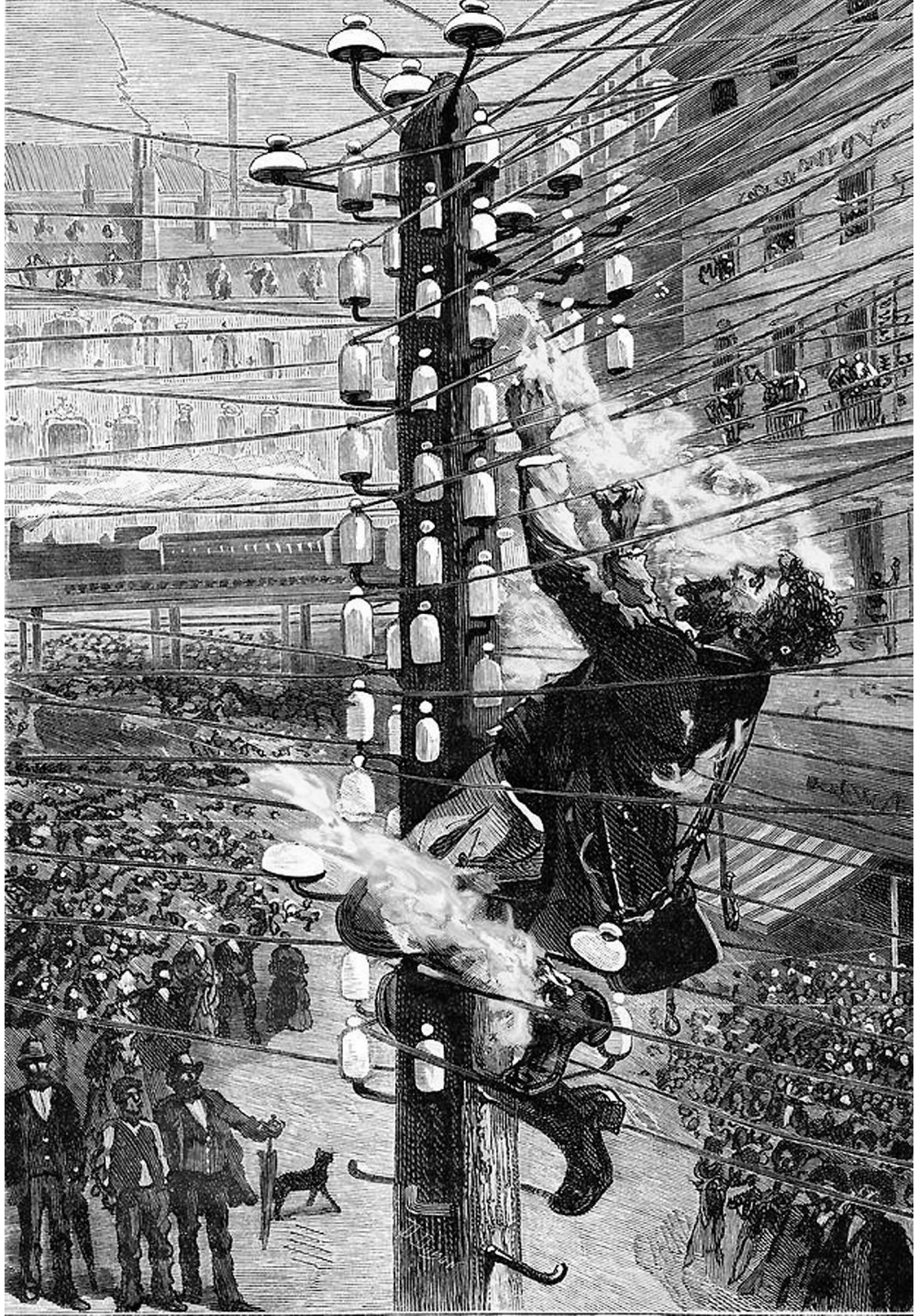
EEE 103

(Part 1)

Electrical Circuits II

Dept. of EEE, BUBT

1889 engraving by William Allen Rogers for Harper's Weekly magazine. GRAND STREET, NEW YORK, AT NIGHT



October 11, 1889. The death of John Feeks. Manhattan, New York.

World's Columbian Exposition, 1893, Chicago

Sinusoids

$$v(t) = V_m \sin(\omega t + \phi)$$

V_m = the *amplitude* of the sinusoid

ω = the *angular frequency* in radians/s

$(\omega t + \phi)$ is the argument

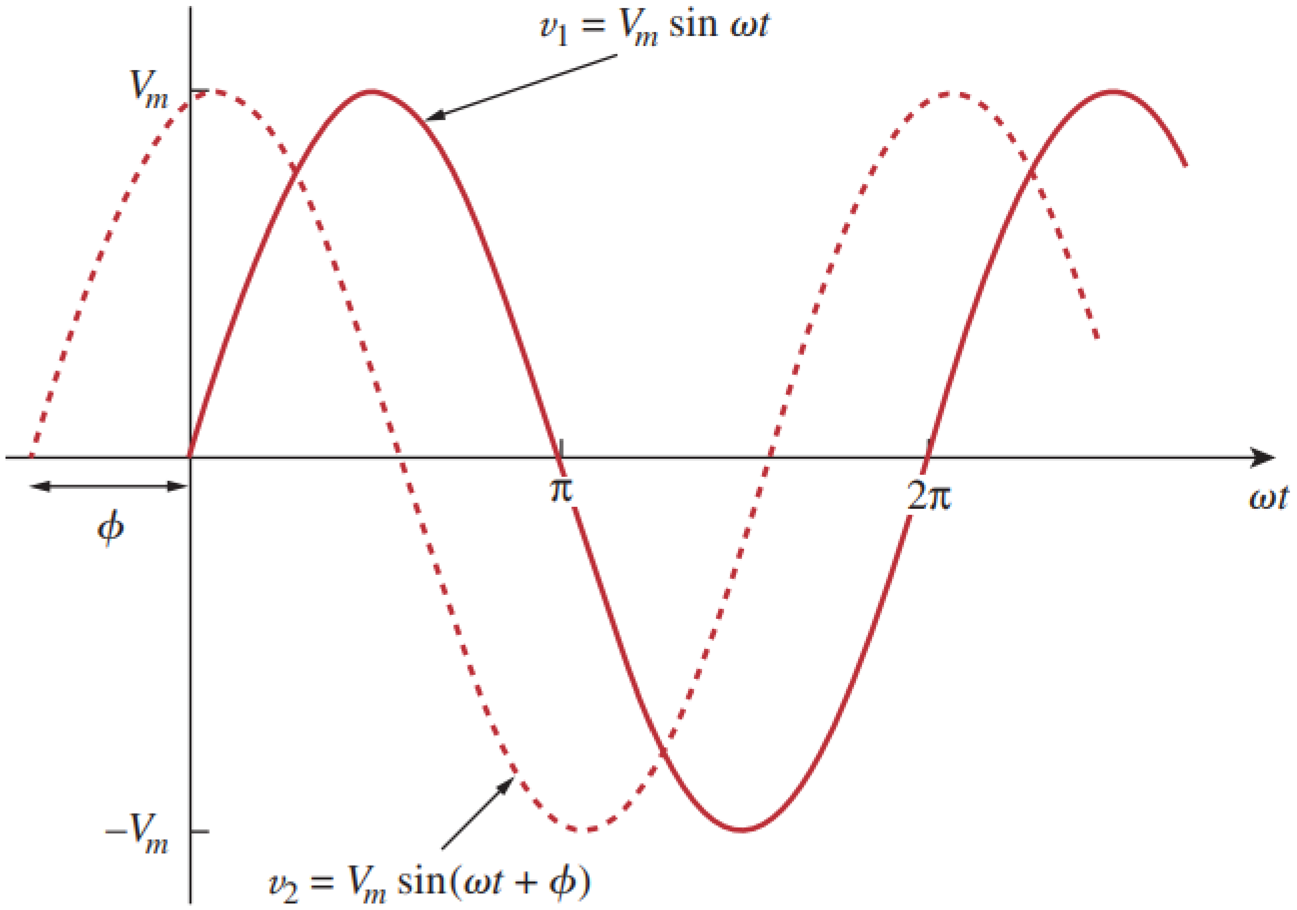
ϕ is the *phase*

$$T = \frac{2\pi}{\omega}$$

$$v(t + T) = v(t)$$

$$f = \frac{1}{T}$$

$$\omega = 2\pi f$$



Example

Find the amplitude, phase, period, and frequency of the sinusoid $v(t) = 12\cos(50t + 10°)$

The amplitude is $V_m = 12$ V.

The phase is $\phi = 10°$.

The angular frequency is $\omega = 50$ rad/s.

The period $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{50} = 0.1257$ s.

The frequency is $f = \dfrac{1}{T} = 7.958$ Hz.

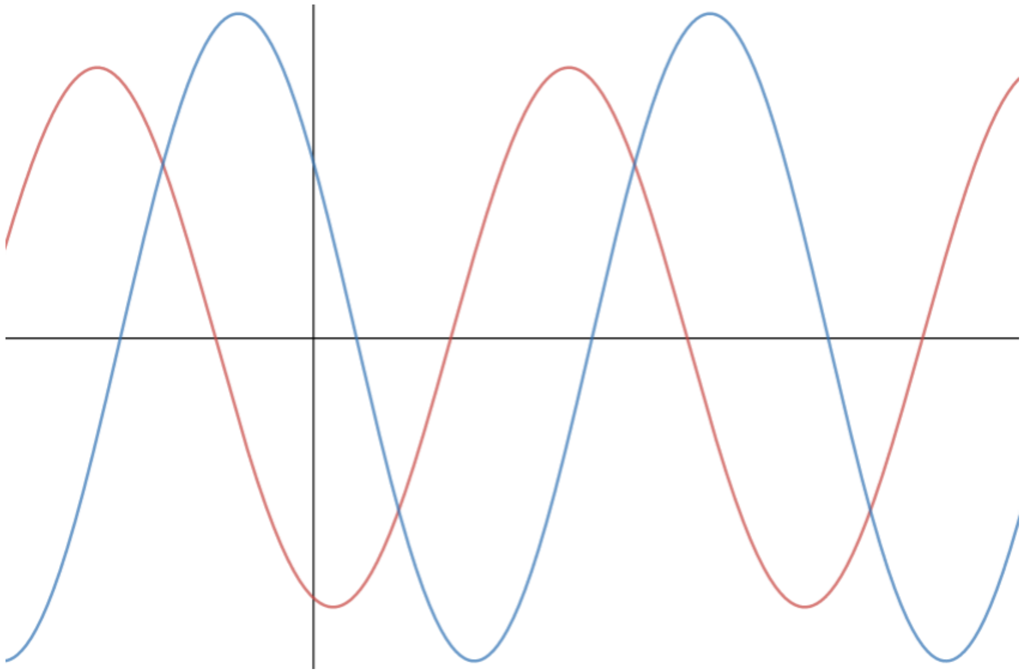
continued...

Calculate the phase angle between $v_1 = -10\cos(\omega t + 50°)$ and $v_2 = 12\sin(\omega t - 10°)$. State which sinusoid is leading.

$$v_1 = -10\cos(\omega t + 50°)$$
$$= 10\sin(\omega t + 50° - 90°)$$
$$= 10\sin(\omega t - 40°)$$
$$v_2 = 12\sin(\omega t - 10°)$$

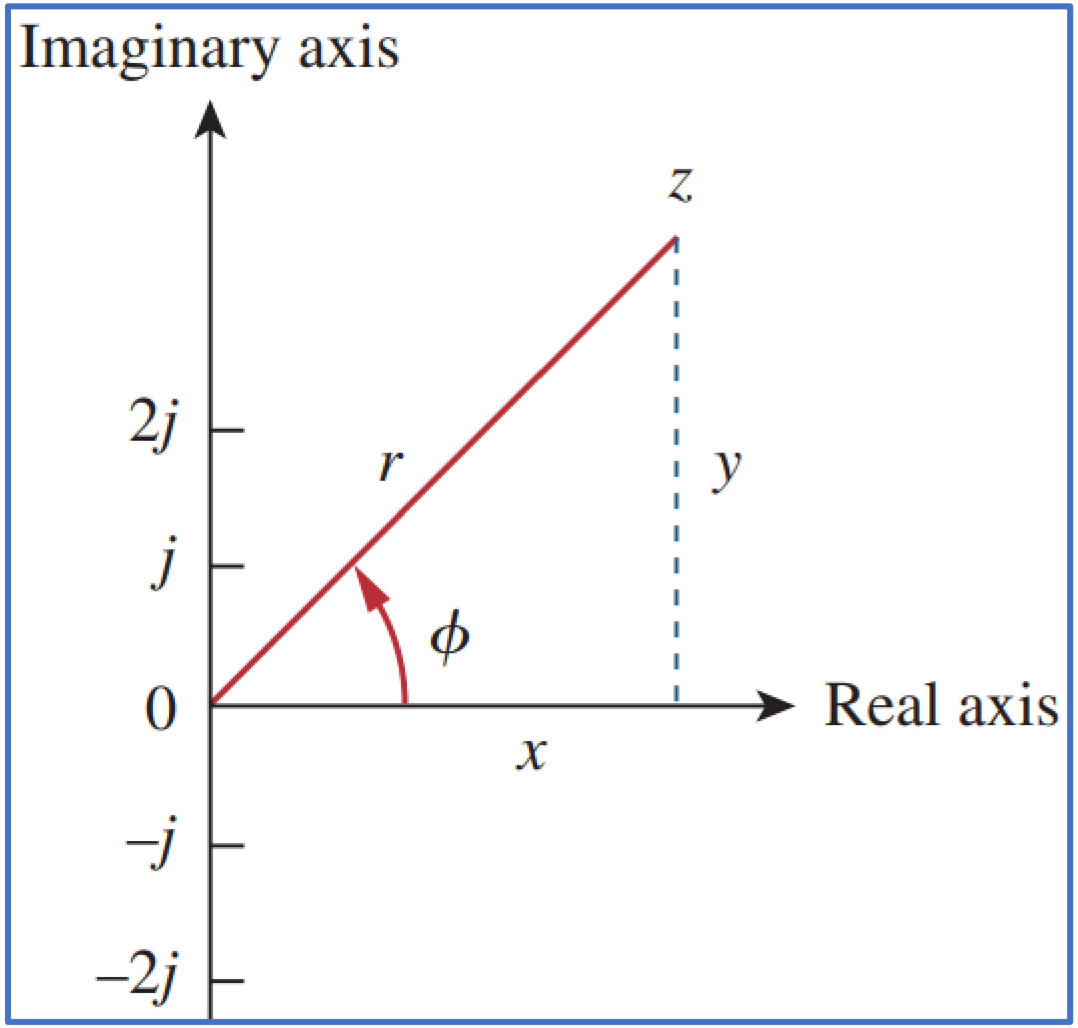
v_2 leads v_1 by 30°.

$$\sin(\omega t \pm 180°) = -\sin\omega t$$
$$\cos(\omega t \pm 180°) = -\cos\omega t$$
$$\sin(\omega t \pm 90°) = \pm\cos\omega t$$
$$\cos(\omega t \pm 90°) = \mp\sin\omega t$$



Red: v1
Blue: v2

Phasors



$$z = x + jy \qquad \text{Rectangular form}$$

$$z = r\angle\phi \qquad \text{Polar form}$$

$$z = re^{j\phi} \qquad \text{Exponential form}$$

$$r = \sqrt{x^2 + y^2}, \qquad \phi = \tan^{-1}\frac{y}{x}$$

$$x = r\cos\phi, \qquad y = r\sin\phi$$

$$\frac{1}{j} = -j$$

Some Basic Operations of **Complex Number**

Given the complex numbers

$$z = x + jy = r\angle\phi$$

$$z_1 = x_1 + jy_1 = r_1\angle\phi_1$$

$$z_2 = x_2 + jy_2 = r_2\angle\phi_2$$

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

$$z_1 z_2 = r_1 r_2 \angle\phi_1 + \phi_2$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle\phi_1 - \phi_2$$

$$\frac{1}{z} = \frac{1}{r}\angle-\phi$$

$$\sqrt{z} = \sqrt{r}\angle\phi/2$$

$$z^* = x - jy = r\angle-\phi = re^{-j\phi}$$

Euler's Identity and Phasor Representation

$$e^{\pm j\phi} = \cos\phi \pm j\sin\phi$$

$$\cos\phi = \mathrm{Re}(e^{j\phi})$$

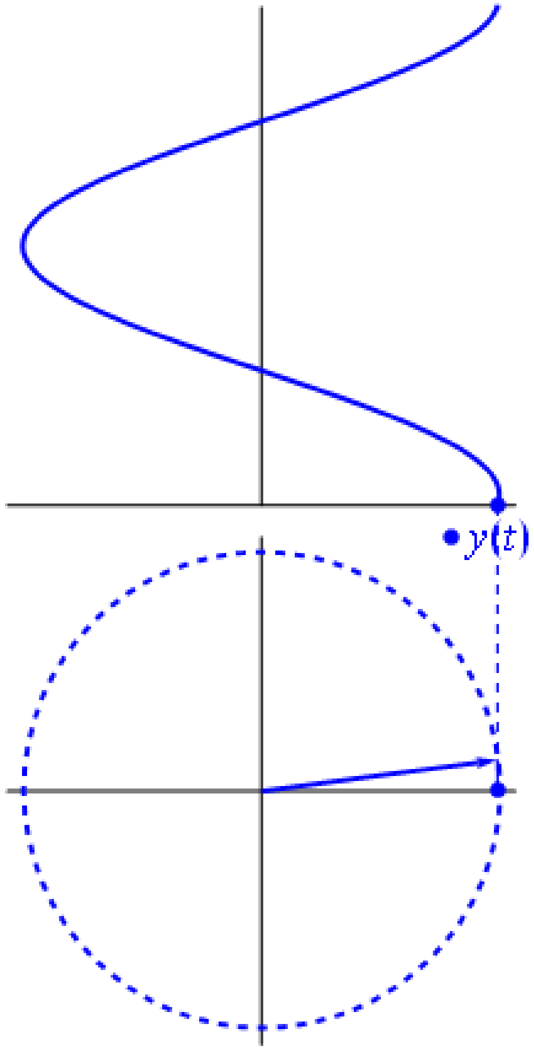
$$\sin\phi = \mathrm{Im}(e^{j\phi})$$

$$v(t) = V_m\cos(\omega t + \phi) = \mathrm{Re}(V_m e^{j(\omega t+\phi)})$$

$$= \mathrm{Re}(V_m e^{j\phi} e^{j\omega t})$$

$$\mathbf{V} = V_m e^{j\phi} = V_m \underline{/\phi}$$

$$v(t) = \mathrm{Re}(\mathbf{V}e^{j\omega t})$$



continued…

$$v(t) = V_m \cos(\omega t + \phi) \quad \Leftrightarrow \quad \mathbf{V} = V_m \angle \phi$$

(Time-domain representation) (Phasor-domain representation)

- $v(t)$ is time-domain representation, while $\mathbf{V}$ is frequency or phasor-domain representation.
- $v(t)$ is time dependent, while $\mathbf{V}$ is not.
- $v(t)$ is always real, while $\mathbf{V}$ is generally complex.

$$\frac{dv}{dt} \quad \Leftrightarrow \quad j\omega\mathbf{V}$$

(Time domain) (Phasor domain)

$$\int v\, dt \quad \Leftrightarrow \quad \frac{\mathbf{V}}{j\omega}$$

(Time domain) (Phasor domain)

Example

Transform these sinusoids to phasors:
(a) $i = 6\cos(50t - 40^\circ)$ A
(b) $v = -4\sin(30t + 50^\circ)$ V

(a) $\mathbf{I} = 6\underline{/-40^\circ}$ A

(b)
$$\begin{aligned} v &= -4\sin(30t + 50^\circ) \\ &= 4\cos(30t + 50^\circ + 90^\circ) \\ &= 4\cos(30t + 140^\circ) \end{aligned}$$

$$\mathbf{V} = 4\underline{/140^\circ}\ \text{V}$$

continued…

Find the sinusoids represented by these phasors:

(a) $\mathbf{I} = -3 + j4$ A

(b) $\mathbf{V} = j8e^{-j20^\circ}$ V

(a) $\mathbf{I} = -3 + j4 = 5\angle 126.87^\circ$

$$i(t) = 5\cos(\omega t + 126.87^\circ) \text{ A}$$

(b) $\mathbf{V} = j8\angle -20^\circ = (1\angle 90^\circ)(8\angle -20^\circ)$

$$= 8\angle 90^\circ - 20^\circ$$

$$= 8\angle 70^\circ \text{ V}$$

$$v(t) = 8\cos(\omega t + 70^\circ) \text{ V}$$

continued…

Using the phasor approach, determine the current $i(t)$ in a circuit described by the integrodifferential equation

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75^\circ)$$

$$4\mathbf{I} + \frac{8\mathbf{I}}{j\omega} - 3j\omega\mathbf{I} = 50\angle 75^\circ$$

But $\omega = 2$

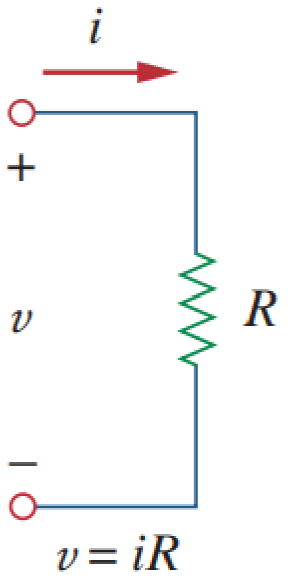
$$\mathbf{I}(4 - j4 - j6) = 50\angle 75^\circ$$

$$\mathbf{I} = \frac{50\angle 75^\circ}{4 - j10} = \frac{50\angle 75^\circ}{10.77\angle -68.2^\circ} = 4.642\angle 143.2^\circ \text{ A}$$

$$i(t) = 4.642\cos(2t + 143.2^\circ) \text{ A}$$

Phasor with Elements

(1) Resistor:



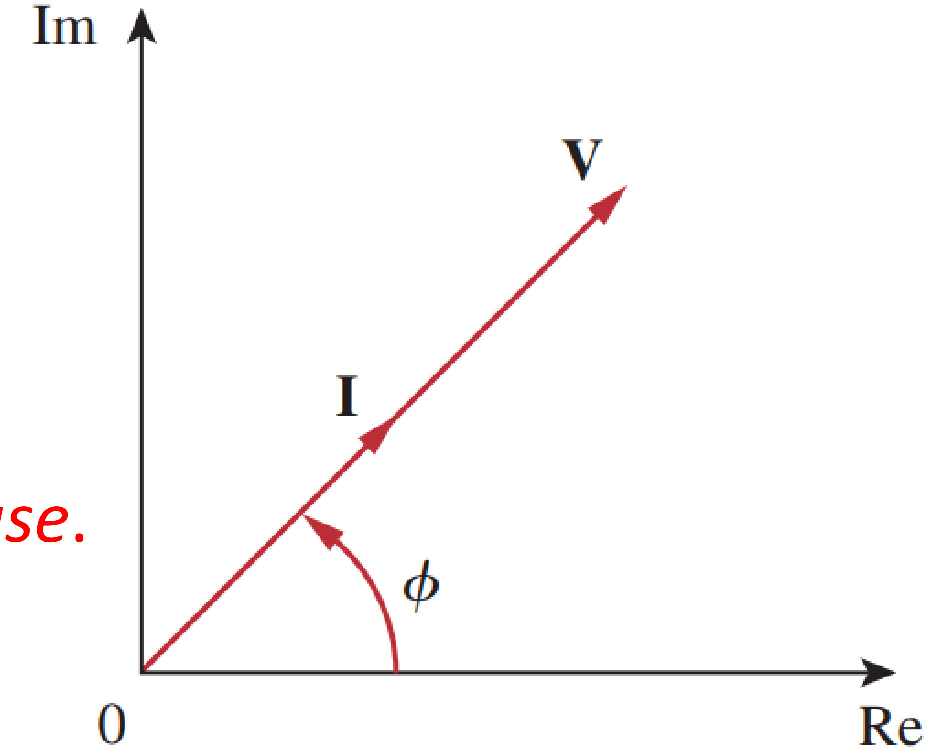
$$i = I_m \cos(\omega t + \phi) \quad \Big| \quad I_m \angle \phi = \mathbf{I}$$

$$v = iR$$

$$= RI_m \cos(\omega t + \phi)$$

$$\mathbf{V} = RI_m \angle \phi$$

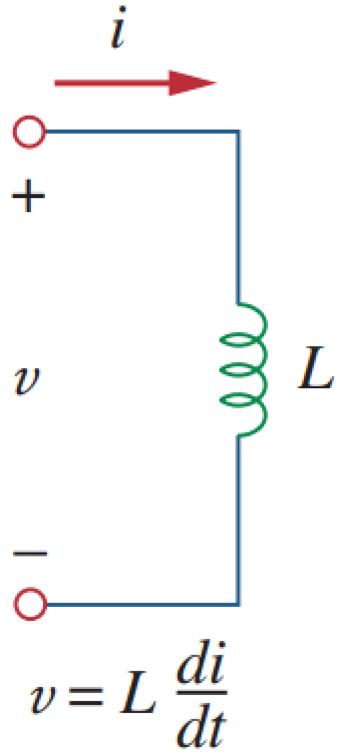
$$\mathbf{V} = R\mathbf{I}$$



Current and Voltage are *in phase*.

