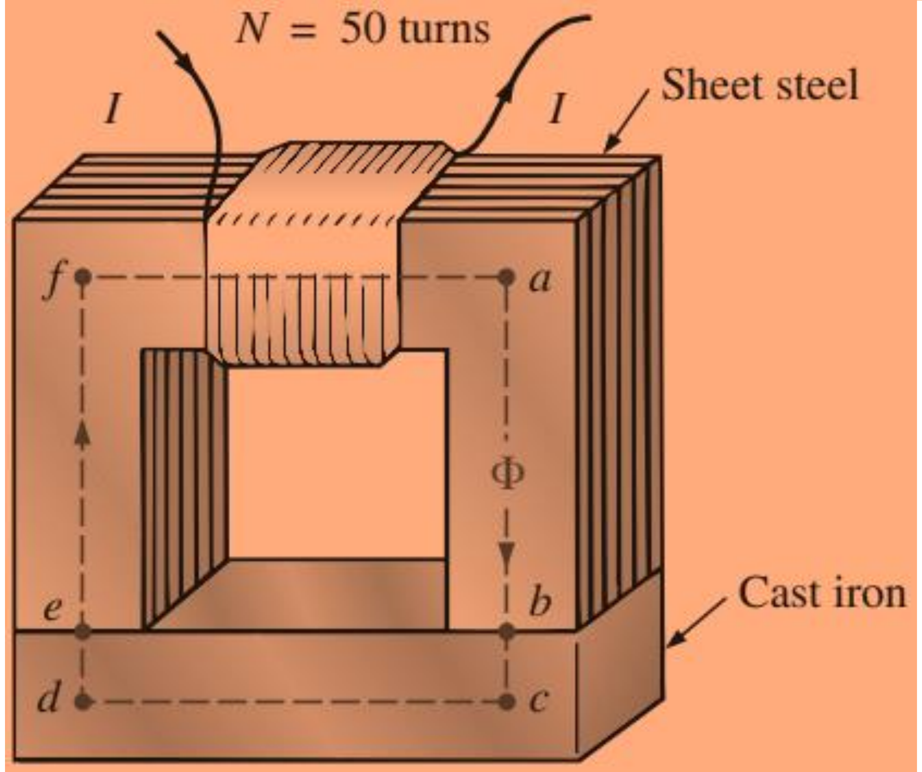
continued…

b. $$\mu = \frac{B}{H} = \frac{0.2\ \text{T}}{170\ \text{At/m}} = \mathbf{1.18 \times 10^{-3}\ Wb/Am}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{1.18 \times 10^{-3}}{4\pi \times 10^{-7}} = \mathbf{939.01}$$

continued…

Determine the current I required to establish the indicated flux in the core.



$l_{ab} = l_{cd} = l_{ef} = l_{fa} = 4$ in.
$l_{bc} = l_{de} = 0.5$ in.
Area (throughout) $= 1$ in.^{2}
$\Phi = 3.5 \times 10^{-4}$ Wb

$$l_{efab} = 4 \text{ in.} + 4 \text{ in.} + 4 \text{ in.} = 12 \text{ in.}$$
$$= 304.8 \times 10^{-3} \text{ m}$$

$$l_{bcde} = 0.5 \text{ in.} + 4 \text{ in.} + 0.5 \text{ in.}$$
$$= 5 \text{ in.} = 127 \times 10^{-3} \text{ m}$$

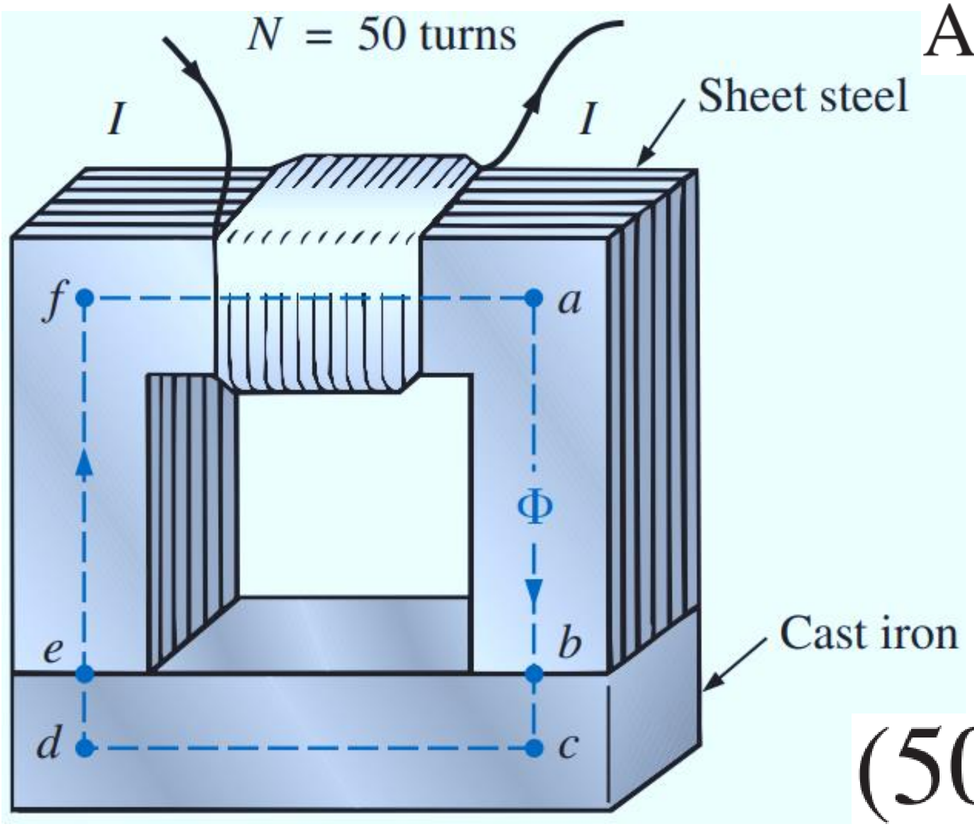
$$\text{Area (throughout)} = 1 \text{ in.}^2$$
$$= 6.45 \times 10^{-4} \text{ m}^2$$