continued…

(2) Inductor:



$$i = I_m \cos(\omega t + \phi)$$

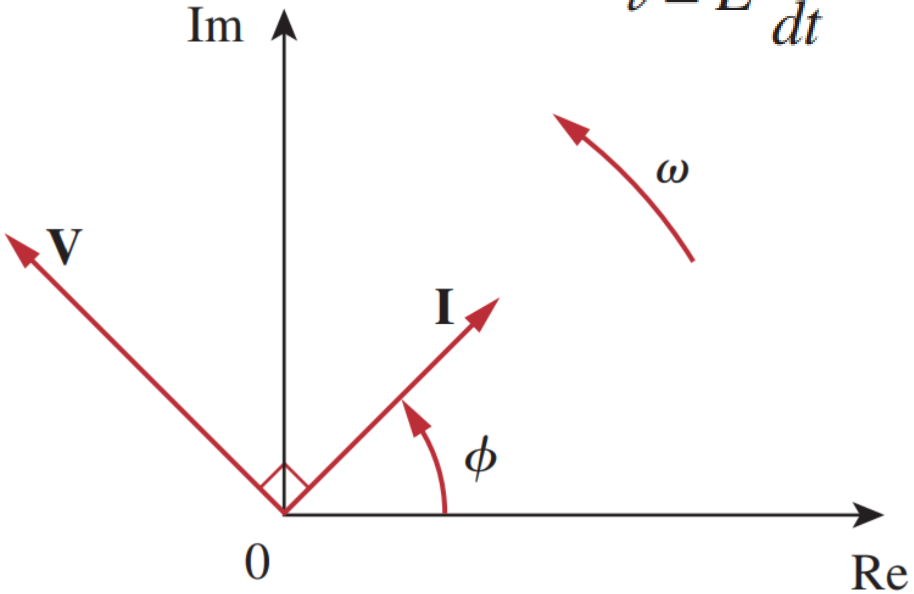
$$v = L\frac{di}{dt}$$

$$= -\omega L I_m \sin(\omega t + \phi)$$

$$= \omega L I_m \cos(\omega t + \phi + 90^\circ)$$

$$= \omega L I_m \angle \phi + 90^\circ$$

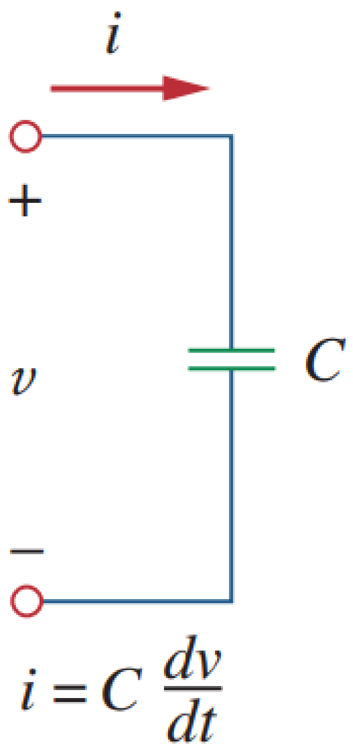
$$\mathbf{V} = j\omega L \mathbf{I}$$



Current *lags* Voltage by 90°

continued…

(3) Capacitor:

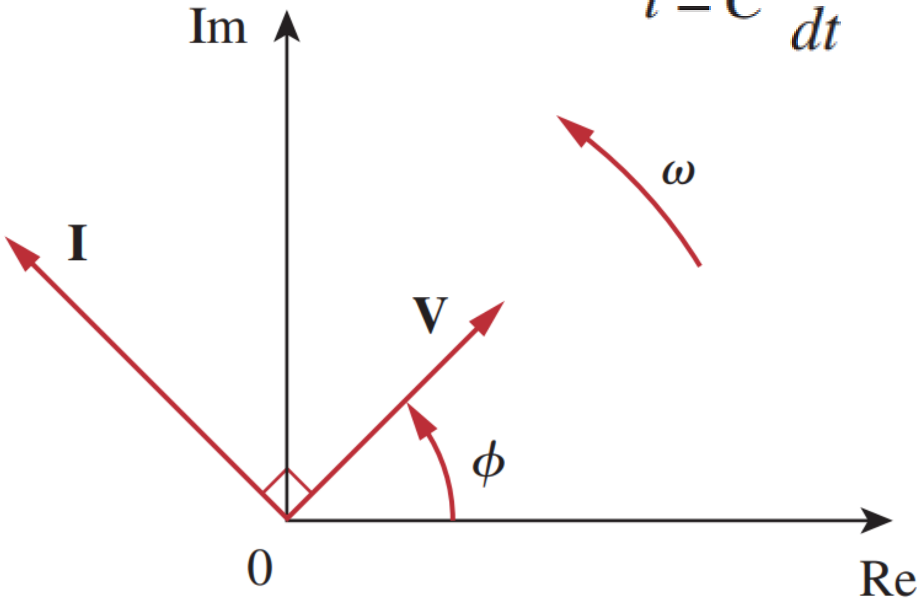


$$v(t) = V_m \sin(\omega t + \phi)$$

$$i = C\frac{dv}{dt}$$

$$\mathbf{I} = j\omega C\mathbf{V}$$

$$\mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$



Current *leads* Voltage by 90°

Example

The voltage $v = 12\cos(60t + 45°)$ is applied to a 0.1-H inductor. Find the steady-state current through the inductor.

$\omega = 60$ rad/s $\qquad\qquad \mathbf{V} = 12\underline{/45°}$ V

For the inductor, $\mathbf{V} = j\omega L\mathbf{I}$

$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{12\underline{/45°}}{j60 \times 0.1} = \frac{12\underline{/45°}}{6\underline{/90°}}$$

$$= 2\underline{/-45°} \text{ A}$$

$$i(t) = 2\cos(60t - 45°) \text{ A}$$

continued...

If voltage $v = 6\cos(100t - 30^\circ)$ is applied to a 50 μF capacitor, calculate the current through the capacitor.

$\omega = 100$ $\mathbf{V} = 6\angle{-30^\circ}$

For the capacitor, $\mathbf{V} = \mathbf{I} / (j\omega C)$

$$\mathbf{I} = j\omega C\,\mathbf{V}$$

$$= (j100)(50\text{x}10^{-6})(6\angle{-30^\circ})$$

$$= 30\angle 60^\circ \text{ mA}$$

$$i(t) = 30\cos(100t + 60^\circ)\text{ mA}$$

Impedance

$$\mathbf{V} = R\mathbf{I}, \qquad \mathbf{V} = j\omega L\mathbf{I}, \qquad \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$

$$\frac{\mathbf{V}}{\mathbf{I}} = R, \qquad \frac{\mathbf{V}}{\mathbf{I}} = j\omega L, \qquad \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{or} \quad \mathbf{V} = \mathbf{Z}\mathbf{I}$$

Impedance (*frequency-dependent*)
Unit: Ohm

continued…

$$\mathbf{Z} = R + jX$$

$R = \mathrm{Re}\,\mathbf{Z}$ is the *resistance*

$X = \mathrm{Im}\,\mathbf{Z}$ is the *reactance*

- Impedance $\mathbf{Z}$ is inductive (*lagging*) when X is positive.
- Impedance $\mathbf{Z}$ is capacitive (*leading*) when X is negative.

$$\mathbf{Z} = R + jX = |\mathbf{Z}|\underline{/\theta}$$

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}, \qquad \theta = \tan^{-1}\frac{X}{R}$$

$$R = |\mathbf{Z}|\cos\theta, \qquad X = |\mathbf{Z}|\sin\theta$$

KCL, KVL, Voltage-division, Current-division – all hold in **Phasor** domain too.

Admittance

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}}$$

$$\mathbf{Y} = G + jB$$

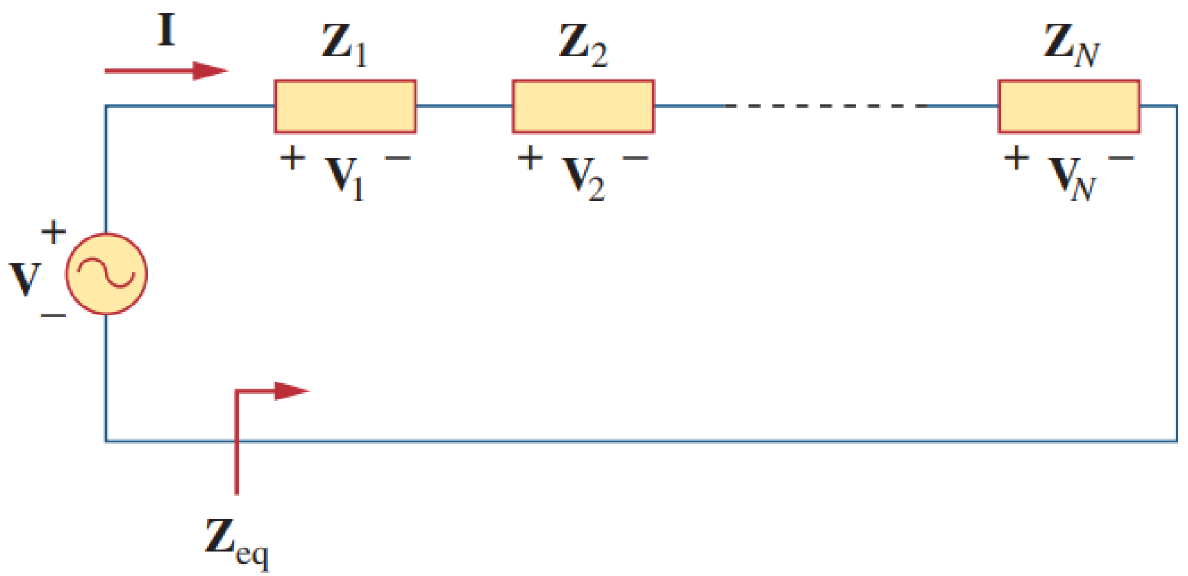
$G = \operatorname{Re} \mathbf{Y}$ is *conductance*

$B = \operatorname{Im} \mathbf{Y}$ is *susceptance*

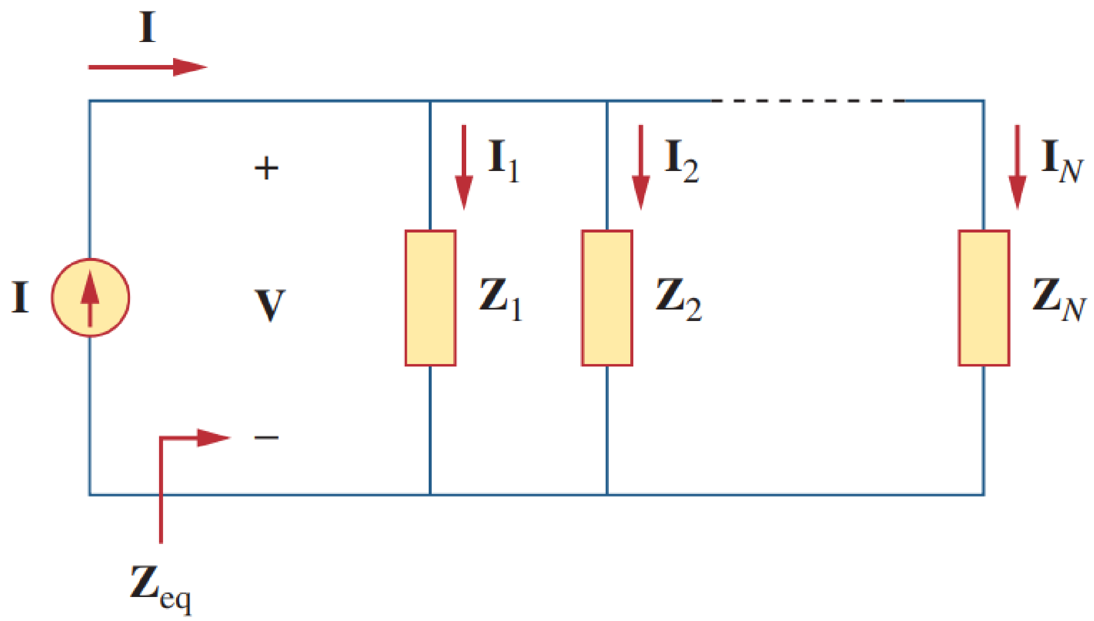
Unit: Siemens (S) *or* Mho (℧)

$$G + jB = \frac{1}{R + jX}$$

Series and Parallel Impedances



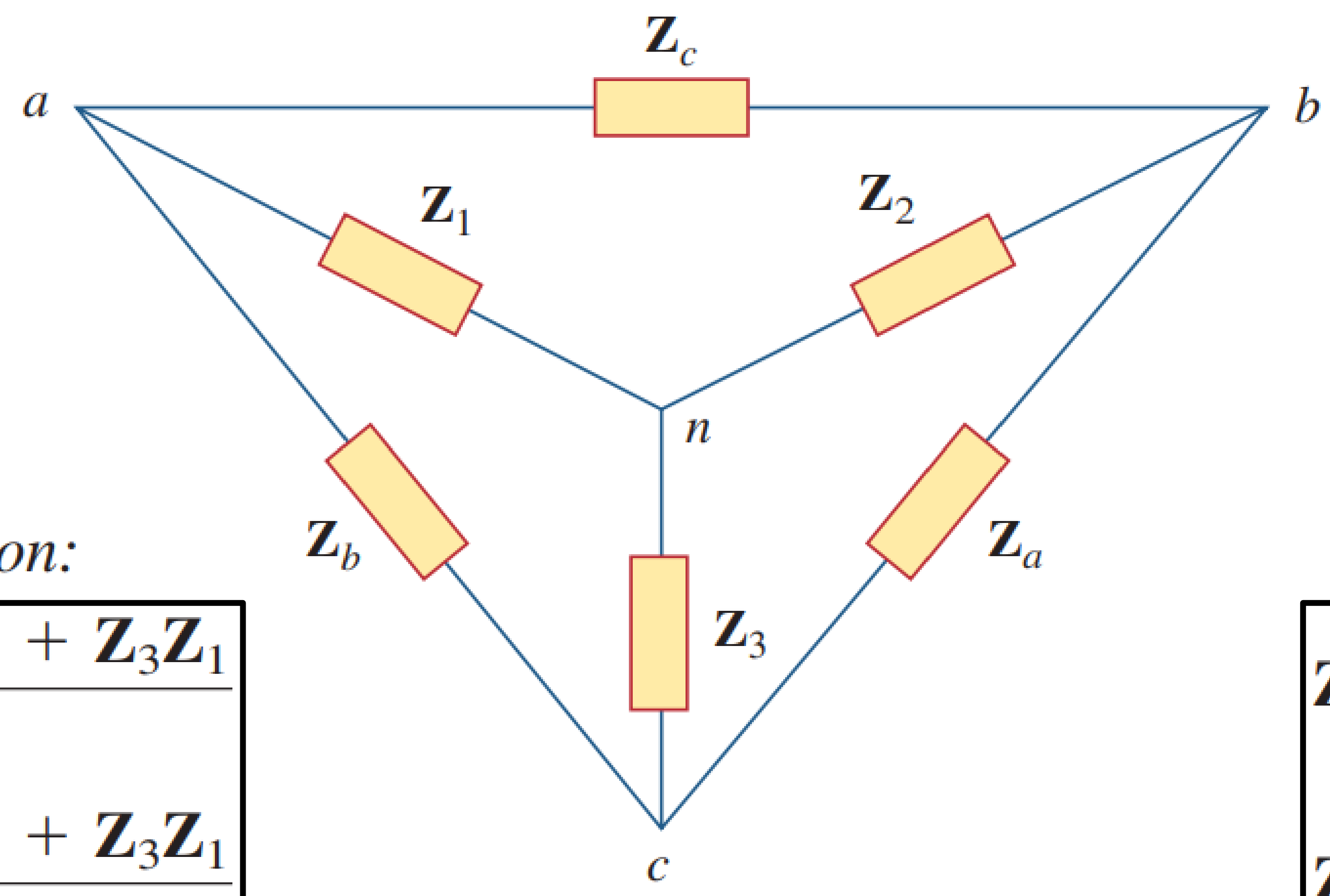
$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N$$



$$\frac{1}{\mathbf{Z}_{eq}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N}$$

$$\mathbf{Y}_{eq} = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_N$$

Y-Δ Transformations



Y-Δ Conversion:

$$\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$

$$\mathbf{Z}_c = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

Δ-Y Conversion:

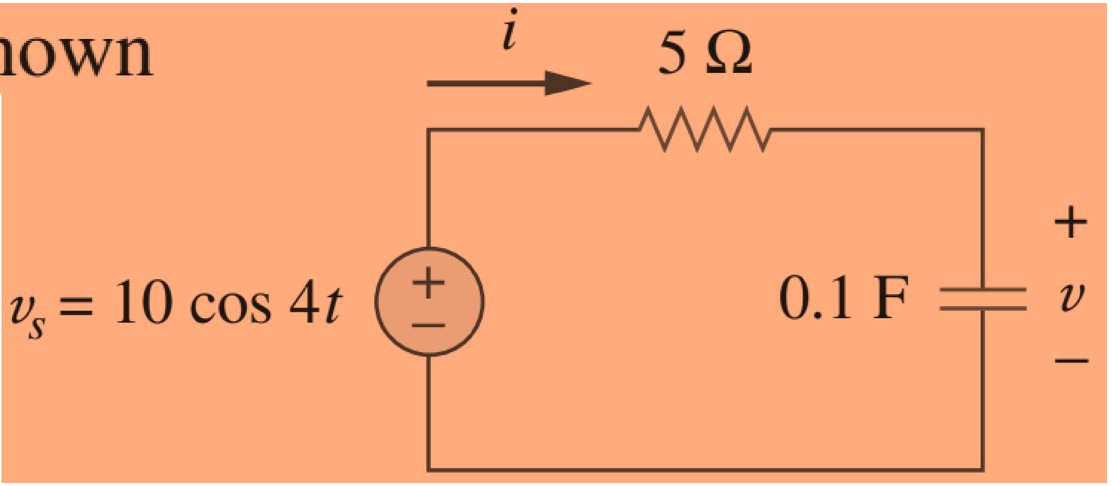
$$\mathbf{Z}_1 = \frac{\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_a\mathbf{Z}_b}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

Example

Find $v(t)$ and $i(t)$ in the circuit shown



$$\omega = 4$$

$$\mathbf{V}_s = 10\underline{/0^\circ}\text{ V}$$

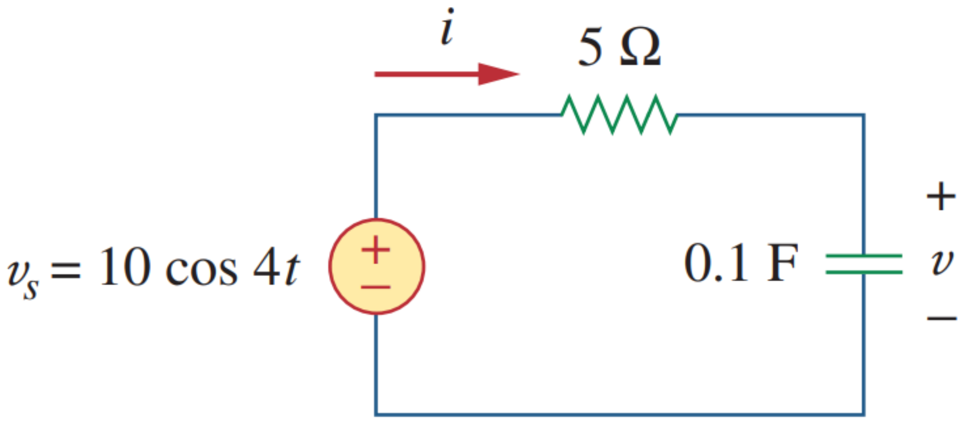
$$\mathbf{Z} = 5 + \frac{1}{j\omega C}$$

$$= 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5\ \Omega$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10\underline{/0^\circ}}{5 - j2.5} = 1.6 + j0.8$$

$$= 1.789\underline{/26.57^\circ}\text{ A}$$

continued...



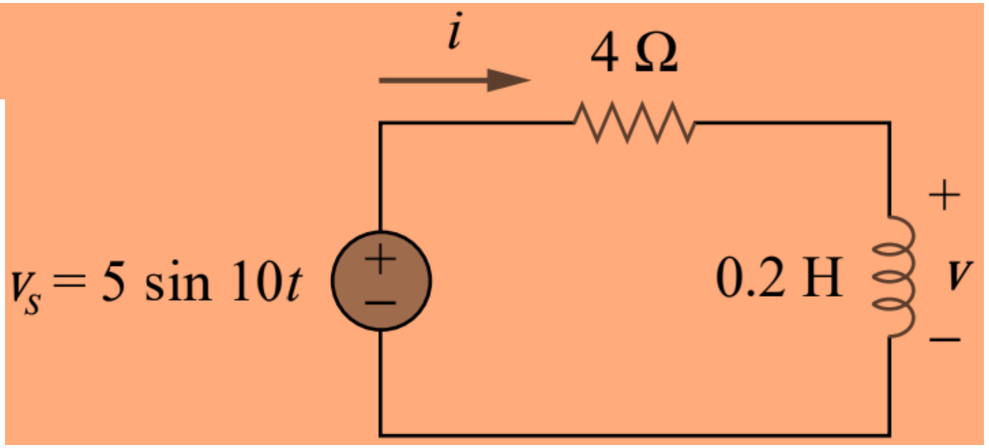
$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = \frac{\mathbf{I}}{j\omega C} = \frac{1.789\angle 26.57^\circ}{j4 \times 0.1} = \frac{1.789\angle 26.57^\circ}{0.4\angle 90^\circ} = 4.47\angle -63.43^\circ \text{ V}$$

$$i(t) = 1.789 \cos(4t + 26.57^\circ) \text{ A}$$

$$v(t) = 4.47 \cos(4t - 63.43^\circ) \text{ V}$$

continued…

Determine $v(t)$ and $i(t)$.



$$\omega = 10$$

$$\mathbf{V}_s = 5\angle 0°$$

$$\mathbf{Z} = 4 + j\omega L$$

$$= 4 + j2$$

$$\mathbf{I} = \mathbf{V}_s / \mathbf{Z} = \frac{5\angle 0°}{4 + j2} = 1.118\angle\text{-}26.57°$$

$$\mathbf{V} = j\omega L\,\mathbf{I} = j2\,\mathbf{I} = (2\angle 90°)(1.118\angle\text{-}26.57°)$$

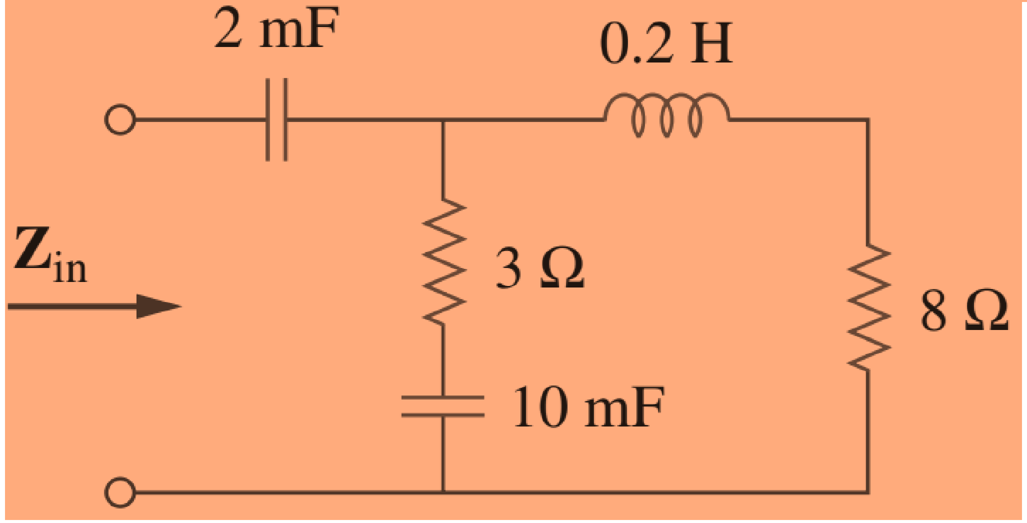
$$= 2.236\angle 63.43°$$

$$v(t) = 2.236 \sin(10t + 63.43°)\ \text{V},$$

$$i(t) = 1.118 \sin(10t - 26.57°)\ \text{A}.$$

continued...

Find the input impedance of the circuit . Assume that the circuit operates at $\omega = 50$ rad/s.



$\mathbf{Z}_1$ = Impedance of the 2-mF capacitor

$\mathbf{Z}_2$ = Impedance of the 3-Ω resistor in series with the10-mF capacitor

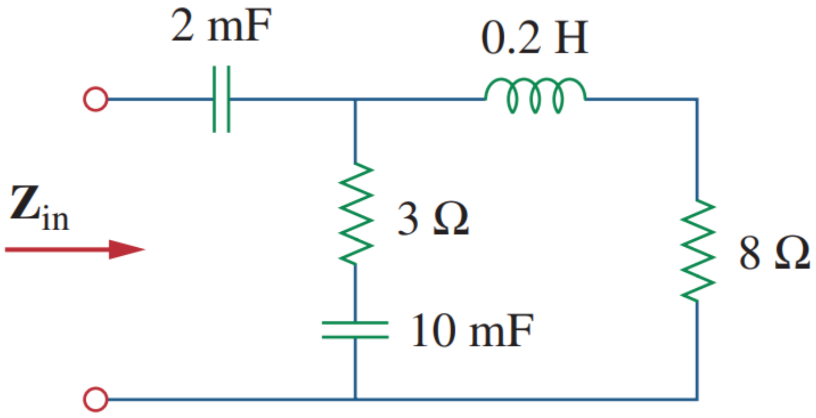
$\mathbf{Z}_3$ = Impedance of the 0.2-H inductor in series with the 8-Ω resistor

$$\mathbf{Z}_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\ \Omega$$

$$\mathbf{Z}_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\ \Omega$$

$$\mathbf{Z}_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\ \Omega$$

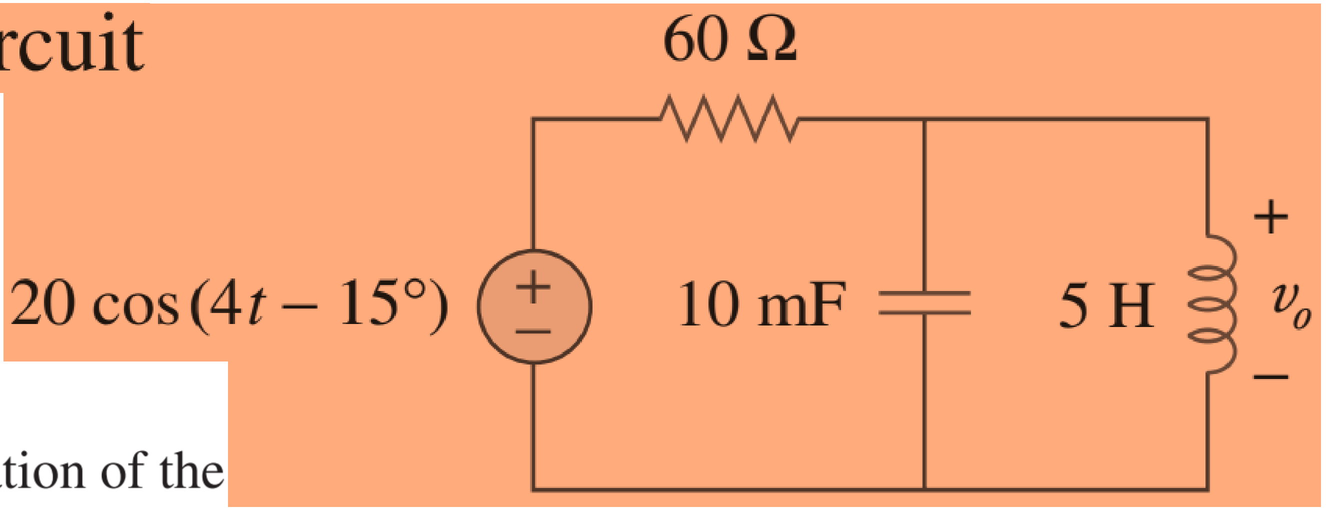
continued…



$$
\begin{aligned}
\mathbf{Z}_{in} &= \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 \\
&= -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8} \\
&= -j10 + 3.22 - j1.07\ \Omega \\
&= 3.22 - j11.07\ \Omega
\end{aligned}
$$

continued...

Determine $v_o(t)$ in the circuit



$$\mathbf{V}_s = 20\angle{-15^\circ}\ \text{V}$$

$$\omega = 4$$

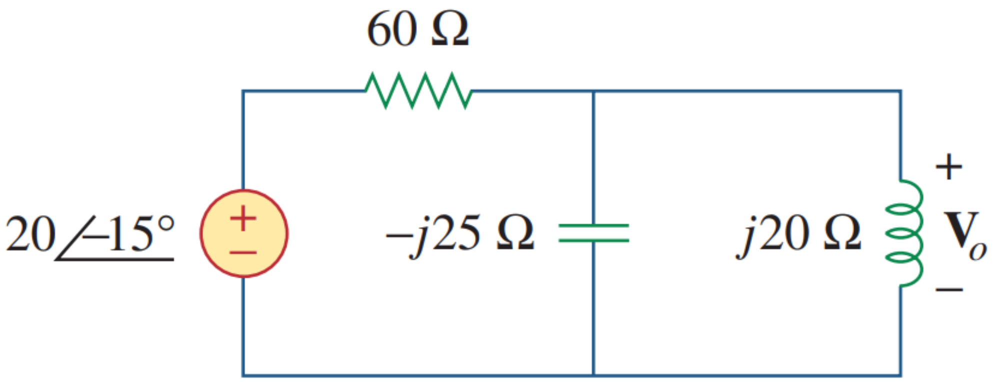
$\mathbf{Z}_1$ = Impedance of the 60-Ω resistor

$\mathbf{Z}_2$ = Impedance of the parallel combination of the 10-mF capacitor and the 5-H inductor

$$10\ \text{mF} \quad \Rightarrow \quad \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}} = -j25\ \Omega$$

$$5\ \text{H} \quad \Rightarrow \quad j\omega L = j4 \times 5 = j20\ \Omega$$

$$\mathbf{Z}_2 = -j25 \parallel j20 = \frac{-j25 \times j20}{-j25 + j20} = j100\ \Omega$$



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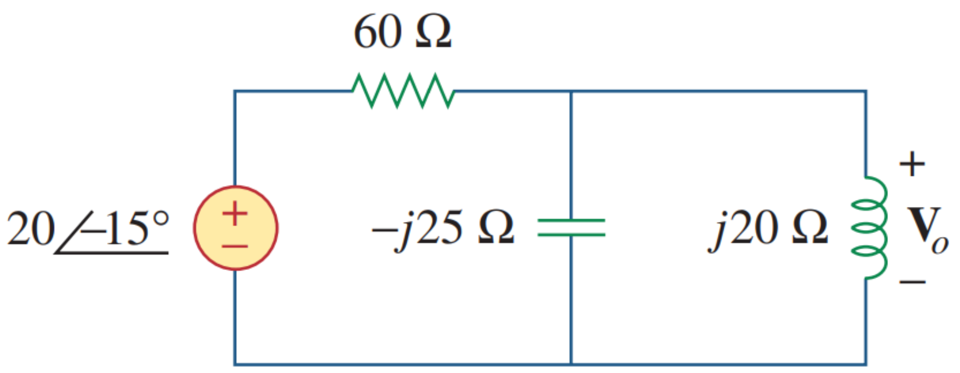


$$\mathbf{V}_o = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s = \frac{j100}{60 + j100}(20\angle{-15^\circ})$$

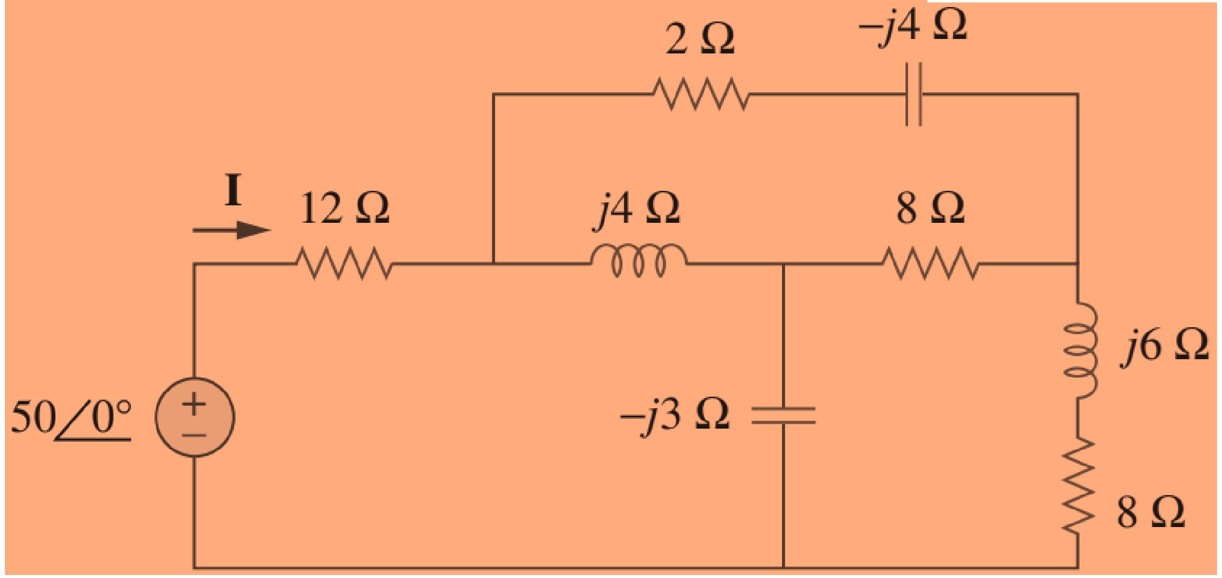
$$= (0.8575\angle{30.96^\circ})(20\angle{-15^\circ})$$

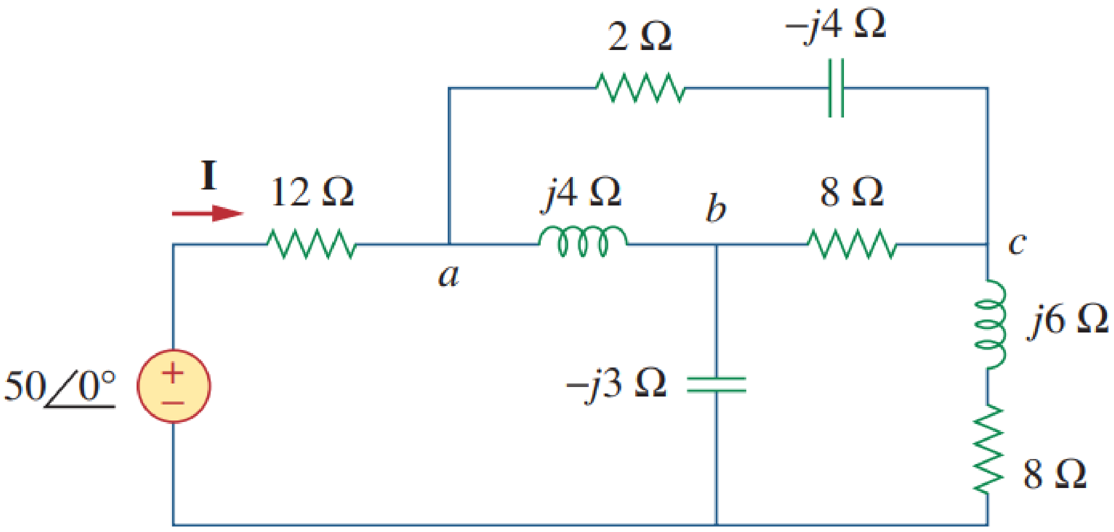
$$= 17.15\angle{15.96^\circ}\ \text{V}$$

$$v_o(t) = 17.15\cos(4t + 15.96^\circ)\ \text{V}$$

continued…

Find current **I** in the circuit

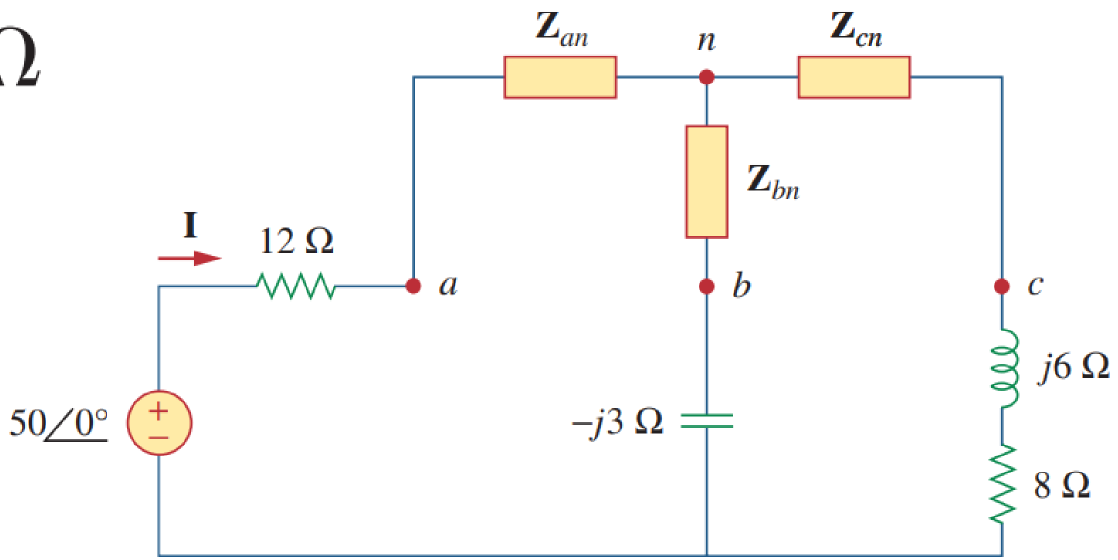




$$\mathbf{Z}_{an} = \frac{j4(2 - j4)}{j4 + 2 - j4 + 8} = (1.6 + j0.8)\ \Omega$$

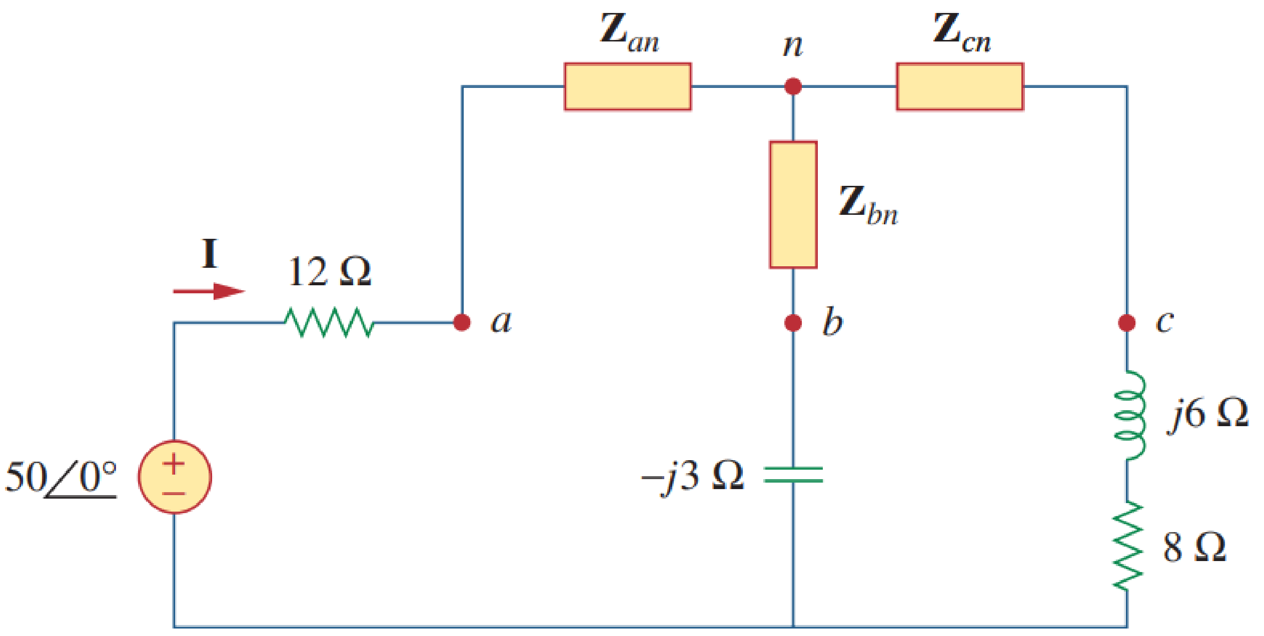
$$\mathbf{Z}_{bn} = \frac{j4(8)}{10} = j3.2\ \Omega$$

$$\mathbf{Z}_{cn} = \frac{8(2 - j4)}{10} = (1.6 - j3.2)\ \Omega$$



continued…

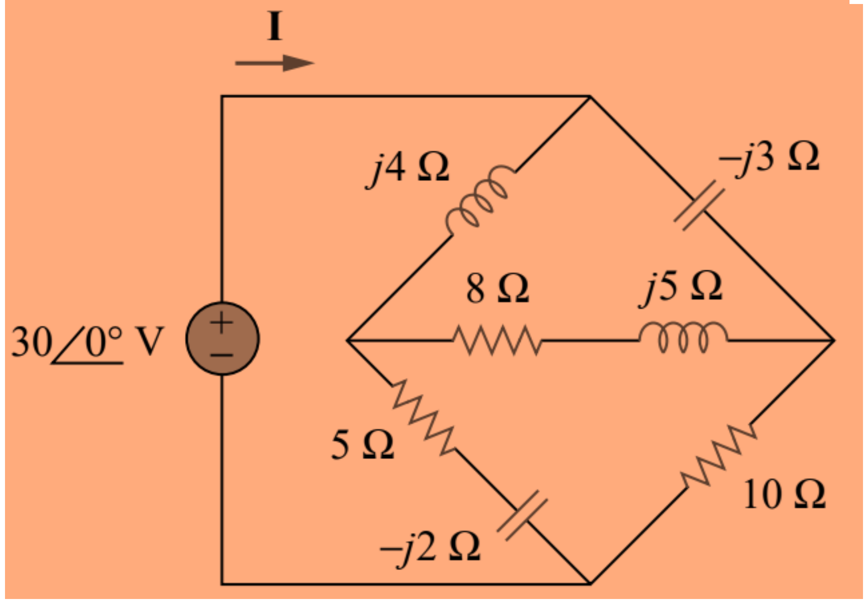
$$\mathbf{Z} = 12 + \mathbf{Z}_{an} + (\mathbf{Z}_{bn} - j3) \,\|\, (\mathbf{Z}_{cn} + j6 + 8)$$
$$= 12 + 1.6 + j0.8 + (j0.2) \,\|\, (9.6 + j2.8)$$
$$= 13.6 + j0.8 + \frac{j0.2(9.6 + j2.8)}{9.6 + j3} = 13.6 + j1$$
$$= 13.64\angle 4.204^\circ\ \Omega$$

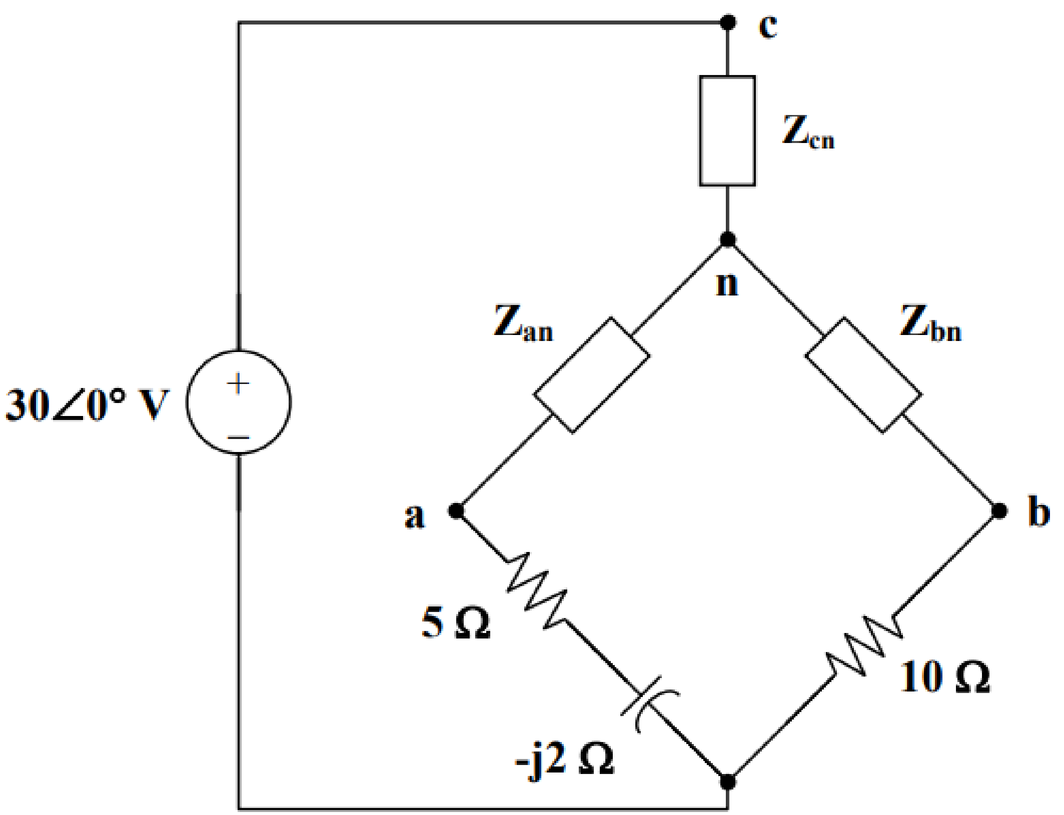


$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\angle 0^\circ}{13.64\angle 4.204^\circ}$$
$$= 3.666\angle -4.204^\circ\ \text{A}$$

continued…

Find I in the circuit





$$\mathbf{Z}_{an} = \frac{j4(8+j5)}{j4+8+j5-j3} = 0.32 + j3.76$$

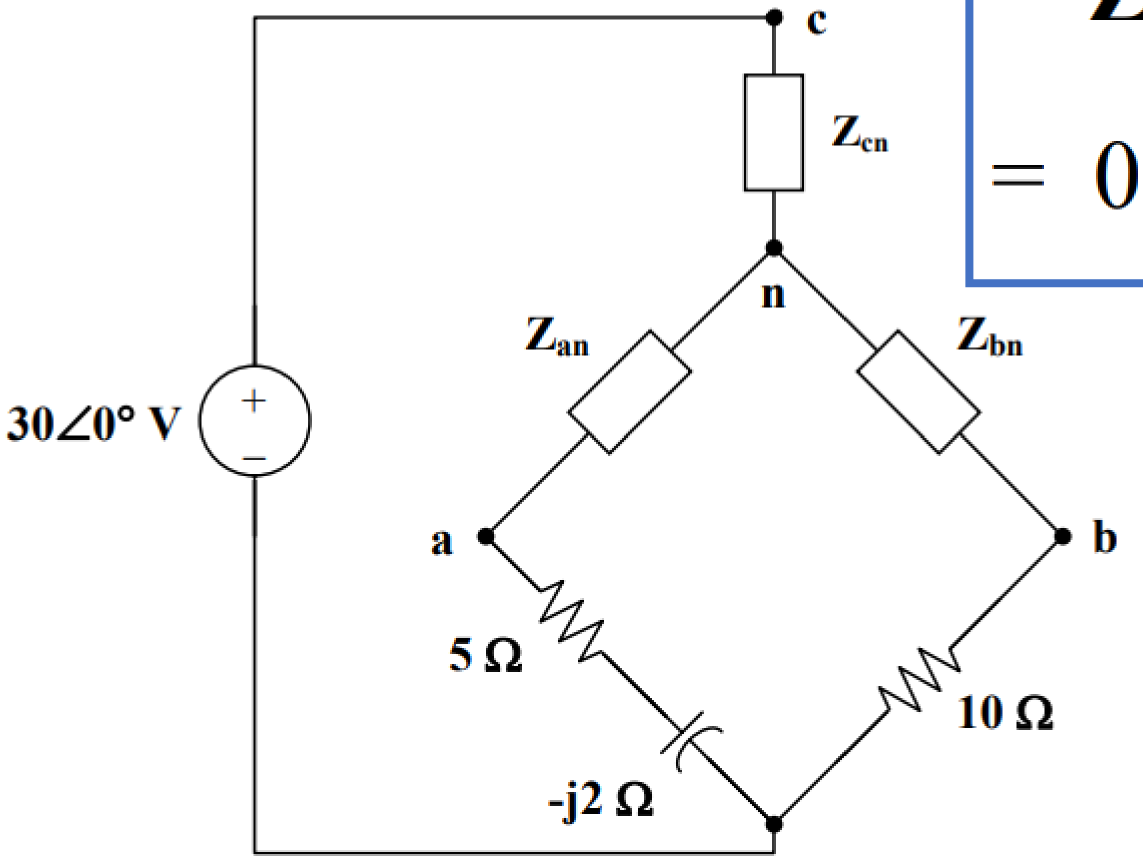
$$\mathbf{Z}_{bn} = \frac{-j3(8+j5)}{8+j6} = -0.24 - j2.82$$