$$B = \frac{\Phi}{A} = \frac{3.5 \times 10^{-4} \text{ Wb}}{6.45 \times 10^{-4} \text{ m}^2} = 0.54 \text{ T}$$

continued…

H (sheet steel, Fig. *B-H*) $\cong$ 70 At/m

H (cast iron, Fig. *B-H*) $\cong$ 1600 At/m

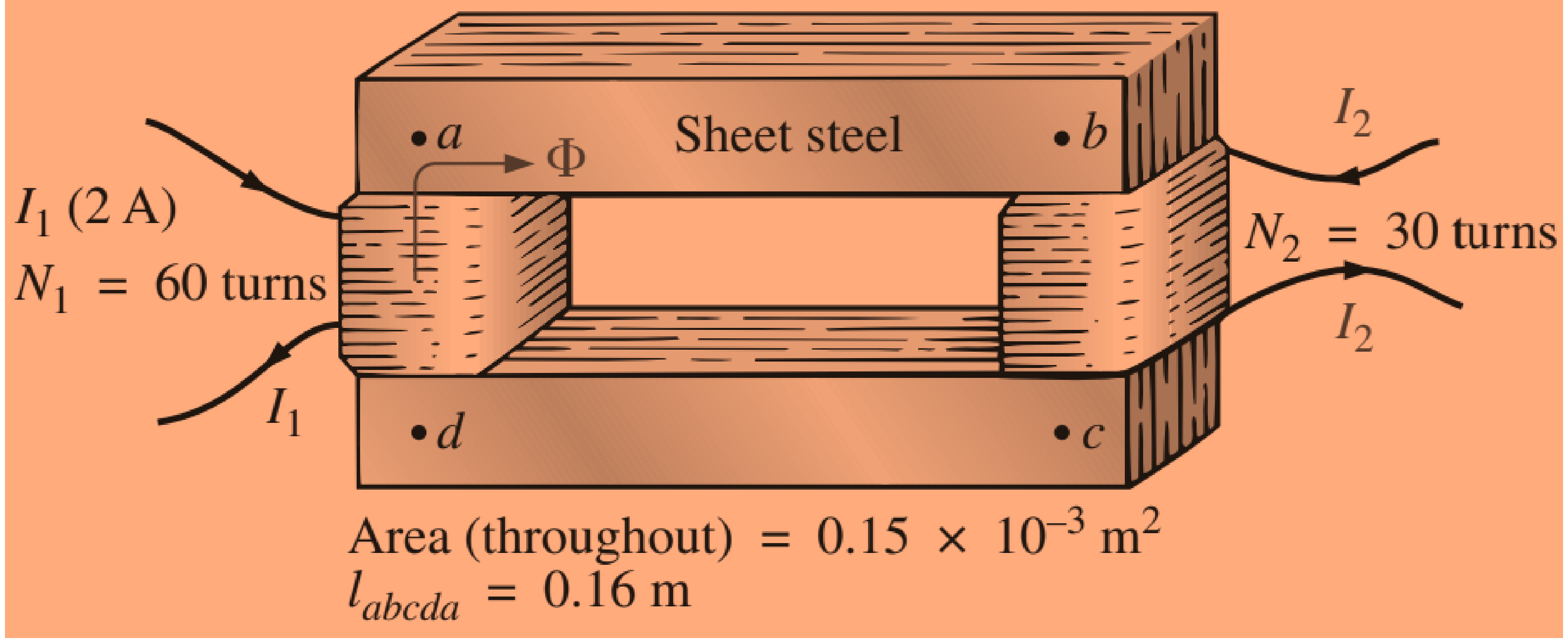


Applying Ampère's circuital law

$$
\begin{aligned}
NI &= H_{efab}l_{efab} + H_{bcde}l_{bcde} \\
&= (70\ \text{At/m})(304.8 \times 10^{-3}\ \text{m}) \\
&\quad + (1600\ \text{At/m})(127 \times 10^{-3}\ \text{m}) \\
&= 21.34\ \text{At} + 203.2\ \text{At} \\
&= 224.54\ \text{At} \\
(50\ \text{t})I &= 224.54\ \text{At} \\
I &= \frac{224.54\ \text{At}}{50\ \text{t}} = \mathbf{4.49\ A}
\end{aligned}
$$

continued…

Determine the secondary current I_2 for the transformer if the resultant clockwise flux in the core is 1.5×10^{-5} Wb.