$$\mathbf{Z}_{cn} = \frac{j4(-j3)}{8+j6} = 0.96 - j0.72$$

continued…

$$\mathbf{Z} = \mathbf{Z}_{cn} + (\mathbf{Z}_{an} + 5 - j2) \parallel (\mathbf{Z}_{bn} + 10)$$

$$= \mathbf{Z}_{cn} + (5.32 + j1.76) \parallel (9.76 - j2.82)$$



$$= \mathbf{Z}_{cn} + \frac{(5.32 + j1.76)(9.76 - j2.82)}{(5.32 + j1.76) + (9.76 - j2.82)}$$

$$= 0.96 - j0.72 + 3.744 + j0.4074$$

$$= 4.704 - j0.3126$$

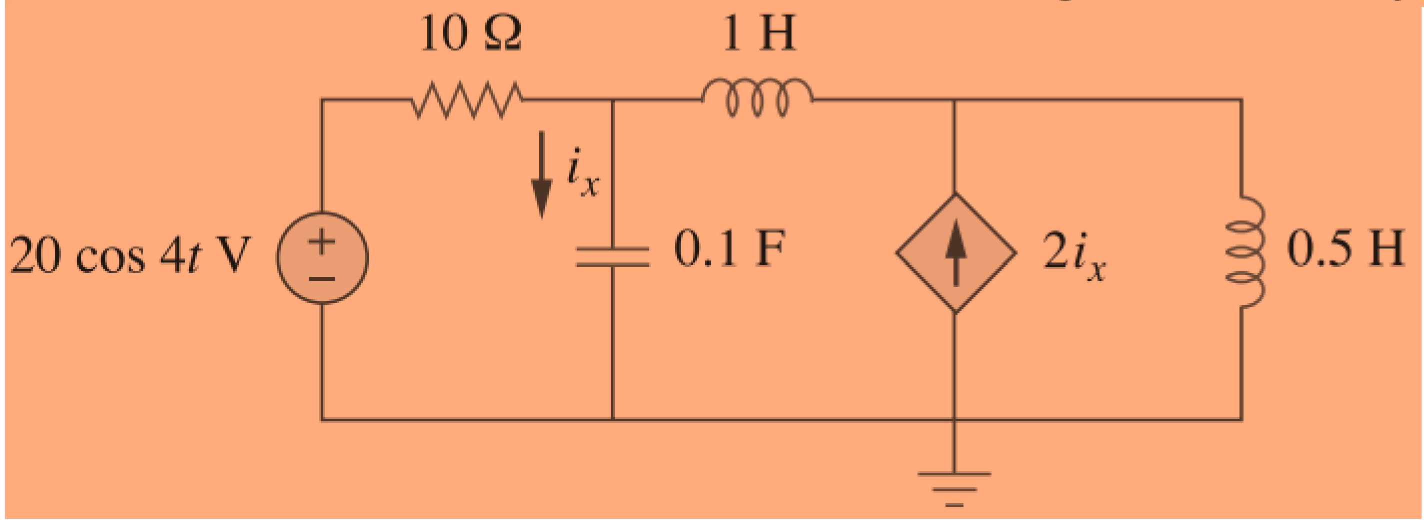
$$= 4.714\angle\text{-}3.802^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{30\angle 0^\circ}{4.714\angle -3.802^\circ}$$

$$= 6.364\angle 3.802^\circ \text{ A.}$$

Nodal Analysis Example

Find i_x in the circuit using nodal analysis.



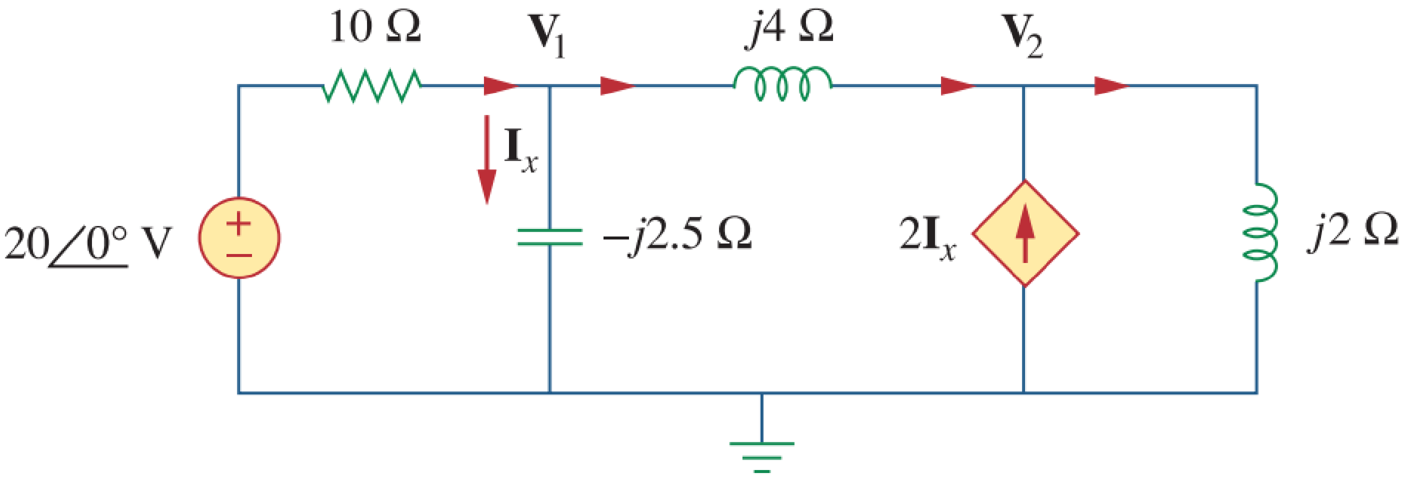
$$20 \cos 4t \quad \Rightarrow \quad 20\angle 0^\circ$$

$$\omega = 4 \text{ rad/s}$$

$$1 \text{ H} \quad \Rightarrow \quad j\omega L = j4$$

$$0.5 \text{ H} \quad \Rightarrow \quad j\omega L = j2$$

$$0.1 \text{ F} \quad \Rightarrow \quad \frac{1}{j\omega C} = -j2.5$$



at node 1,

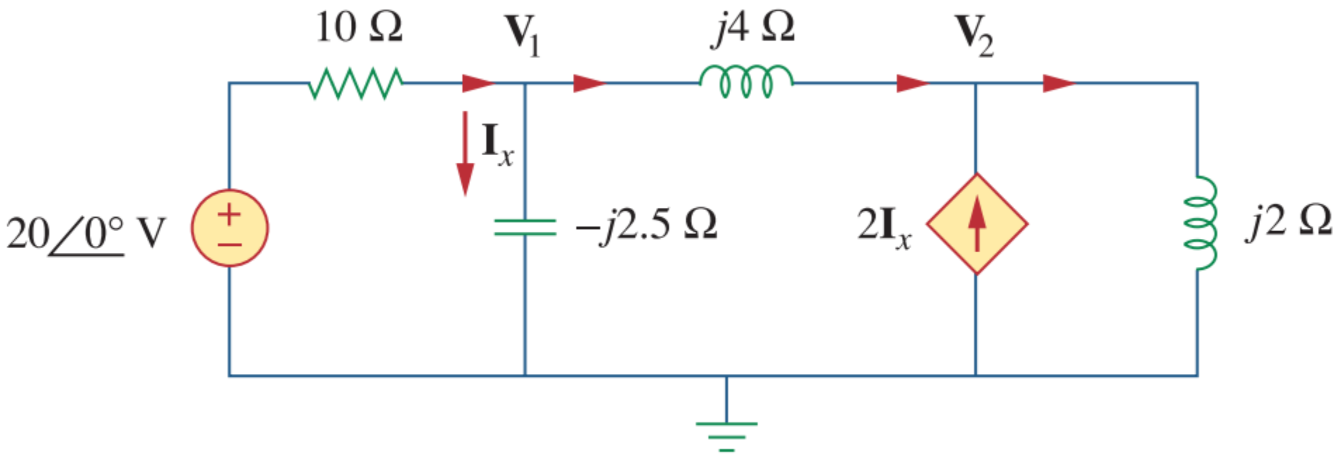
$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

continued...

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$



But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2} \rightarrow 11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$
$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

Nodal Equations (to be solved)

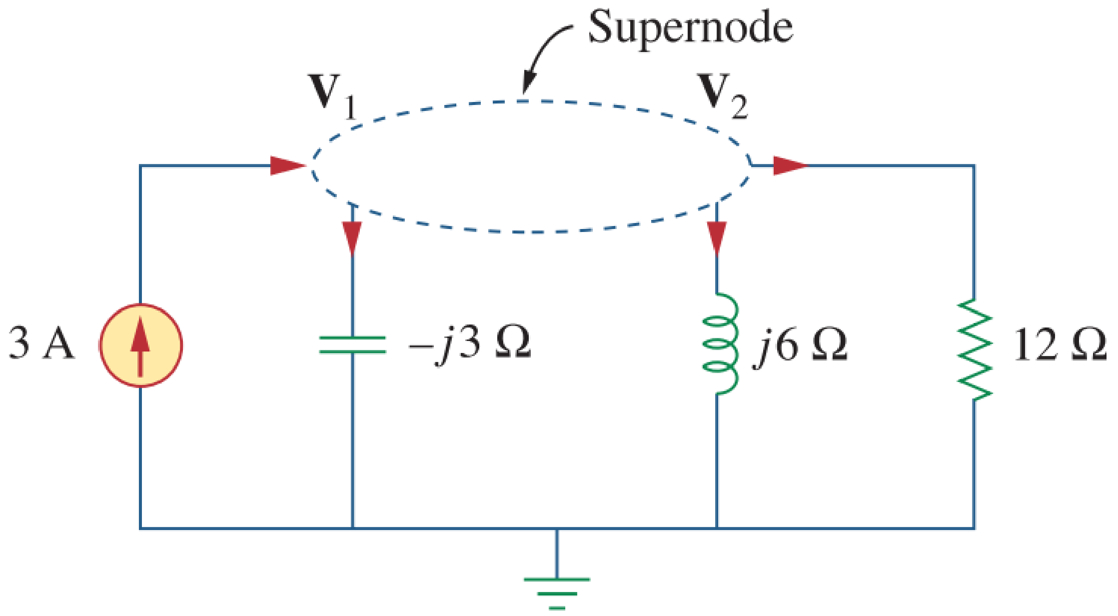
$$\mathbf{V}_1 = 18.97\angle 18.43^\circ \text{ V}$$
$$\mathbf{V}_2 = 13.91\angle 198.3^\circ \text{ V}$$

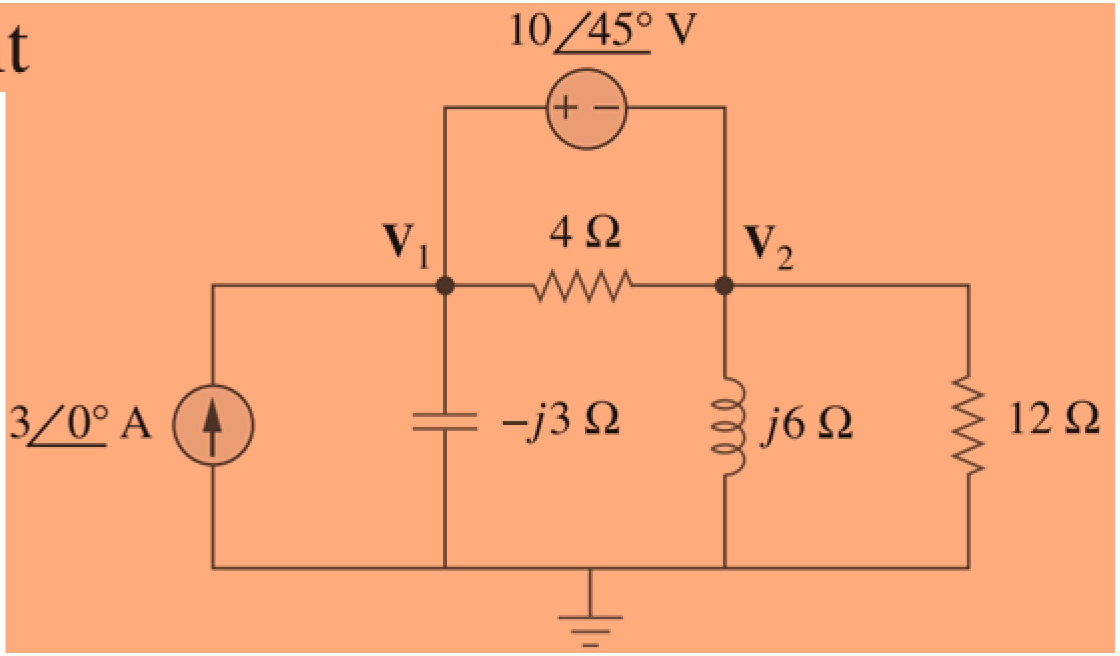
$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\angle 18.43^\circ}{2.5\angle -90^\circ} = 7.59\angle 108.4^\circ \text{ A}$$

$$i_x = 7.59\cos(4t + 108.4^\circ) \text{ A}$$

continued…

Compute $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit





at the supernode

$$3 = \frac{\mathbf{V}_1}{-j3} + \frac{\mathbf{V}_2}{j6} + \frac{\mathbf{V}_2}{12}$$

$$36 = j4\mathbf{V}_1 + (1 - j2)\mathbf{V}_2$$

But $\mathbf{V}_1 - \mathbf{V}_2 = 10\angle 45^\circ \longrightarrow \mathbf{V}_1 = \mathbf{V}_2 + 10\angle 45^\circ$

continued…

After substituting,

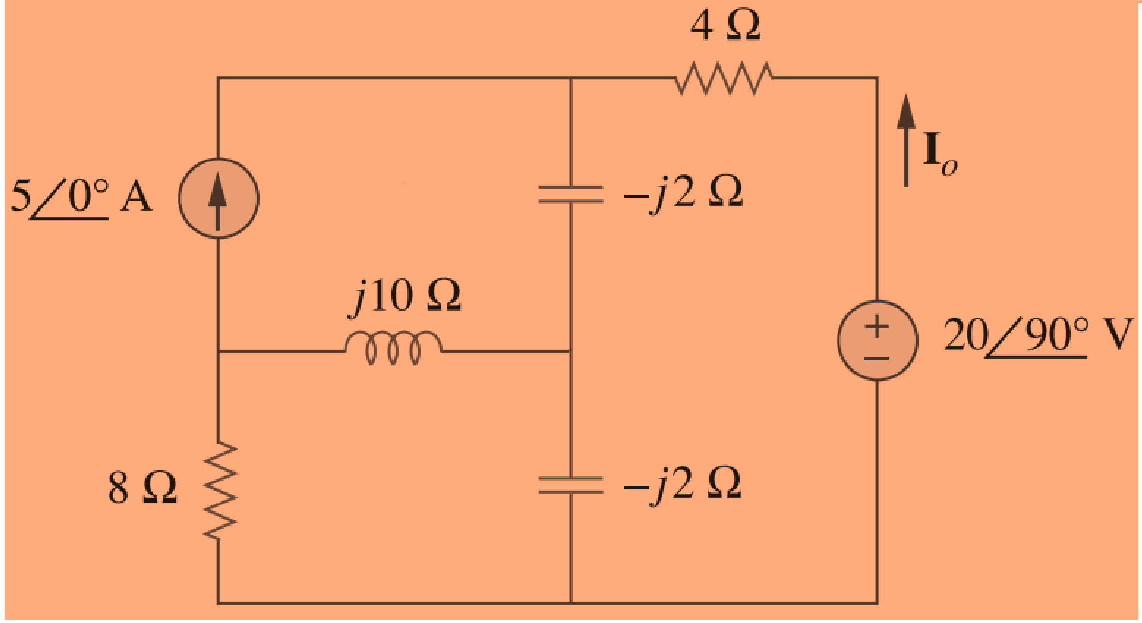
$$36 - 40\angle 135^\circ = (1 + j2)\mathbf{V}_2$$

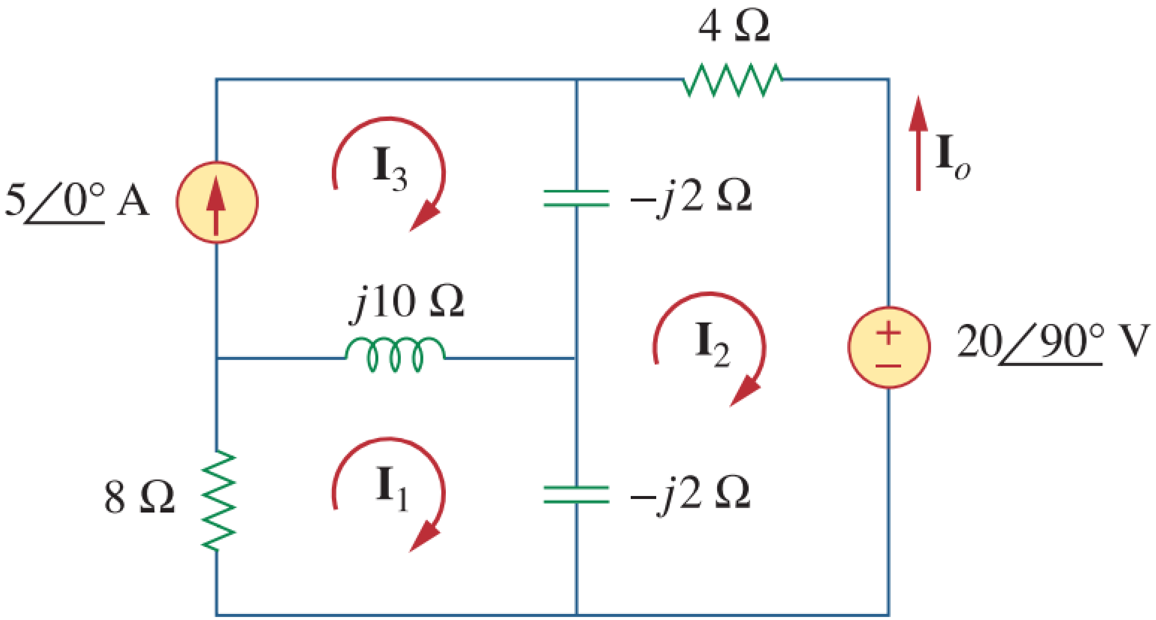
$$\mathbf{V}_2 = 31.41\angle -87.18^\circ \text{ V}$$

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\angle 45^\circ$$
$$= 25.78\angle -70.48^\circ \text{ V}$$

Mesh Analysis Example

Determine current $\mathbf{I}_o$ in the circuit using mesh analysis.





mesh 1 → $(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$

mesh 2 → $(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\angle 90^\circ = 0$

mesh 3 → $\mathbf{I}_3 = 5$

continued…

After substituting,

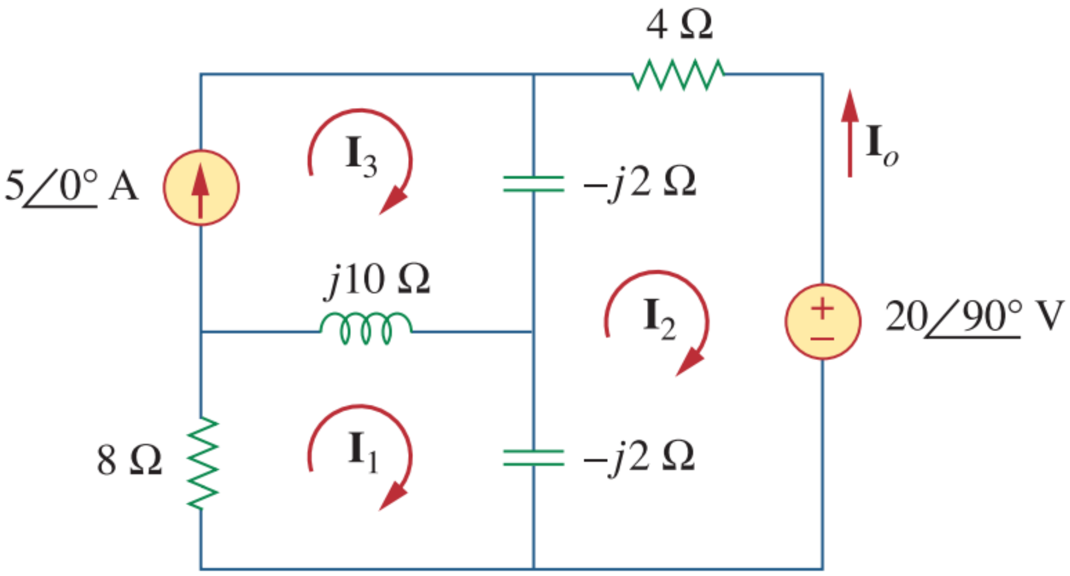
$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50$$

$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10$$

$$\mathbf{I}_2 = 6.12\angle{-35.22^\circ} \text{ A}$$

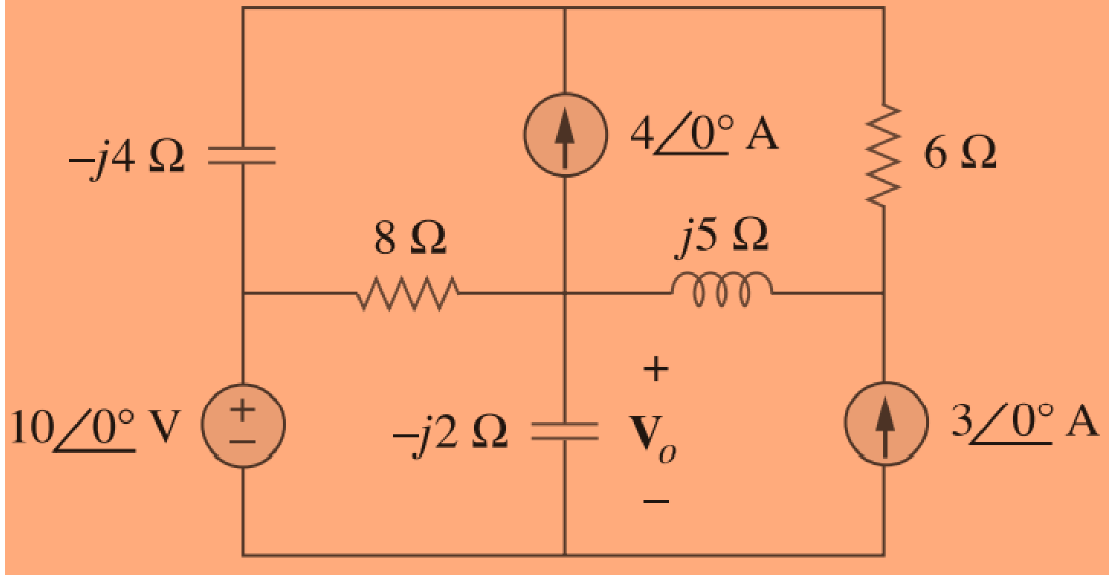
$$\mathbf{I}_o = -\mathbf{I}_2$$

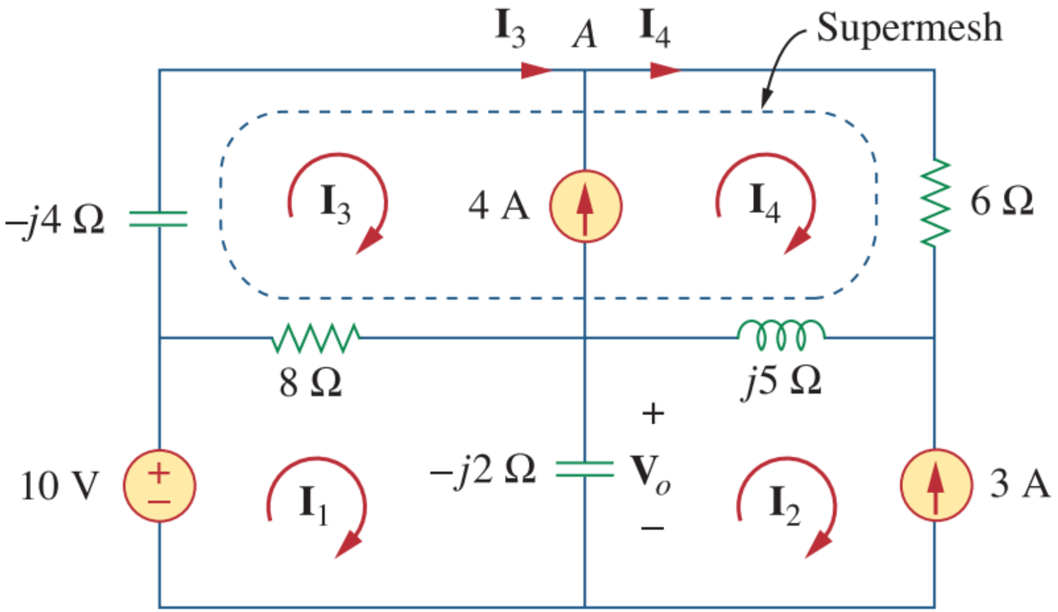
$$= 6.12\angle{144.78^\circ} \text{ A}$$



continued…

Solve for $\mathbf{V}_o$ in the circuit using mesh analysis.





For mesh 1,

$$-10 + (8 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - 8\mathbf{I}_3 = 0$$

$$(8 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 - 8\mathbf{I}_3 = 10$$

For the supermesh,

$$(8 - j4)\mathbf{I}_3 - 8\mathbf{I}_1 + (6 + j5)\mathbf{I}_4 - j5\mathbf{I}_2 = 0$$

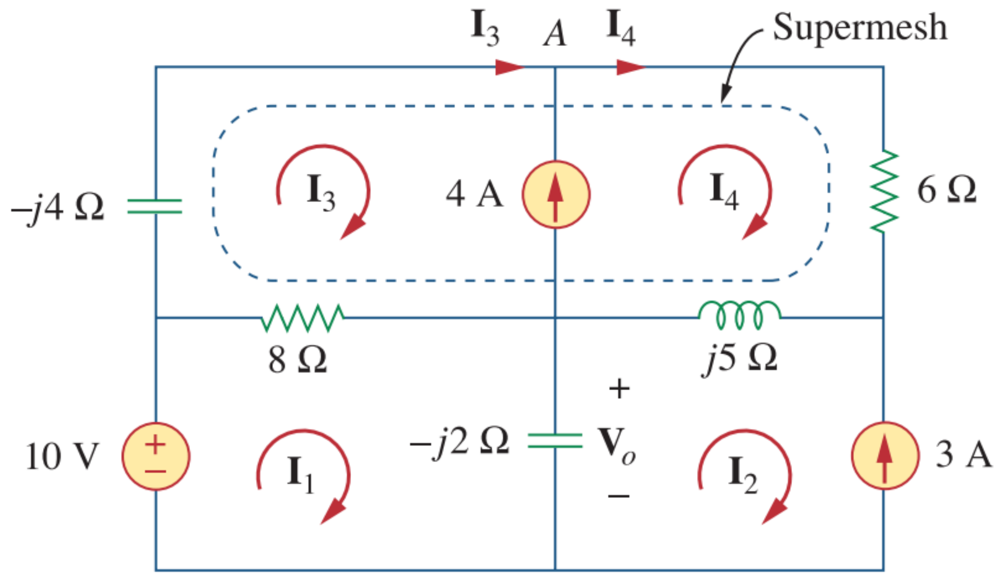
For mesh 2,

$$\mathbf{I}_2 = -3$$

at node *A*,

$$\mathbf{I}_4 = \mathbf{I}_3 + 4$$

continued…



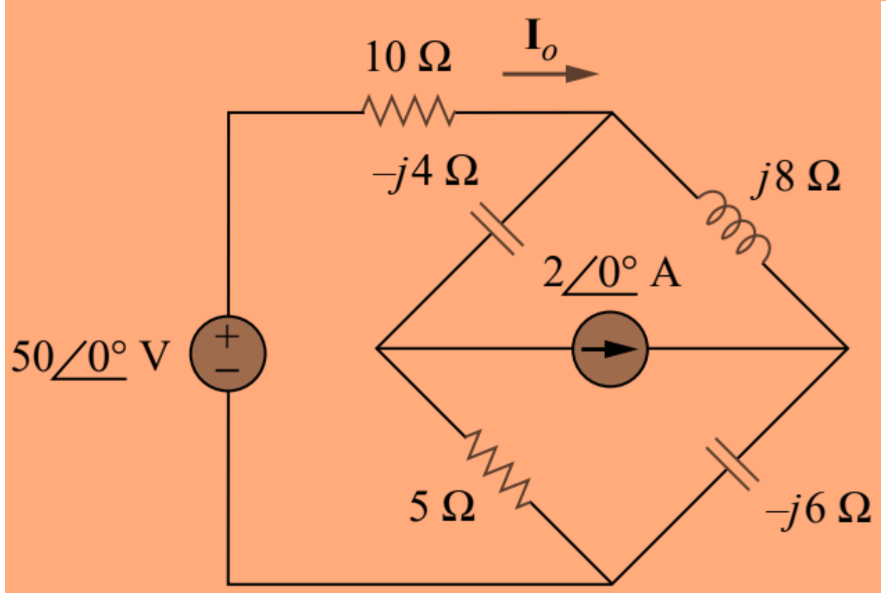
After solving those 4 equations,

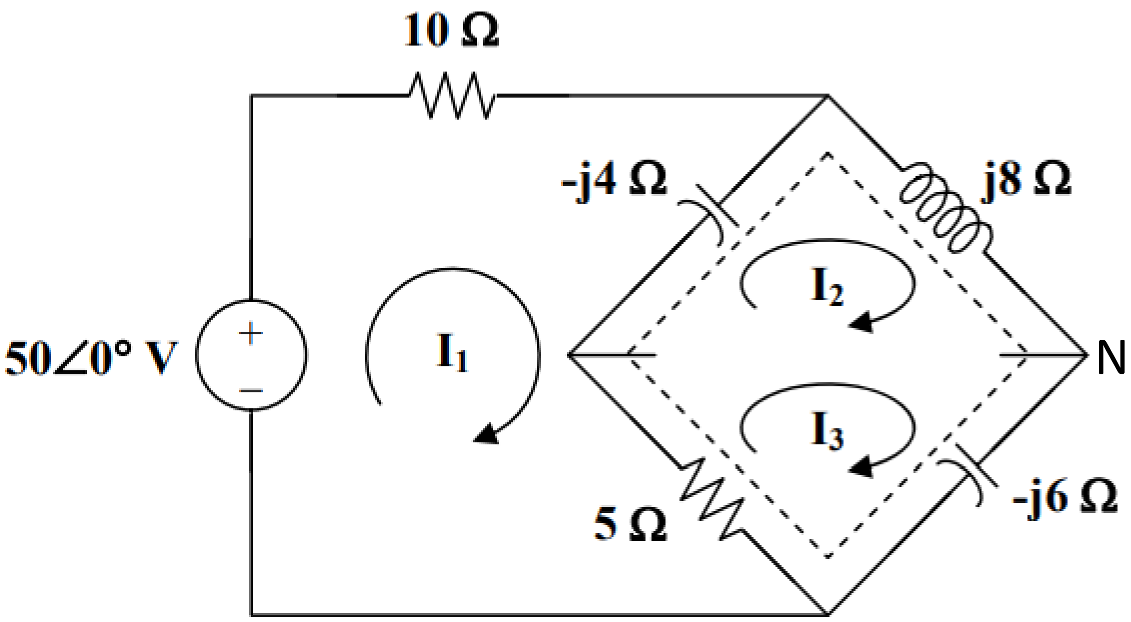
$$\mathbf{I}_1 = 3.618\angle 274.5^\circ \text{ A}$$

$$\begin{aligned}\mathbf{V}_o &= -j2(\mathbf{I}_1 - \mathbf{I}_2) \\ &= -j2(3.618\angle 274.5^\circ + 3) \\ &= -7.2134 - j6.568 \\ &= 9.756\angle 222.32^\circ \text{ V}\end{aligned}$$

continued...

Calculate current $\mathbf{I}_o$ in the circuit





For mesh 1, $-50 + (15 - j4)\mathbf{I}_1 - (-j4)\mathbf{I}_2 - 5\mathbf{I}_3 = 0$

$(15 - j4)\mathbf{I}_1 + j4\mathbf{I}_2 - 5\mathbf{I}_3 = 50$

For the supermesh, $(j8 - j4)\mathbf{I}_2 + (5 - j6)\mathbf{I}_3 - (5 - j4)\mathbf{I}_1 = 0$

At node N, $\mathbf{I}_3 = \mathbf{I}_2 + 2$

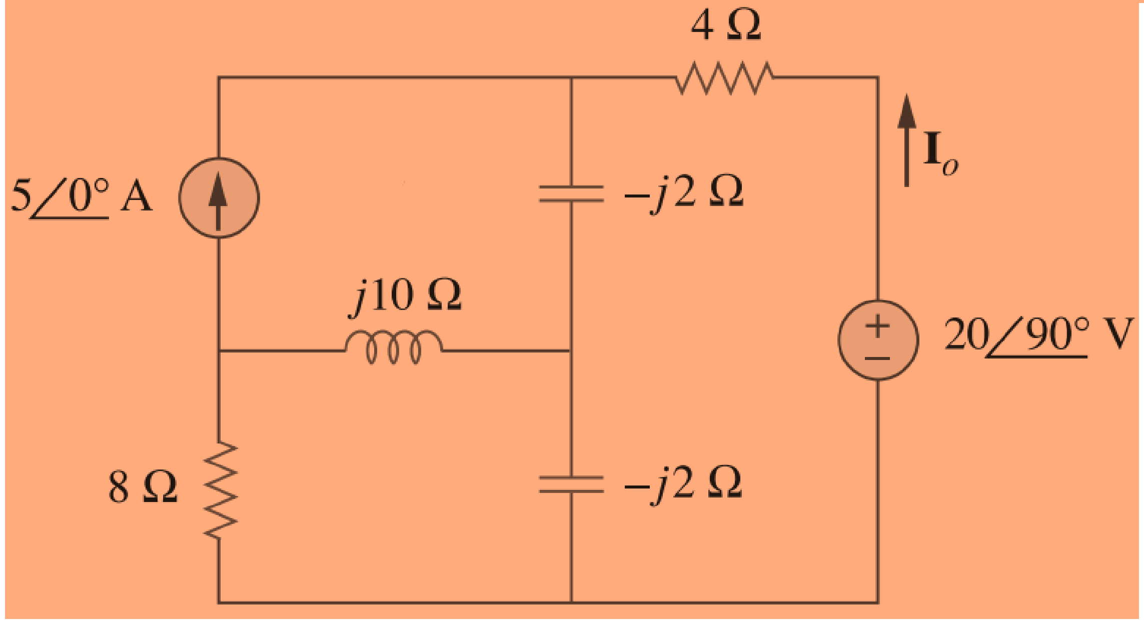
After solving, $\mathbf{I}_1 = 5.074 \angle 5.94° \text{ A}$

But, $\mathbf{I}_o = \mathbf{I}_1$

$\Rightarrow \mathbf{I}_o = 5.074 \angle 5.94° \text{ A}$

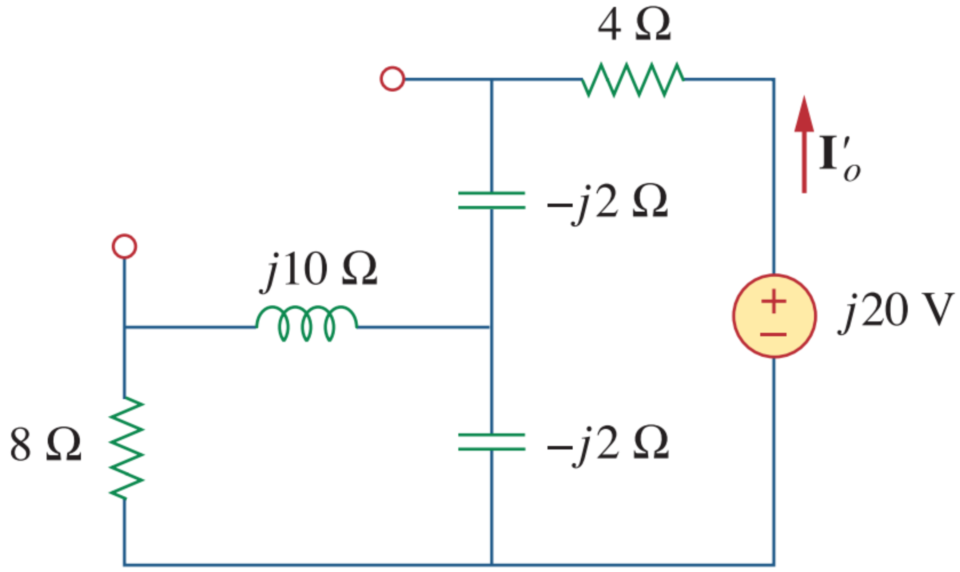
Superposition Example

Use the superposition theorem to find $\mathbf{I}_o$ in the circuit



$$\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o$$

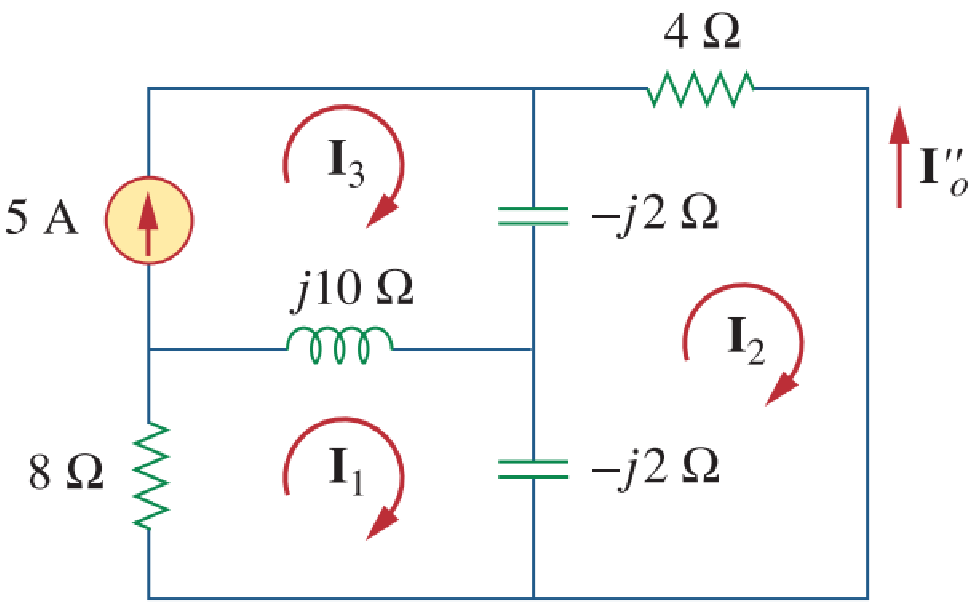
due to the voltage and current sources, respectively



$$\mathbf{Z} = \frac{-j2(8 + j10)}{-2j + 8 + j10} = 0.25 - j2.25$$

$$\mathbf{I}'_o = \frac{j20}{4 - j2 + \mathbf{Z}} = \frac{j20}{4.25 - j4.25} = -2.353 + j2.353$$

continued...



For mesh 1,

$$(8 + j8)\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_2 = 0$$

For mesh 2,

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j2\mathbf{I}_3 = 0$$

For mesh 3,

$$\mathbf{I}_3 = 5$$

After solving,

$$\mathbf{I}_2 = 2.647 - j1.176$$

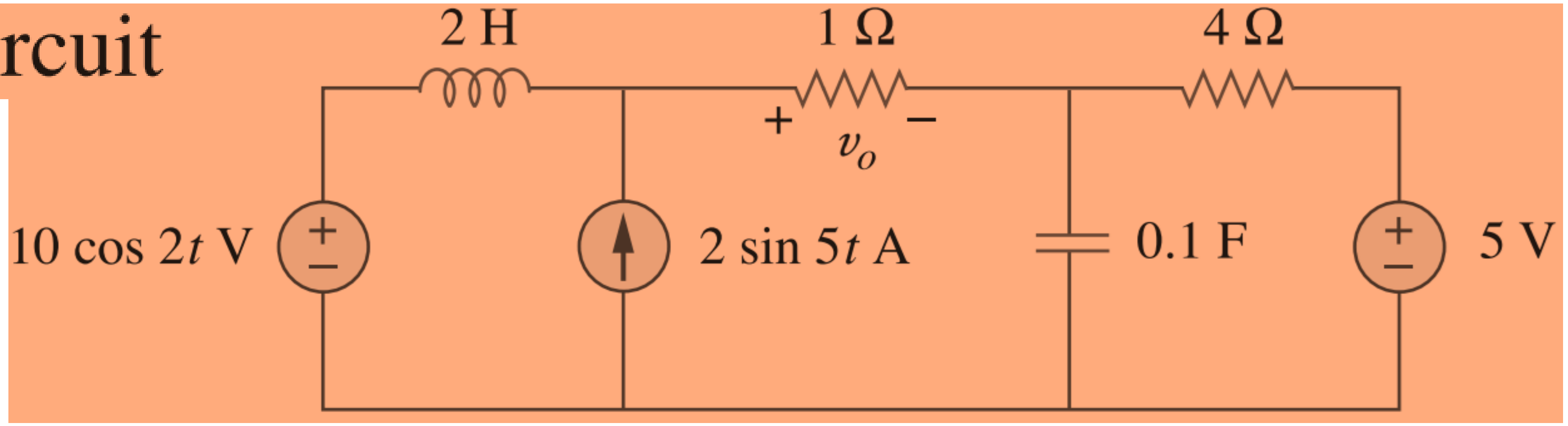
$$\mathbf{I}_o'' = -\mathbf{I}_2 = -2.647 + j1.176$$

$$\mathbf{I}_o = \mathbf{I}_o' + \mathbf{I}_o'' = -5 + j3.529$$

$$= 6.12\angle 144.78^\circ \text{ A}$$

continued...

Find v_o of the circuit

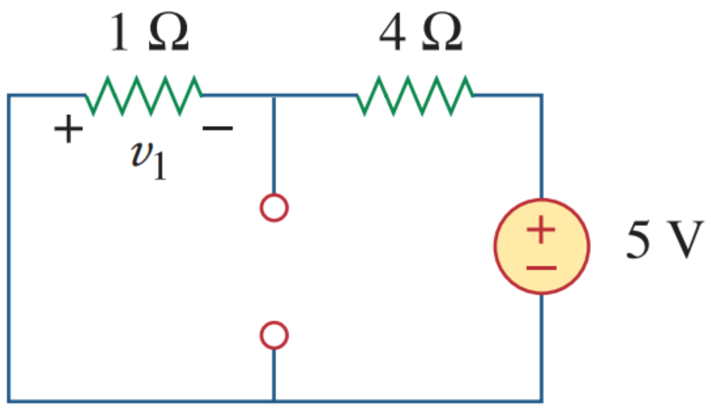


$$v_o = v_1 + v_2 + v_3$$

v_1 is due to the 5-V

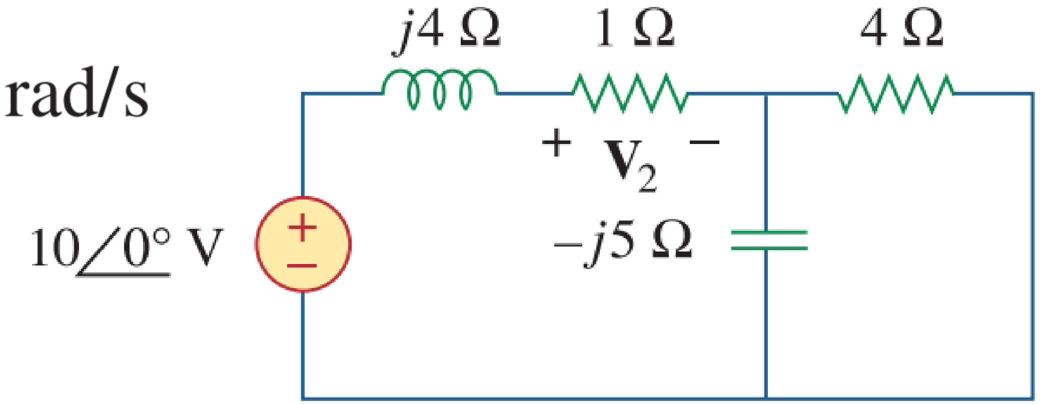
v_2 is due to the 10 cos 2t

v_3 is due to the 2 sin 5t



$$-v_1 = \frac{1}{1+4}(5) = 1 \text{ V}$$

$$\omega = 2 \text{ rad/s}$$



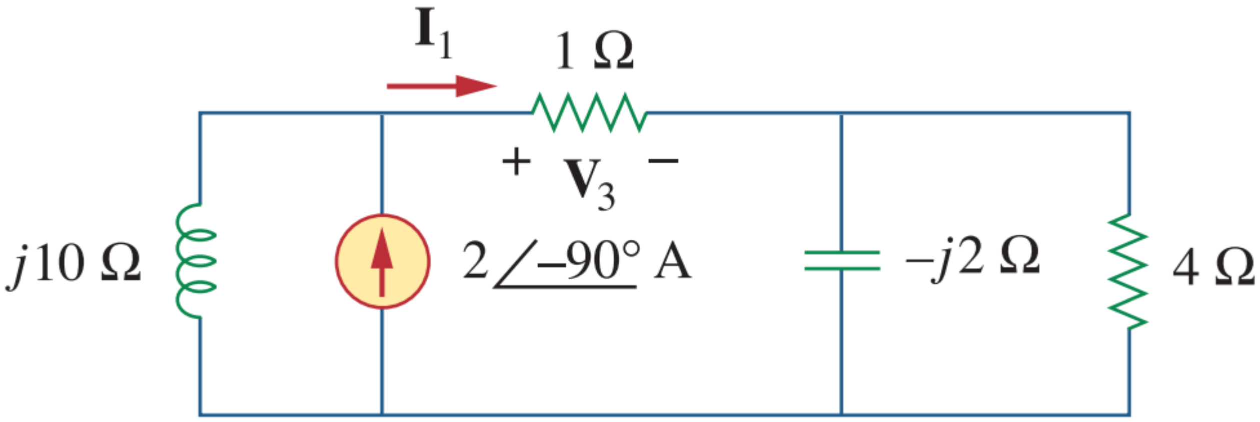
$$\mathbf{Z} = -j5 \parallel 4 = \frac{-j5 \times 4}{4 - j5} = 2.439 - j1.951$$

$$\mathbf{V}_2 = \frac{1}{1 + j4 + \mathbf{Z}}(10\angle 0^\circ) = \frac{10}{3.439 + j2.049}$$

continued…

$$\mathbf{V}_2 = \frac{10}{3.439 + j2.049} = 2.498\angle{-30.79^\circ}$$

$$v_2 = 2.498\cos(2t - 30.79^\circ)$$



$\omega = 5$ rad/s

$2\sin 5t \quad \Rightarrow \quad 2\angle{-90^\circ}$

$$\mathbf{Z}_1 = -j2 \parallel 4 = \frac{-j2 \times 4}{4 - j2} = 0.8 - j1.6\ \Omega$$

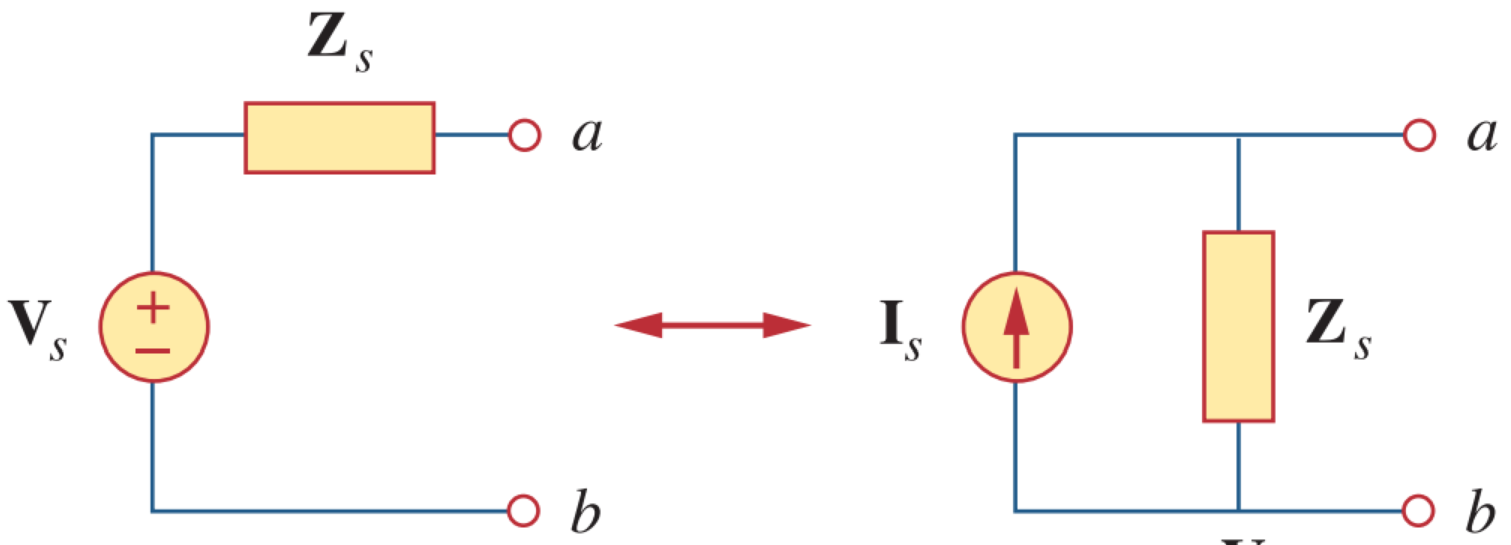
$$\mathbf{I}_1 = \frac{j10}{j10 + 1 + \mathbf{Z}_1}(2\angle{-90^\circ})\text{ A} = \frac{j10}{1.8 + j8.4}(-j2) = 2.328\angle{-80^\circ}\text{ V}$$

$$\mathbf{V}_3 = \mathbf{I}_1 \times 1 = 2.328\angle{-80^\circ}\text{ V} \rightarrow v_3 = 2.33\cos(5t - 80^\circ)$$

continued...