$$B = \frac{\phi}{A} = \frac{1.5 \times 10^{-5}\ \text{Wb}}{0.15 \times 10^{-3}\ \text{m}^2} = 0.10\ \text{T}$$

Using the *B-H* curves → H (Sheet steel) $\cong$ 30 At/m

Applying Ampère's circuital law

$$N_1 I_1 = H_{abcda} l_{abcda} + N_2 I_2$$

$$\Longrightarrow (60\ \text{t})(2\ \text{A}) = (30\ \text{At/m})(0.16\ \text{m}) + (30\ \text{t})(I_2)$$

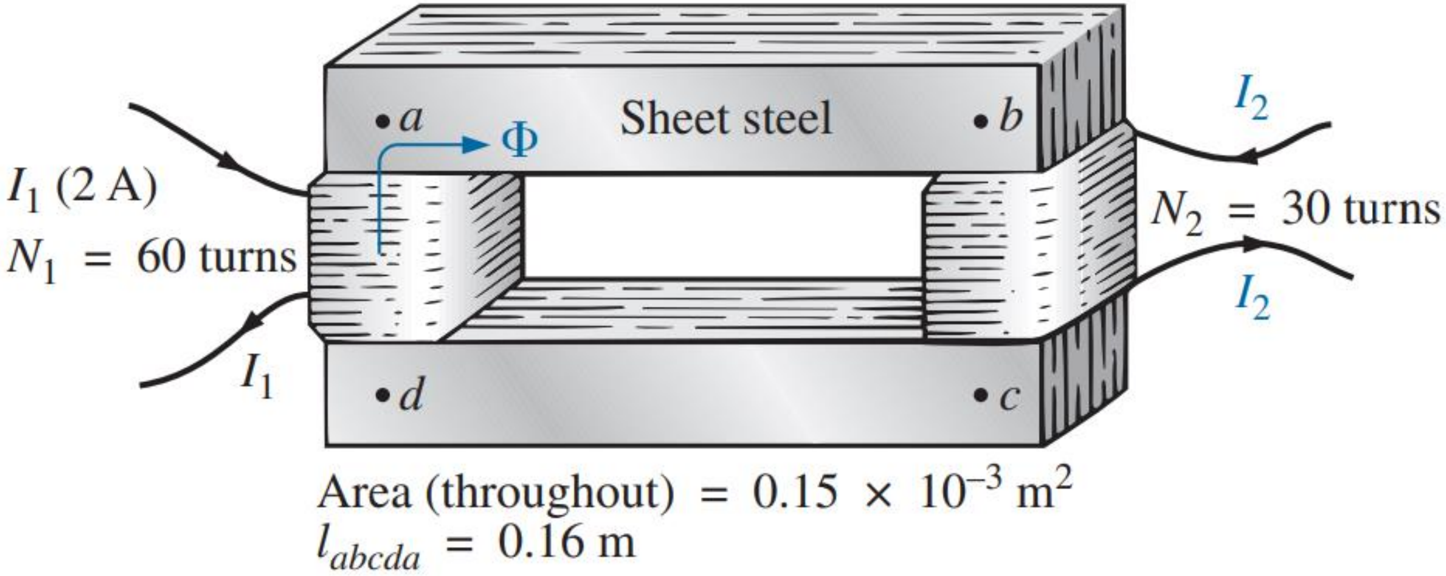
continued…

$$\Longrightarrow 120\text{ At} = 4.8\text{ At} + (30\text{ t})(I_2)$$

$$\Longrightarrow (30\text{ t})(I_2) = 120\text{ At} - 4.8\text{ At}$$

$$\Longrightarrow I_2 = \frac{115.2\text{ At}}{30\text{ t}}$$

$$\Longrightarrow I_2 = 3.84\text{ A}$$



__Note:__ Because of the nonlinearity of the *B-H* curve, it is **not** possible to apply superposition to magnetic circuits.