$$v_o(t) = -1 + 2.498\cos(2t - 30.79°) + 2.33\cos(5t - 80°)$$

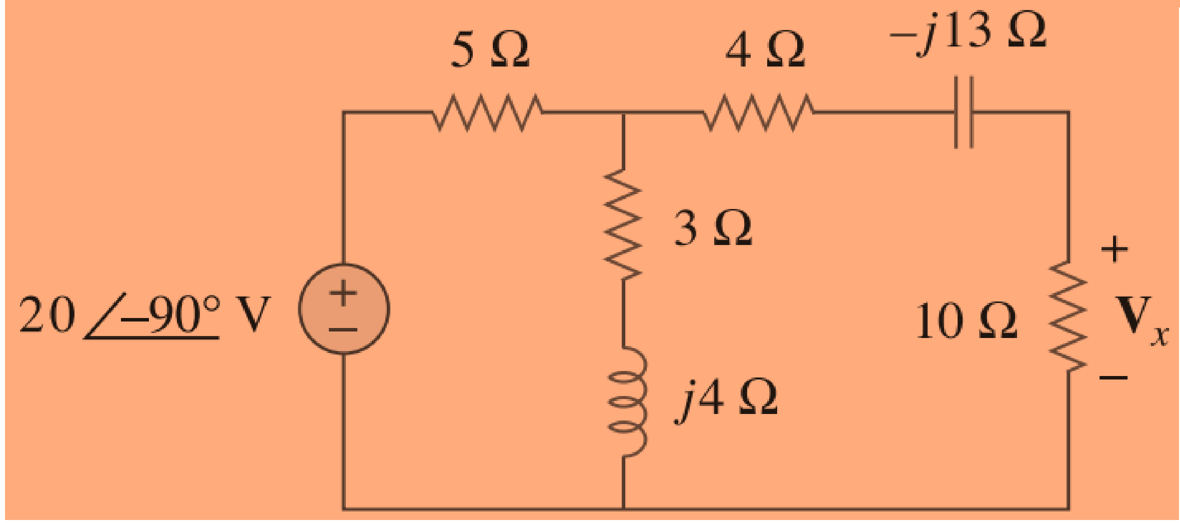
Source Transformation

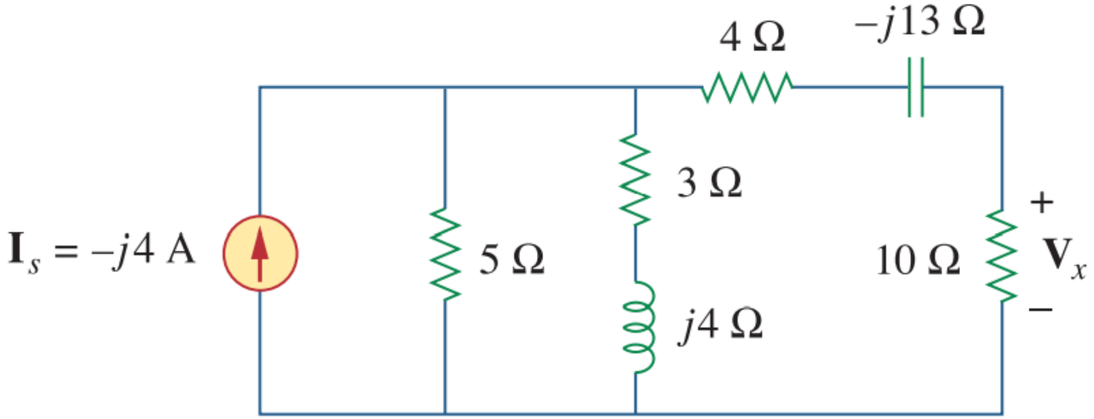


$$\mathbf{V}_s = \mathbf{Z}_s\mathbf{I}_s \qquad \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}$$

Source Transformation Example

Calculate $\mathbf{V}_x$ in the circuit using the method of source transformation.





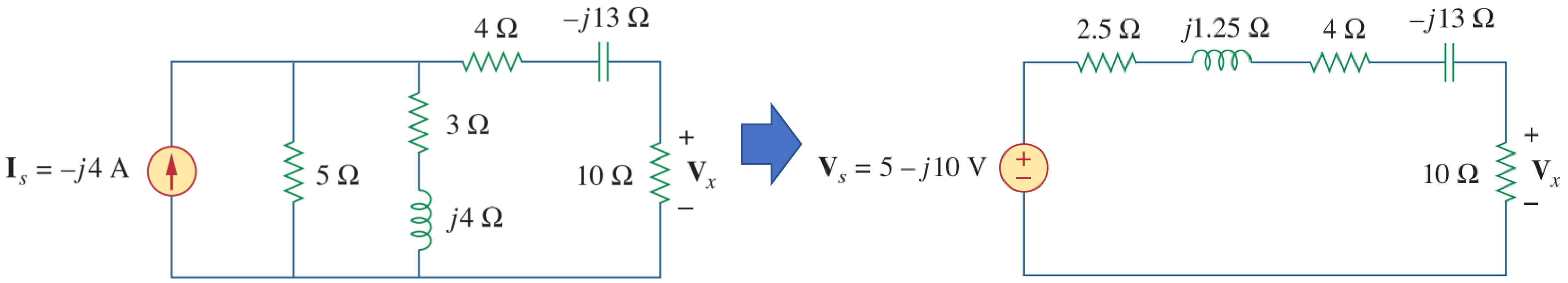
$$\mathbf{I}_s = \frac{20\angle -90^\circ}{5} = 4\angle -90^\circ = -j4 \text{ A}$$

voltage source to a current source

parallel combination of 5-Ω resistance and (3 + j4) impedance

$$\mathbf{Z}_1 = \frac{5(3 + j4)}{8 + j4} = 2.5 + j1.25\ \Omega$$

continued...



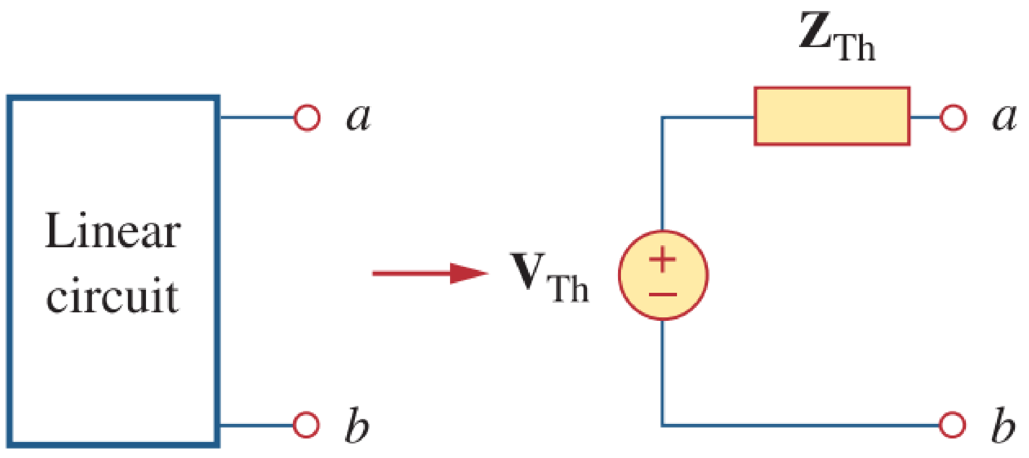
$$\mathbf{V}_s = \mathbf{I}_s\mathbf{Z}_1 = -j4(2.5 + j1.25) = 5 - j10\text{ V}$$

current source to a voltage source

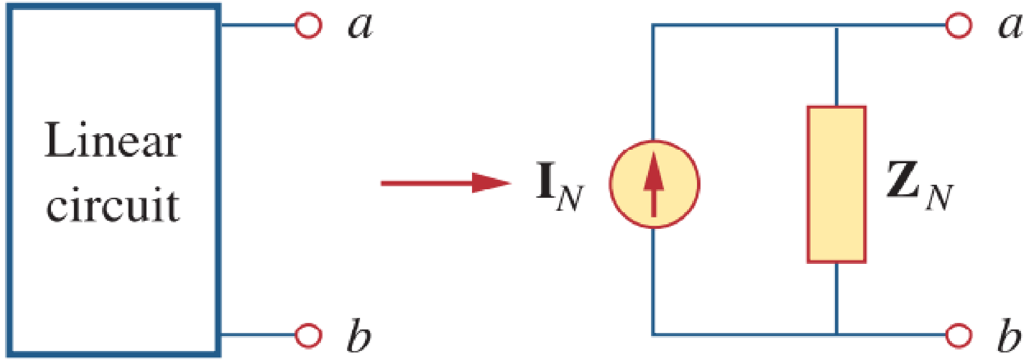
$$\mathbf{V}_x = \frac{10}{10 + 2.5 + j1.25 + 4 - j13}(5 - j10)$$

$$= 5.519\underline{/-28^\circ}\text{ V}$$

Thevenin and Norton Equivalent Circuits



Thevenin equivalent

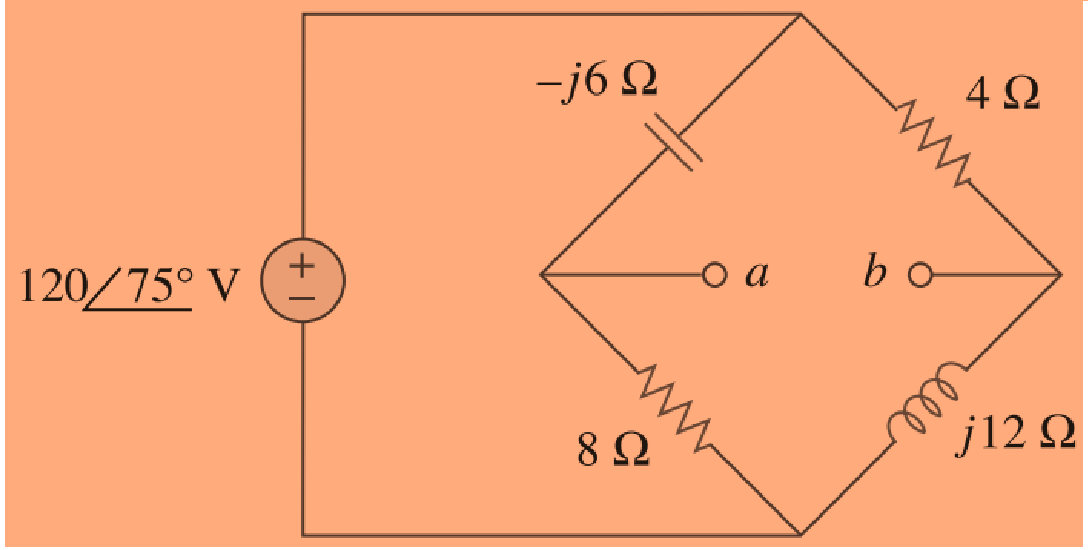


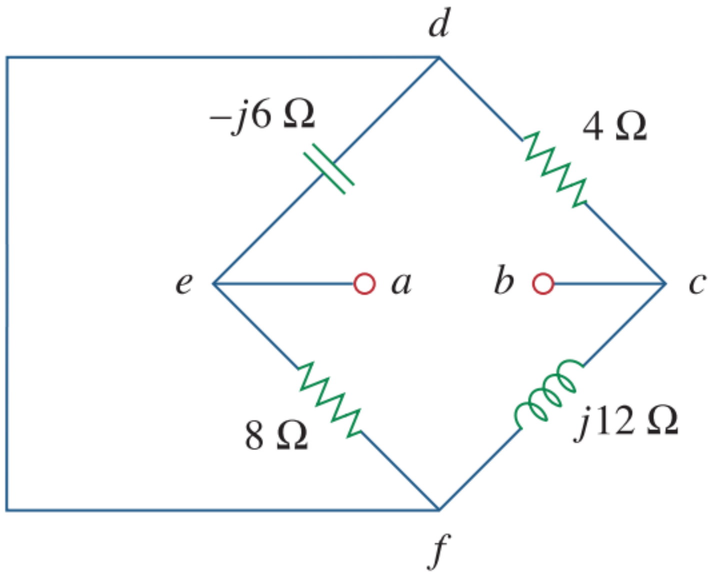
Norton equivalent

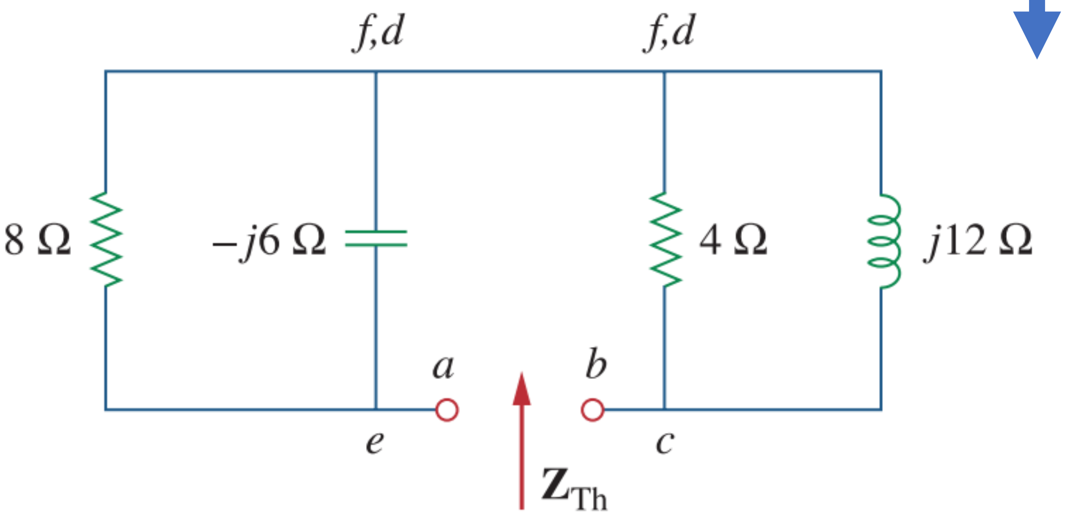
$$\mathbf{V}_{\text{Th}} = \mathbf{Z}_N \mathbf{I}_N, \qquad \mathbf{Z}_{\text{Th}} = \mathbf{Z}_N$$

Example

Obtain the Thevenin equivalent at terminals *a-b* of the circuit





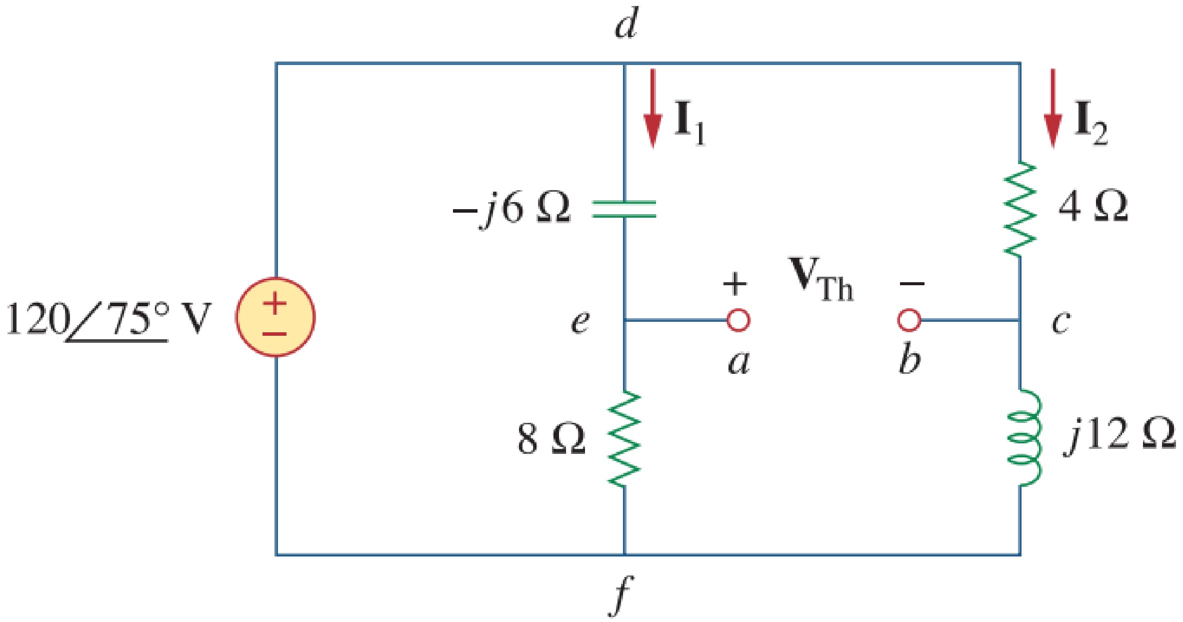


$$\mathbf{Z}_1 = -j6 \parallel 8 = \frac{-j6 \times 8}{8 - j6} = 2.88 - j3.84\ \Omega$$

$$\mathbf{Z}_2 = 4 \parallel j12 = \frac{j12 \times 4}{4 + j12} = 3.6 + j1.2\ \Omega$$

continued…

$$\mathbf{Z}_{Th} = \mathbf{Z}_1 + \mathbf{Z}_2 = 6.48 - j2.64\ \Omega$$



$$\mathbf{I}_1 = \frac{120\angle 75^\circ}{8 - j6}\ \text{A}$$

$$\mathbf{I}_2 = \frac{120\angle 75^\circ}{4 + j12}\ \text{A}$$

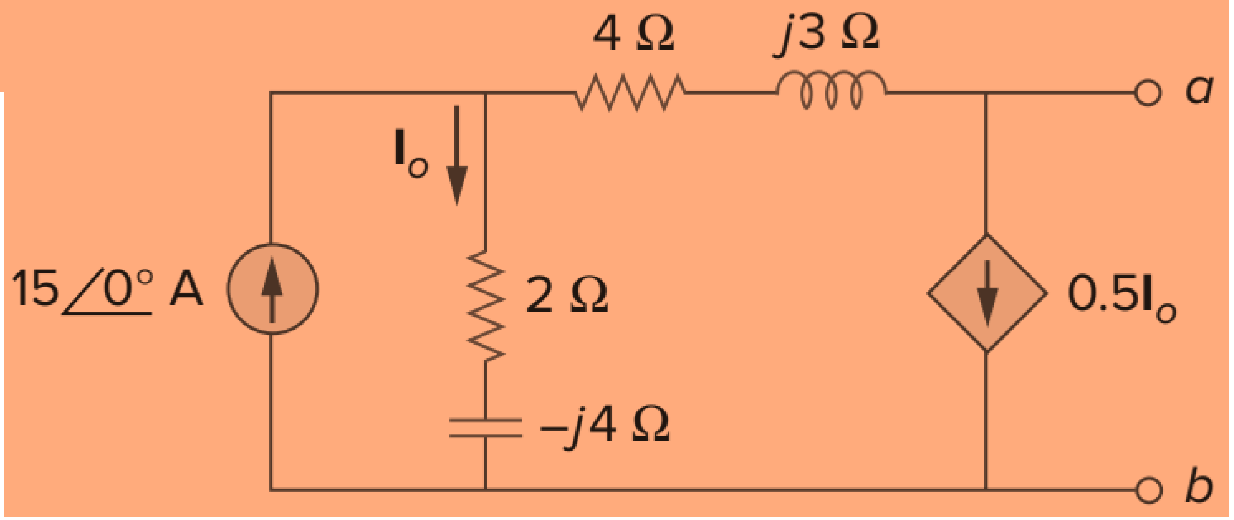
Applying KVL around loop *bcdeab*

$$\mathbf{V}_{Th} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

$$\begin{aligned}\mathbf{V}_{Th} = 4\mathbf{I}_2 + j6\mathbf{I}_1 &= 37.95\angle 3.43^\circ + 72\angle 201.87^\circ \\ &= -28.936 - j24.55 \\ &= 37.95\angle 220.31^\circ\ \text{V}\end{aligned}$$

continued…

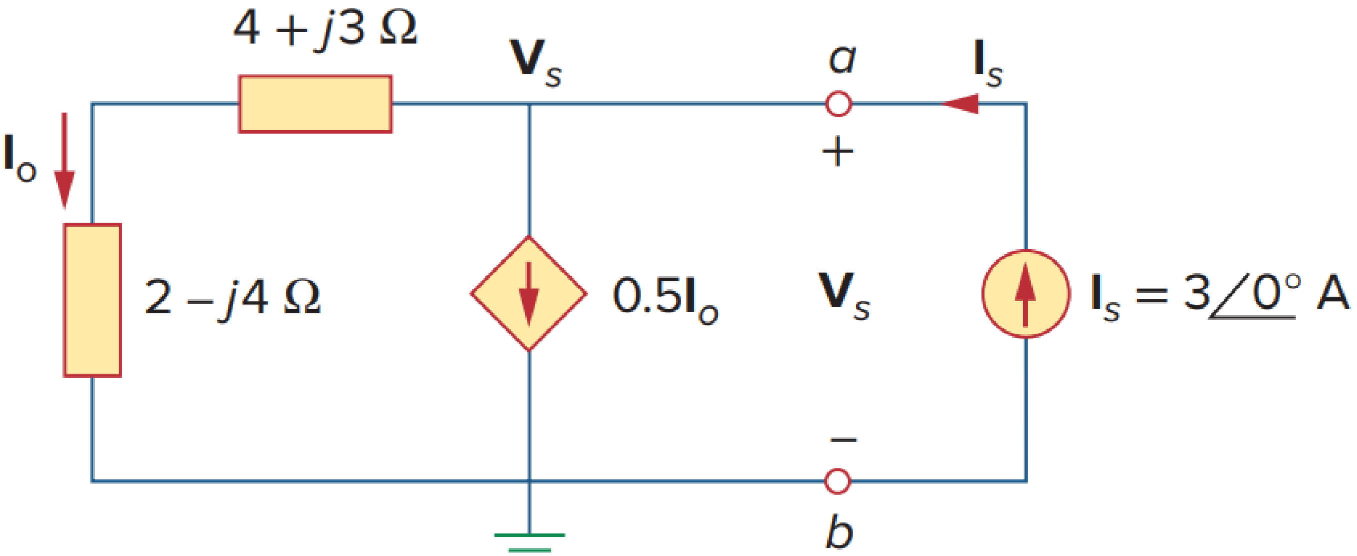
Find the Thevenin equivalent of the circuit as seen from terminals *a-b*.



Finding $\mathbf{Z}_{Th}$:

Due to the presence of the dependent source, we connect a 3-A current source (*or* voltage source if you want).

- 3 is an arbitrary value chosen for convenience here.



KCL gives

$$3 = \mathbf{I}_o + 0.5\mathbf{I}_o$$

$$\Rightarrow \qquad \mathbf{I}_o = 2\text{A}$$

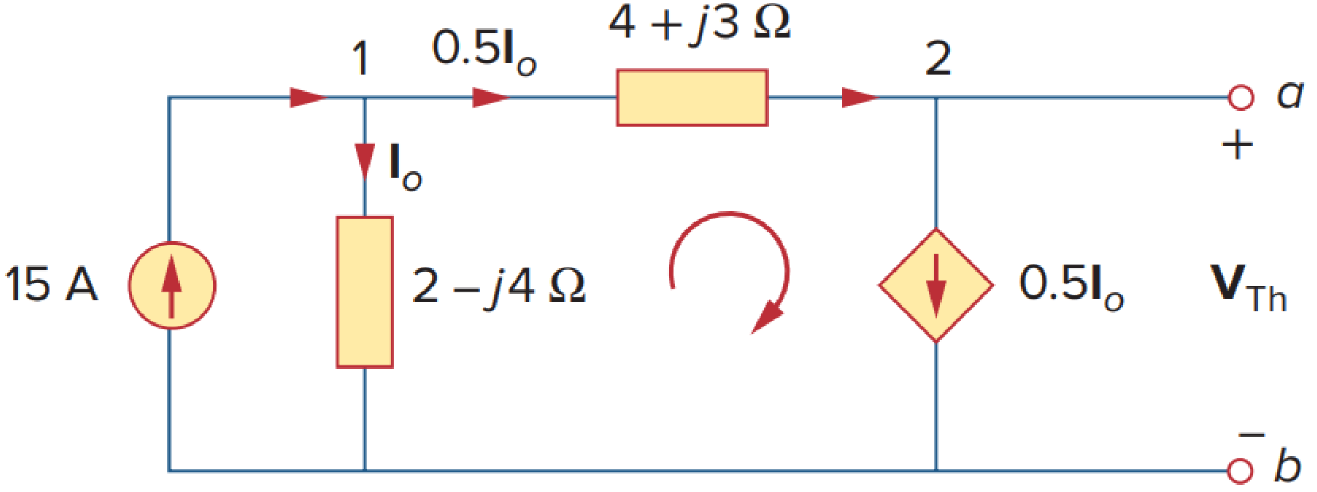
Applying KVL

$$\mathbf{V}_s = \mathbf{I}_o(4 + j3 + 2 - j4)$$

$$= 2(6 - j)$$

continued…

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{2(6 - j)}{3} = 4 - j0.6667\ \Omega$$



KCL at node 1

$$15 = \mathbf{I}_o + 0.5\mathbf{I}_o$$

$$\Rightarrow \quad \mathbf{I}_o = 10\ \text{A}$$

Applying KVL

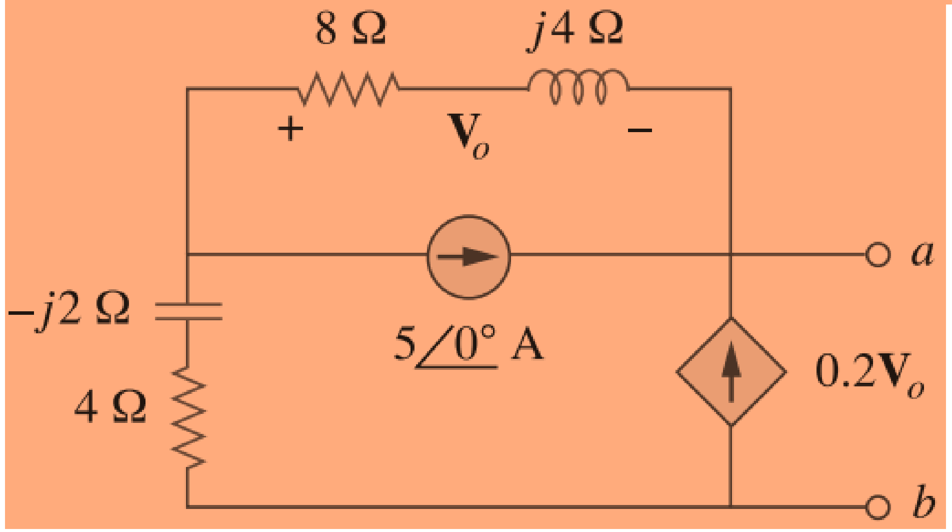
$$-\mathbf{I}_o(2 - j4) + 0.5\mathbf{I}_o(4 + j3) + \mathbf{V}_{\text{Th}} = 0 \longrightarrow \mathbf{V}_{\text{Th}} = 10(2 - j4) - 5(4 + j3)$$

$$= -j55$$

$$= 55\underline{/-90^\circ}\ \text{V}$$

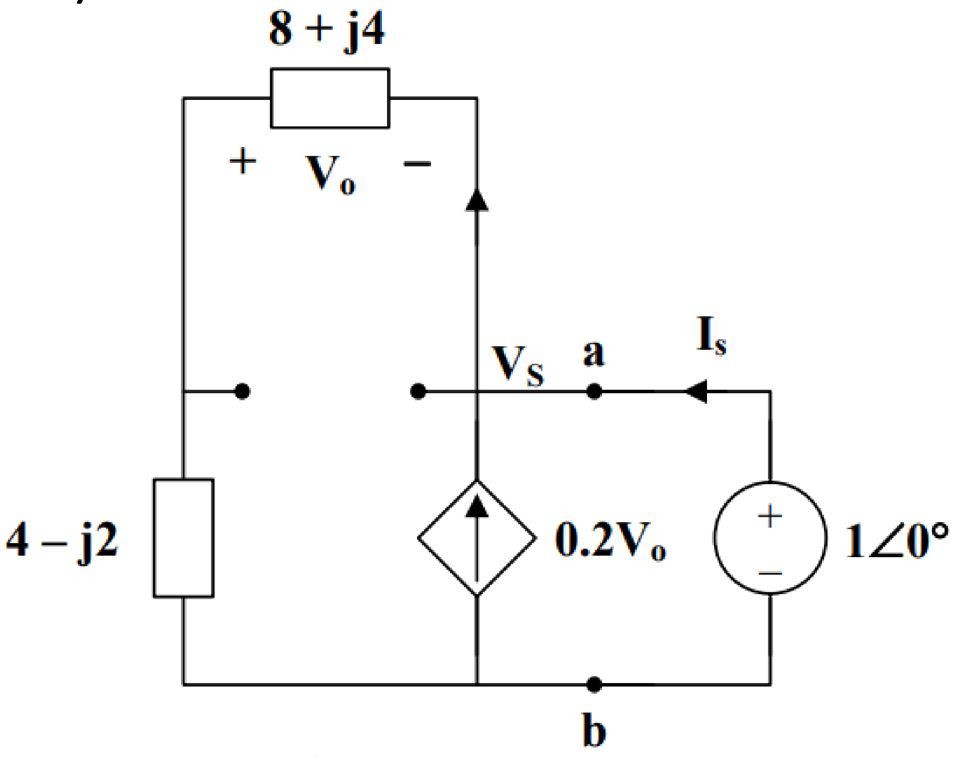
continued…

Determine the Thevenin equivalent of the circuit as seen from the terminals *a-b*.



Finding $\mathbf{Z}_{Th}$:

Due to the presence of the dependent source, we connect a 1-V voltage source (*or* current source if you want).



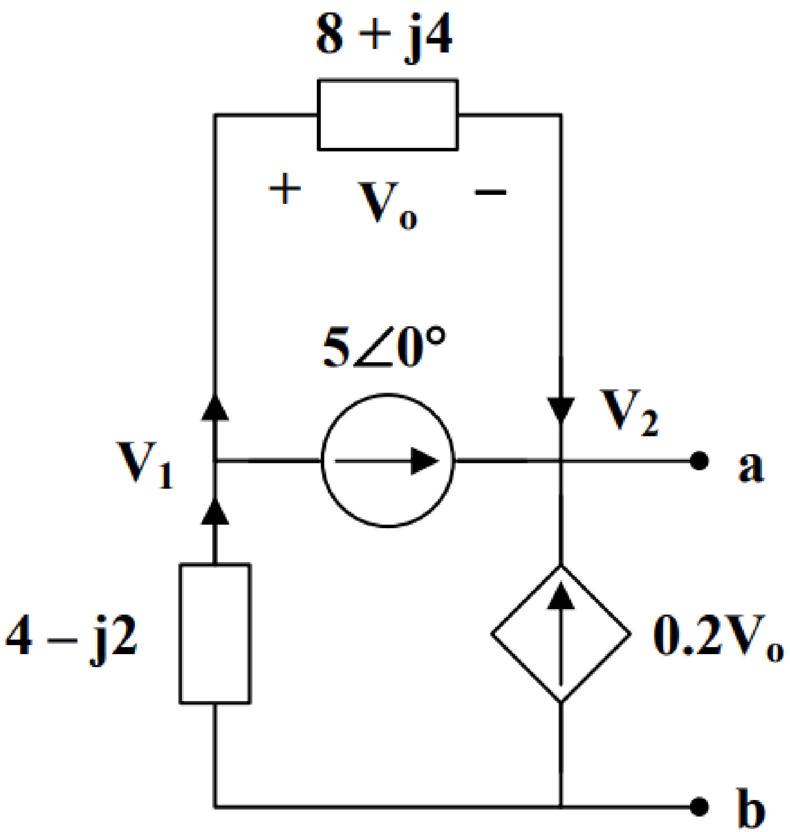
At node a, $$\mathbf{I}_s = -0.2\mathbf{V}_o + \frac{\mathbf{V}_s}{8+j4+4-j2}$$

But, $\mathbf{V}_s = 1$ and $-\mathbf{V}_o = \dfrac{8+j4}{8+j4+4-j2}\mathbf{V}_s$

$$\mathbf{I}_s = (0.2)\frac{8+j4}{12+j2} + \frac{1}{12+j2} = \frac{2.6+j0.8}{12+j2}$$

continued…

$$\mathbf{Z}_{th} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{1}{\mathbf{I}_s} = \frac{12 + j2}{2.6 + j0.8} = 4.473\angle{-7.64^\circ}\ \Omega$$



At node 1, $$\frac{0 - \mathbf{V}_1}{4 - j2} = 5 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j4}$$

$$50 = (1 - j0.5)\mathbf{V}_2 - (3 + j0.5)\mathbf{V}_1$$

At node 2, $$5 + 0.2\mathbf{V}_o + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j4} = 0$$

where $\mathbf{V}_o = \mathbf{V}_1 - \mathbf{V}_2$

$$5 + 0.2(\mathbf{V}_1 - \mathbf{V}_2) + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j4} = 0$$

$$\mathbf{V}_1 = \mathbf{V}_2 - \frac{50}{3 + j0.5}$$

continued…

Substituting this $\mathbf{V}_1$ into *node equation* 1:

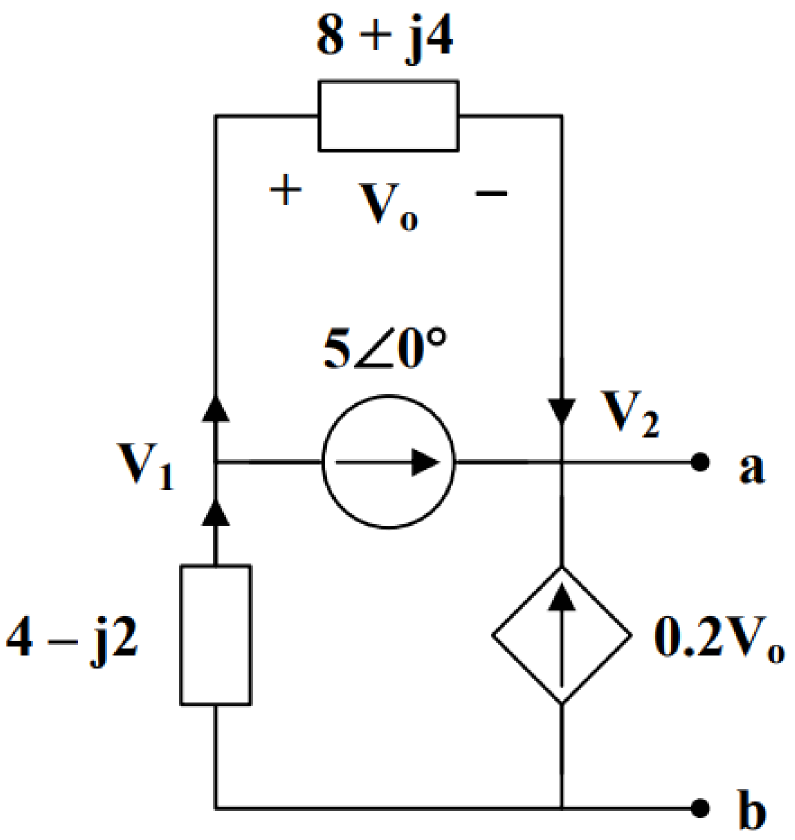
$$50 = (1 - j0.5)\mathbf{V}_2 - (3 + j0.5)\mathbf{V}_2 + (50)\frac{3 + j0.5}{3 - j0.5}$$

$$0 = -50 - (2 + j)\mathbf{V}_2 + \frac{50}{37}(35 + j12)$$

$$\mathbf{V}_2 = \frac{-2.702 + j16.22}{2 + j}$$

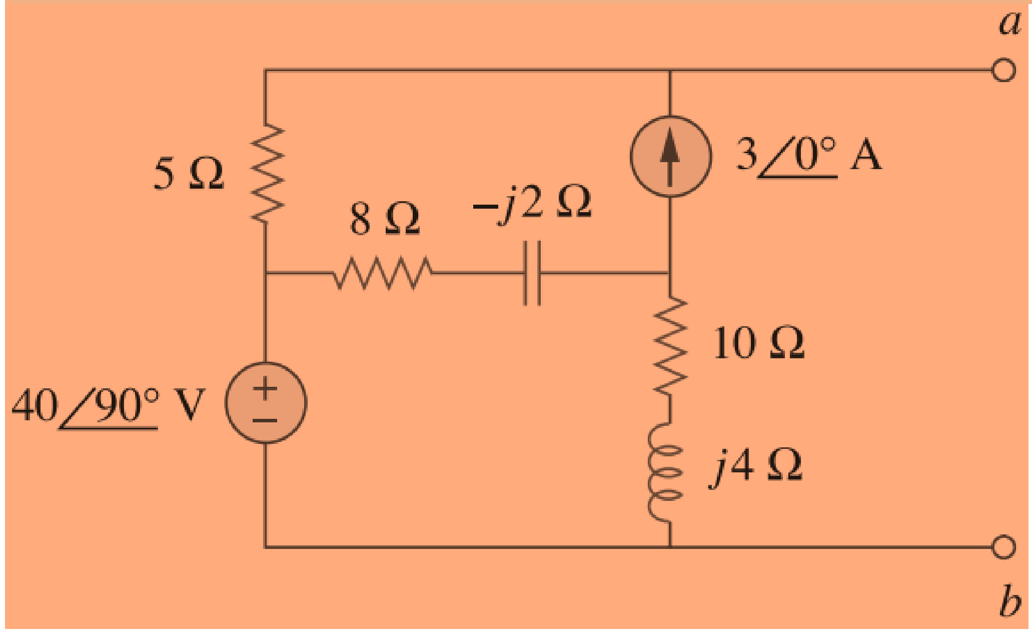
$$= 7.35\angle 72.9°$$

$$\mathbf{V}_{th} = \mathbf{V}_2 = 7.35\underline{/72.9°}\ \mathrm{V}$$

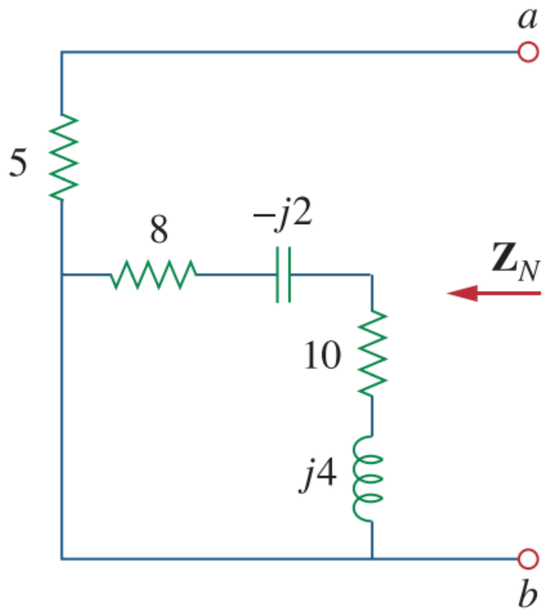


continued…

Obtain the Norton equivalent at terminals *a-b*



$$\mathbf{Z}_N = 5\ \Omega$$



the same way as $\mathbf{Z}_{\text{Th}}$

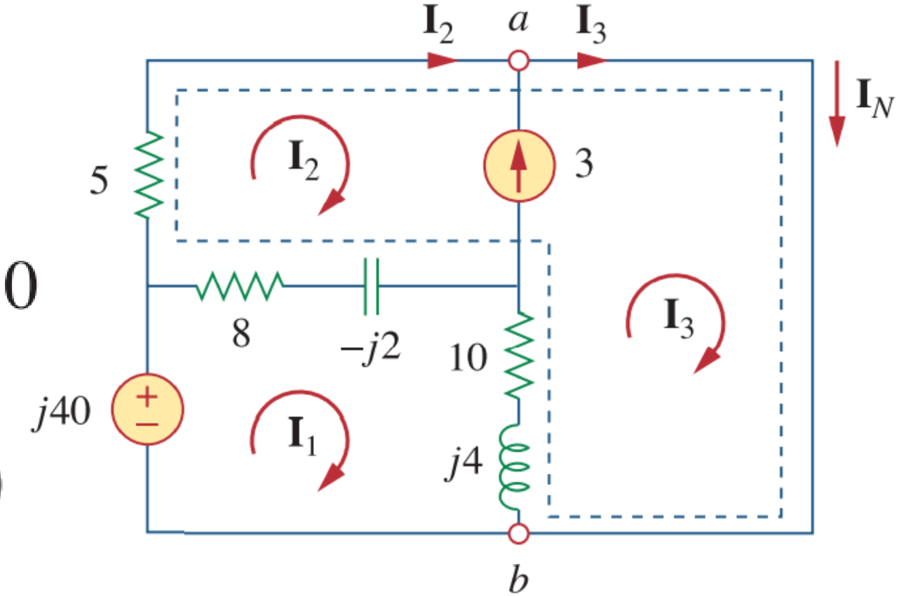
To get $\mathbf{I}_N$, we short-circuit terminals *a-b*

For mesh 1,

$$-j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0$$

For the supermesh,

$$(13 - j2)\mathbf{I}_2 + (10 + j4)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0$$

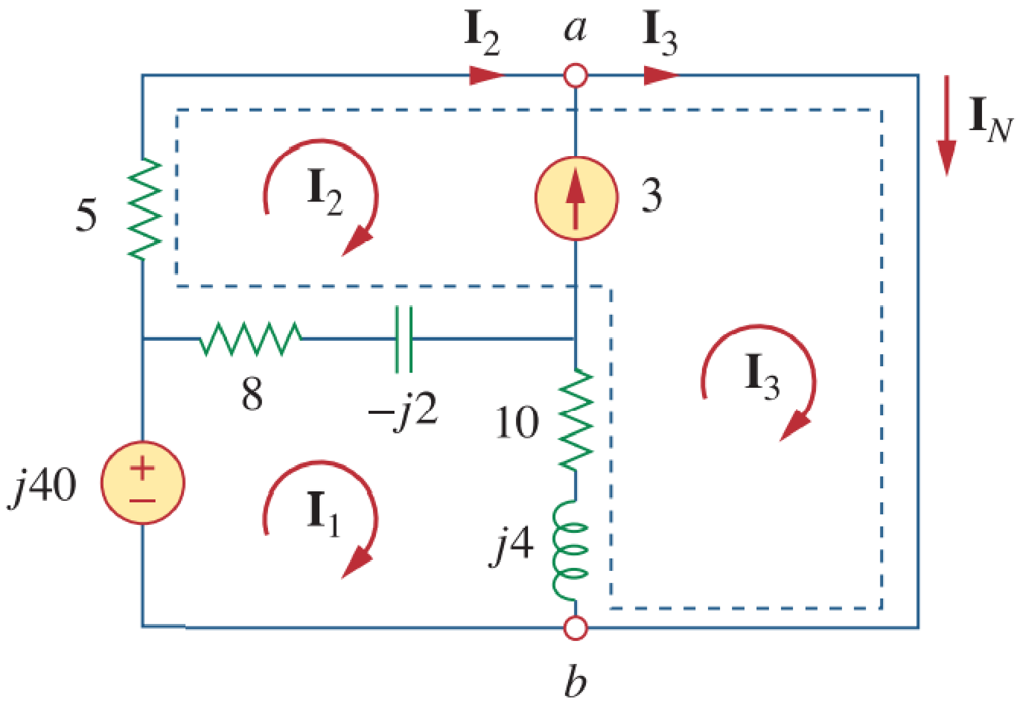


continued…

At node a, $\mathbf{I}_3 = \mathbf{I}_2 + 3$

After solving, $\mathbf{I}_2 = j8$

$$\mathbf{I}_3 = \mathbf{I}_2 + 3 = 3 + j8$$



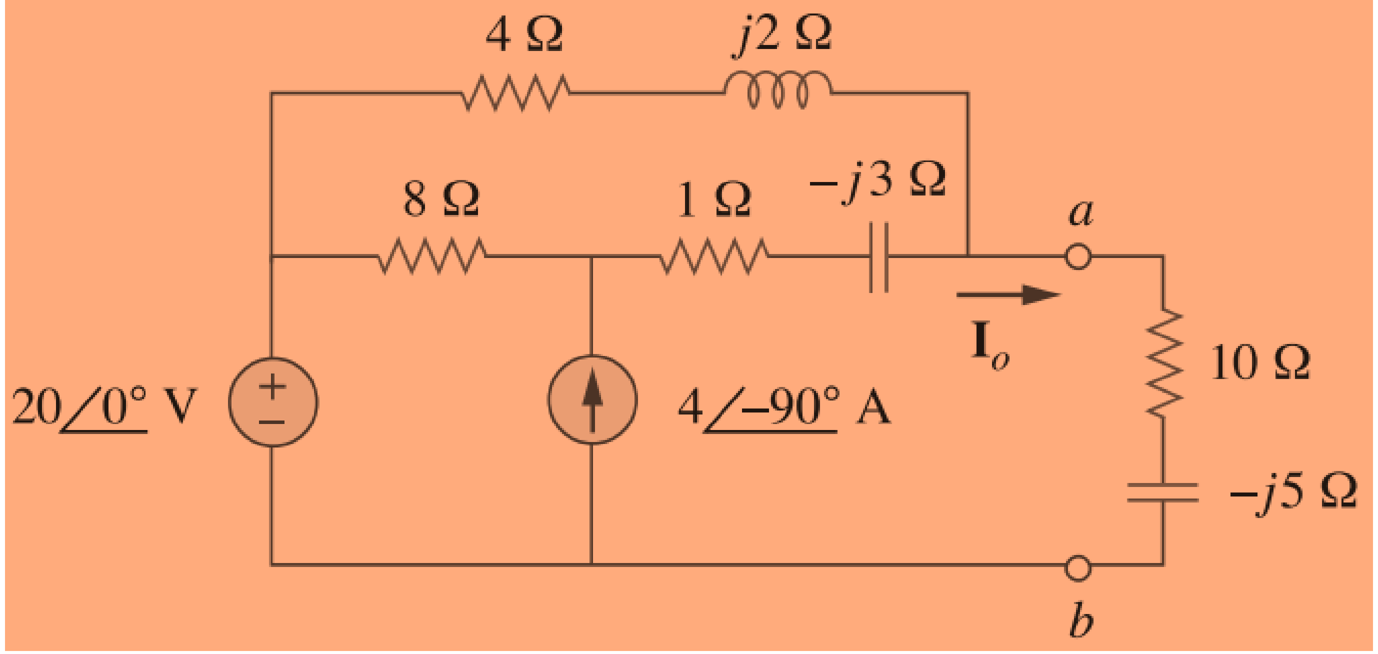
The Norton current $\mathbf{I}_N = \mathbf{I}_3$

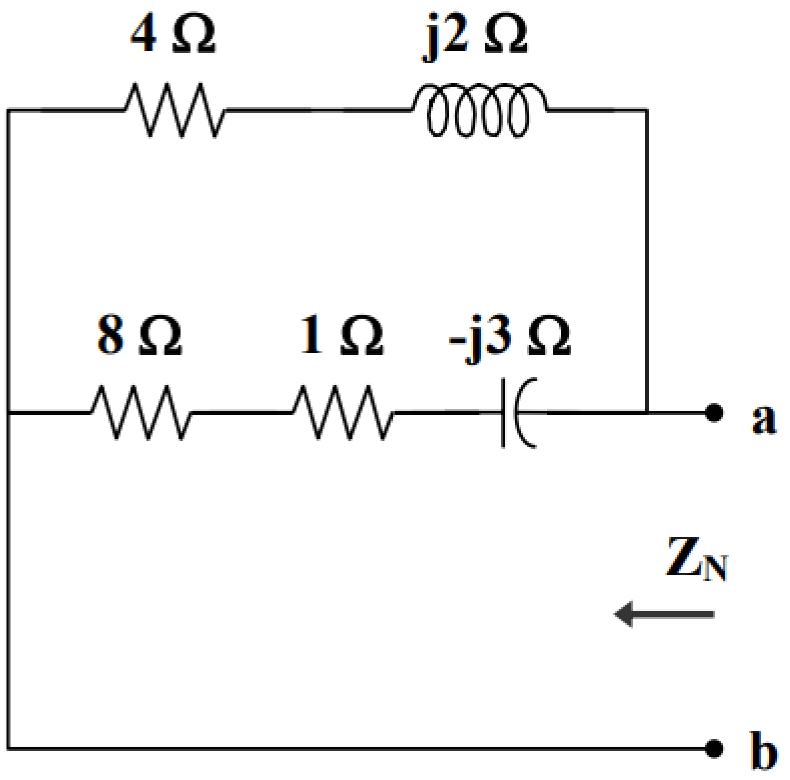
$$= (3 + j8) \text{ A}$$

$$= 8.54 \angle 69.44° \text{ A}$$

continued…

Determine the Norton equivalent of the circuit from terminals *a-b*. Use the equivalent to find $\mathbf{I}_o$.

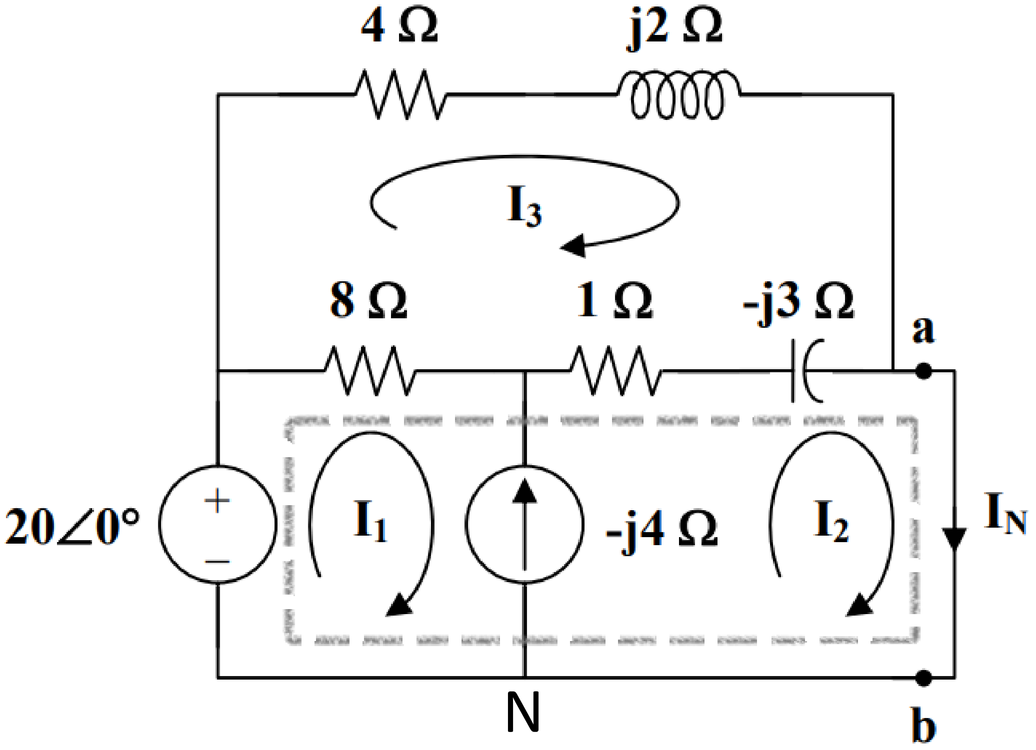




$$\mathbf{Z}_N = (4+j2)\,||\,(9-j3) = \frac{(4+j2)(9-j3)}{13-j}$$

$$= 3.176 + j0.706\ \Omega$$

continued…



For the supermesh,

$$-20 + 8\mathbf{I}_1 + (1 - j3)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 0$$

For mesh 3,

$$(13 - j)\mathbf{I}_3 - 8\mathbf{I}_1 - (1 - j3)\mathbf{I}_2 = 0$$

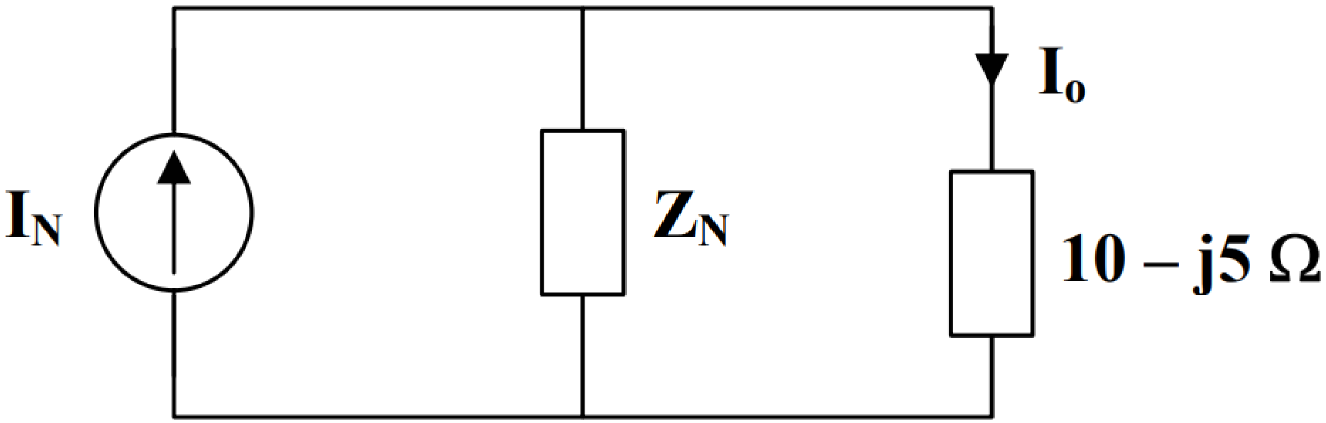
At node N,

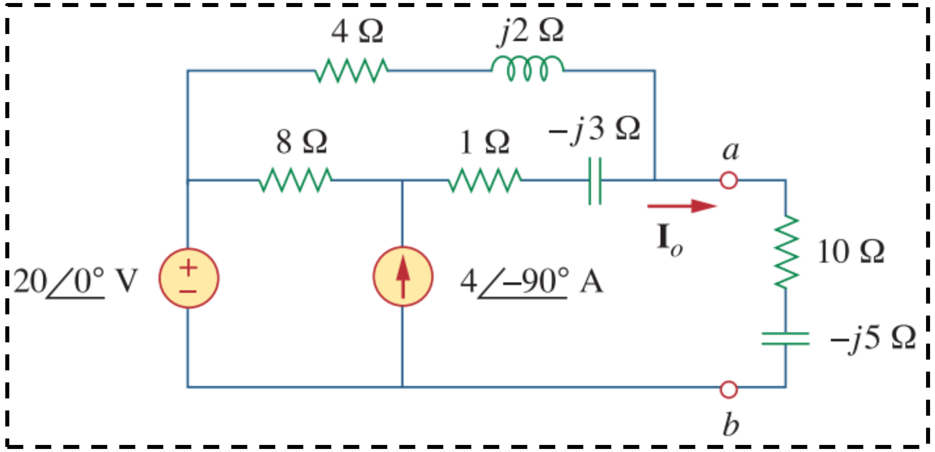
$$\mathbf{I}_2 = \mathbf{I}_1 + (-j4)$$

After solving, $\mathbf{I}_2 = \dfrac{50 - j62}{9 - j3}$

$$\mathbf{I}_N = \mathbf{I}_2 = \frac{50 - j62}{9 - j3} = \frac{79.65\angle -51.11^\circ}{9.487\angle -18.43^\circ} = 8.396\angle{-32.68^\circ}\ \text{A}$$

continued...





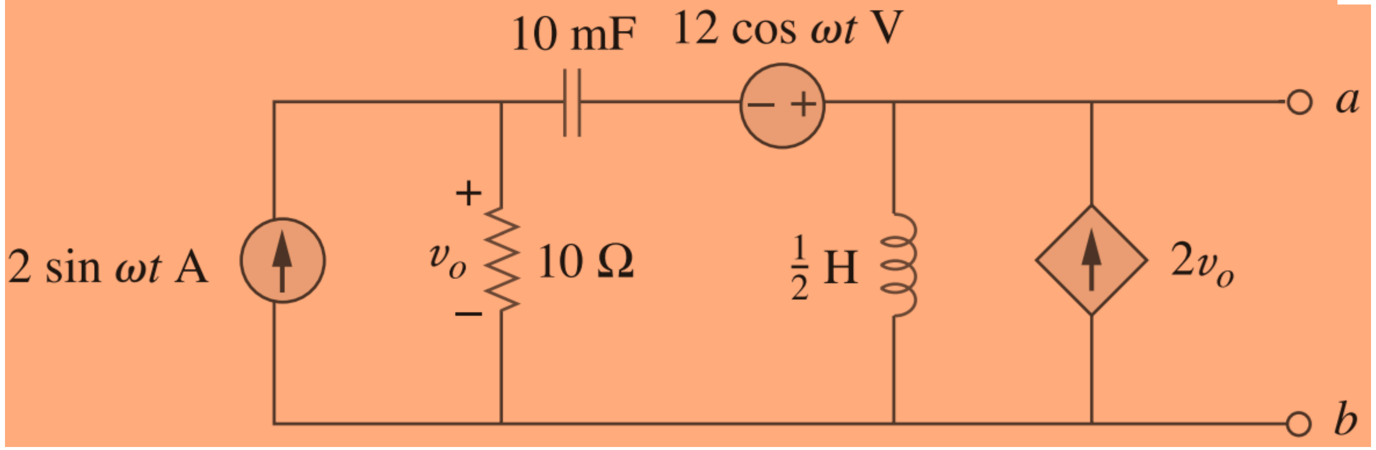
$$\mathbf{I}_o = \frac{\mathbf{Z}_N}{\mathbf{Z}_N + 10 - j5}\mathbf{I}_N = \frac{3.176 + j0.706}{13.176 - j4.294}(8.396\angle -32.68^\circ)$$

$$= \frac{(3.254\angle 12.53^\circ)(8.396\angle -32.68^\circ)}{13.858\angle -18.05^\circ}$$

$$= 1.9714\angle -2.10^\circ \text{ A}$$

continued...

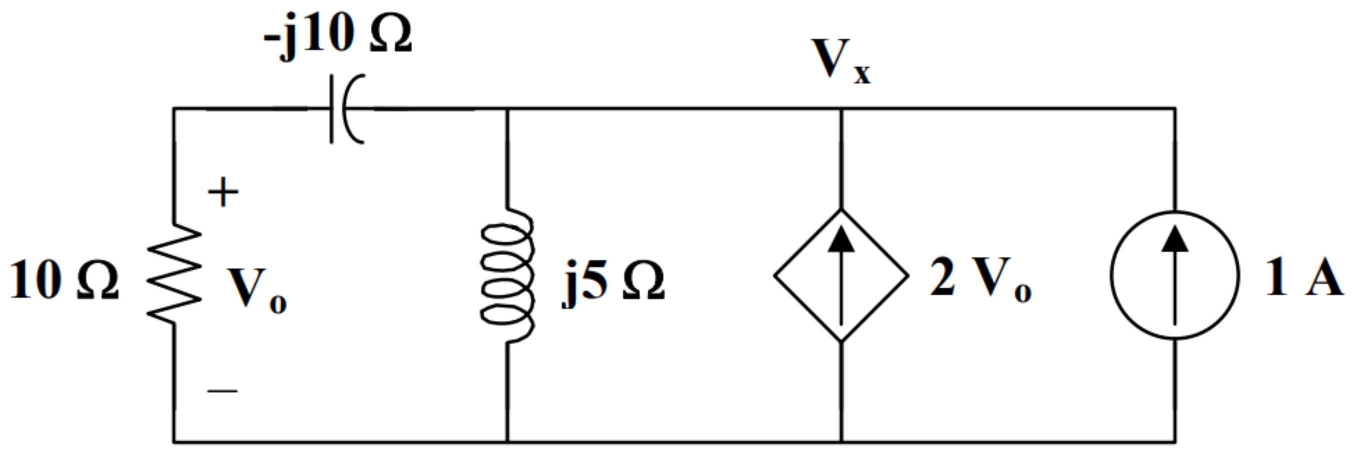
At terminals *a-b*, obtain Thevenin and Norton equivalent circuits for the network depicted Take $\omega = 10$ rad/s.



$$\omega = 10$$

$$0.5\text{ H} \longrightarrow j\omega L = j(10)(0.5) = j5$$

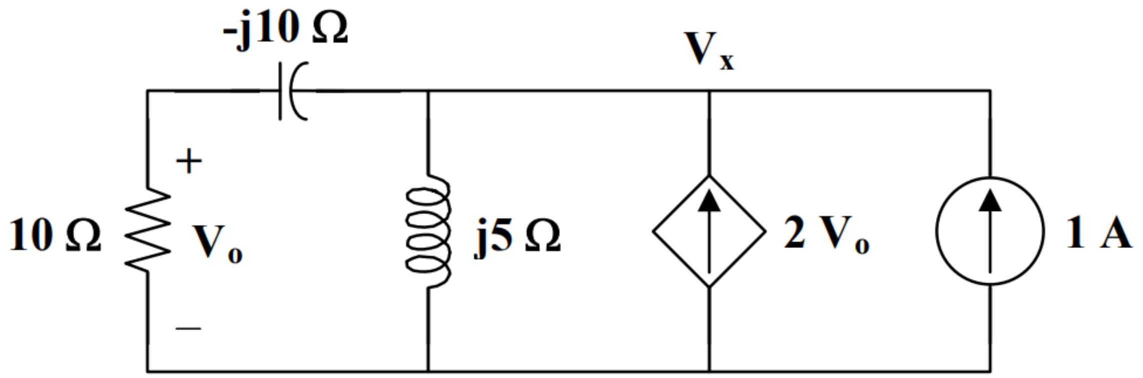
$$10\text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(10\times10^{-3})} = -j10$$



Finding $\mathbf{Z}_{Th}$:

Due to the presence of the dependent source, we connect a 1-A current source (*or* voltage source if you want).

continued…

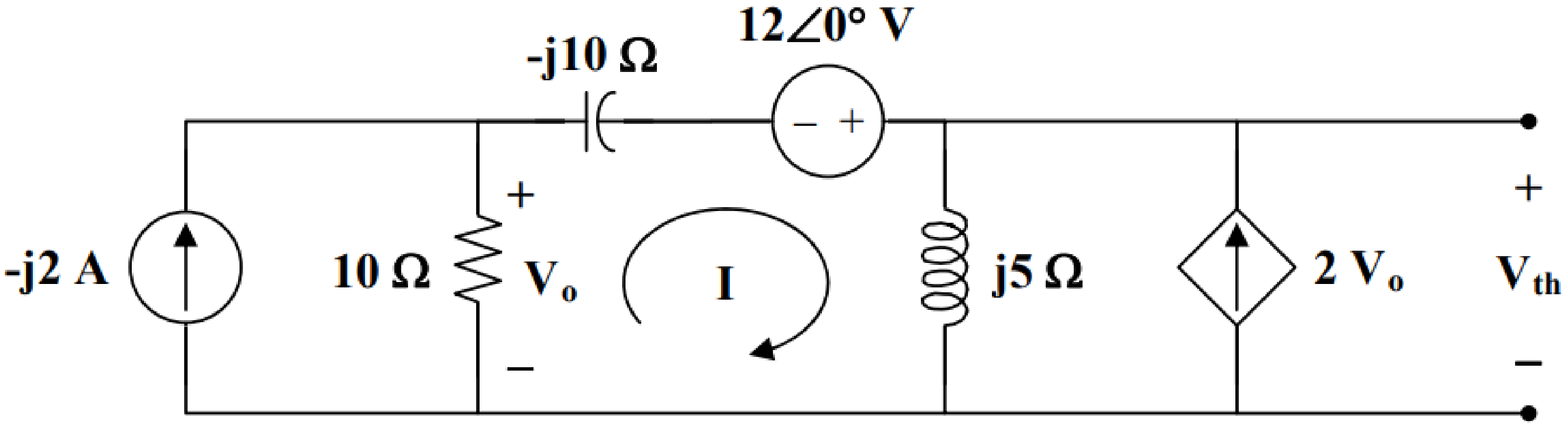


$$1 + 2\mathbf{V}_o = \frac{\mathbf{V}_x}{j5} + \frac{\mathbf{V}_x}{10 - j10}$$

where $\mathbf{V}_o = \dfrac{10\mathbf{V}_x}{10 - j10}$

$$1 + \frac{19\,\mathbf{V}_x}{10 - j10} = \frac{\mathbf{V}_x}{j5} \longrightarrow \mathbf{V}_x = \frac{-10 + j10}{21 + j2}$$

$$\mathbf{Z}_N = \mathbf{Z}_{th} = \frac{\mathbf{V}_x}{1} = -0.42 + 0.51j\ \Omega$$



Finding $\mathbf{V}_{Th}$

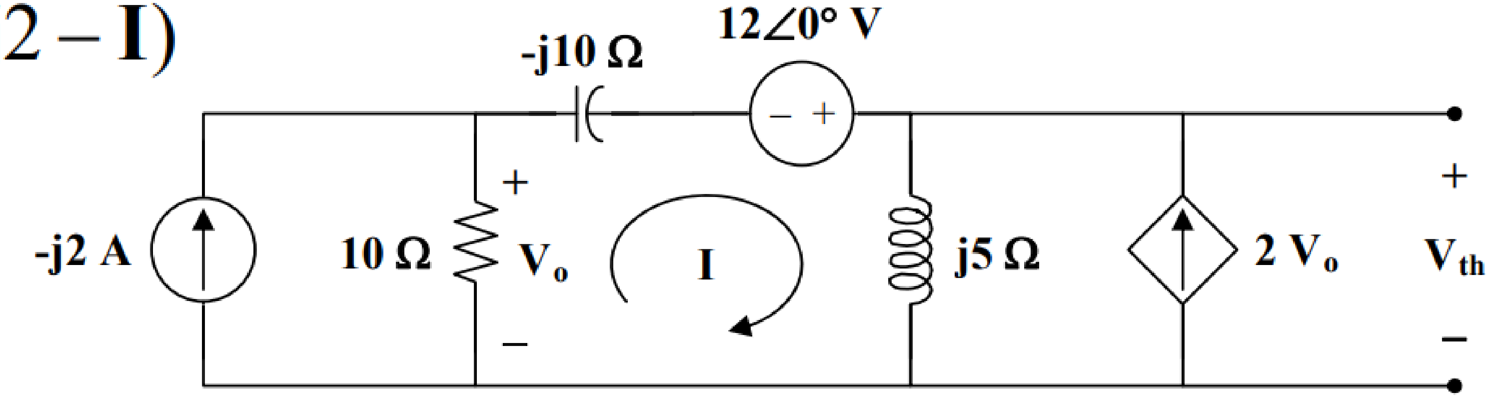
continued…

$$(10 - j10 + j5)\mathbf{I} - (10)(-j2) + j5(2\mathbf{V}_o) - 12 = 0$$

where $\mathbf{V}_o = (10)(-j2 - \mathbf{I})$

$$(10 - j105)\mathbf{I} = -188 - j20$$

$$\mathbf{I} = \frac{188 + j20}{-10 + j105}$$



$$\mathbf{V}_{th} = j5(\mathbf{I} + 2\mathbf{V}_o) = j5(-19\mathbf{I} - j40) = -j95\mathbf{I} + 200$$

$$= \frac{-j95(188 + j20)}{-10 + j105} + 200 = 29.73 - j1.8723$$

$$= \mathbf{29.79\angle{-3.6^\circ}\ V}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{29.79\angle -3.6^\circ}{0.67\angle 129.56^\circ} = \mathbf{44.46\angle{-133.16^\circ}\ A}$$

Instantaneous Power

The instantaneous power (in watts) is the power at any instant of time.

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

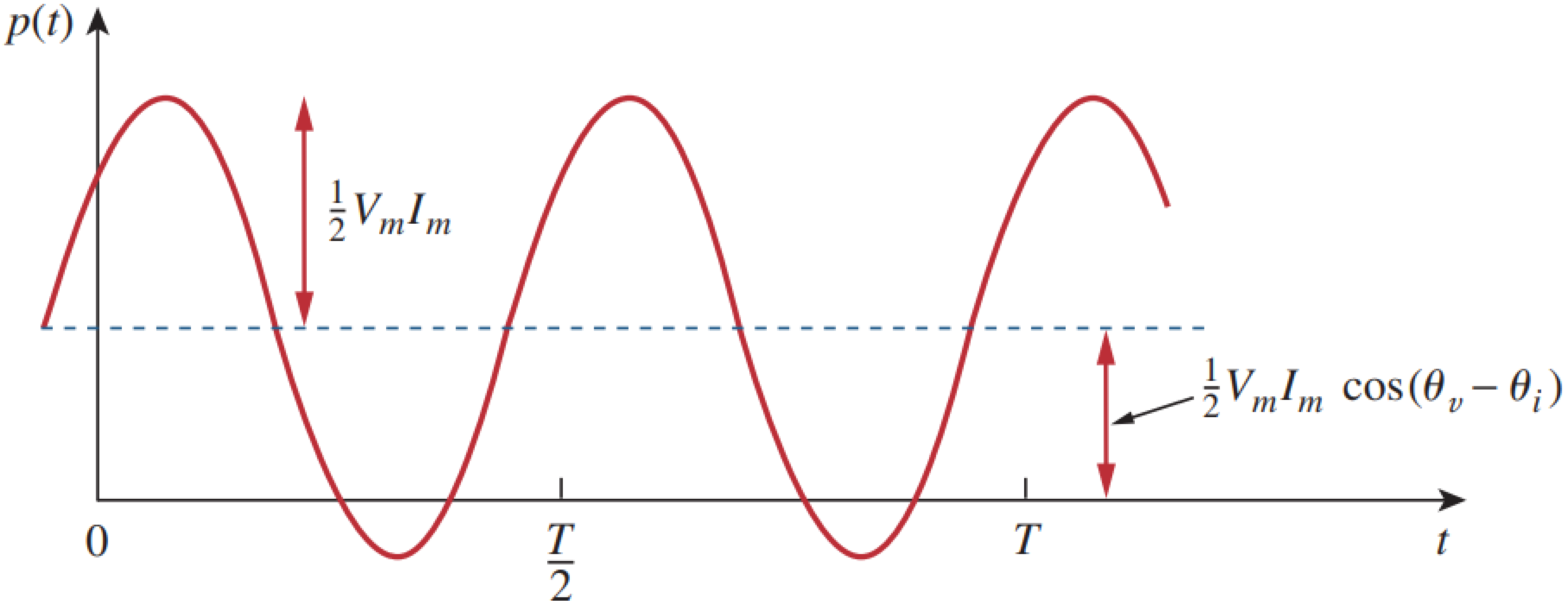
$$p(t) = v(t)i(t)$$

$$= V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

continued…



$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

- The first part is constant (time independent). Its value depends on the phase difference between voltage & current.
- The second part is a sinusoidal function whose frequency is 2ω.

Average Power

The average power (in watts) is the average of the instantaneous power over one period.

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\,dt$$

$$+ \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt$$

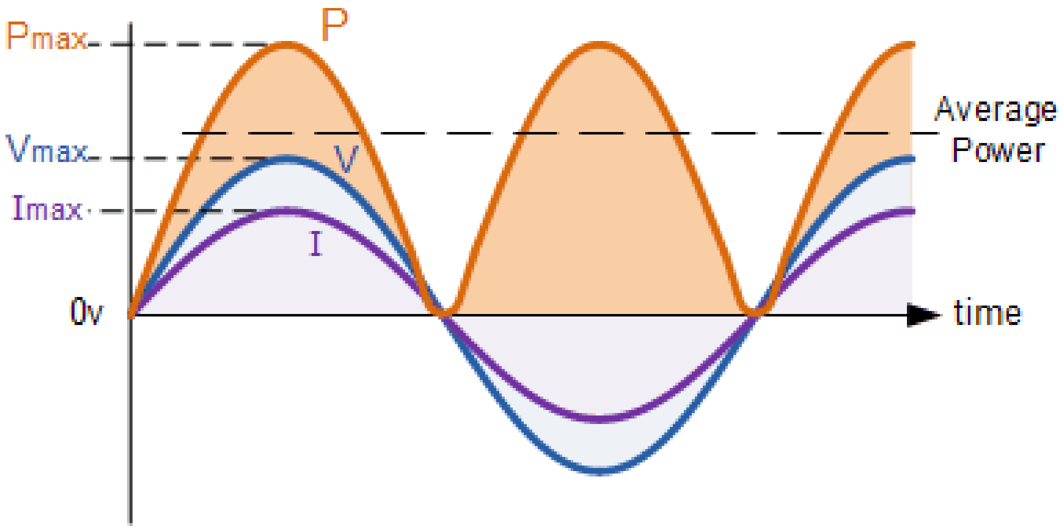
$$= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

We know, $\frac{1}{2}\mathbf{VI}^* = \frac{1}{2}V_m I_m \underline{/\theta_v - \theta_i} = \frac{1}{2}V_m I_m[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)]$

Thus, $P = \frac{1}{2}\mathrm{Re}[\mathbf{VI}^*]$

continued…

- For purely resistive circuit, current & voltage are in phase.



$$P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = \frac{1}{2}|\mathbf{I}|^2 R$$

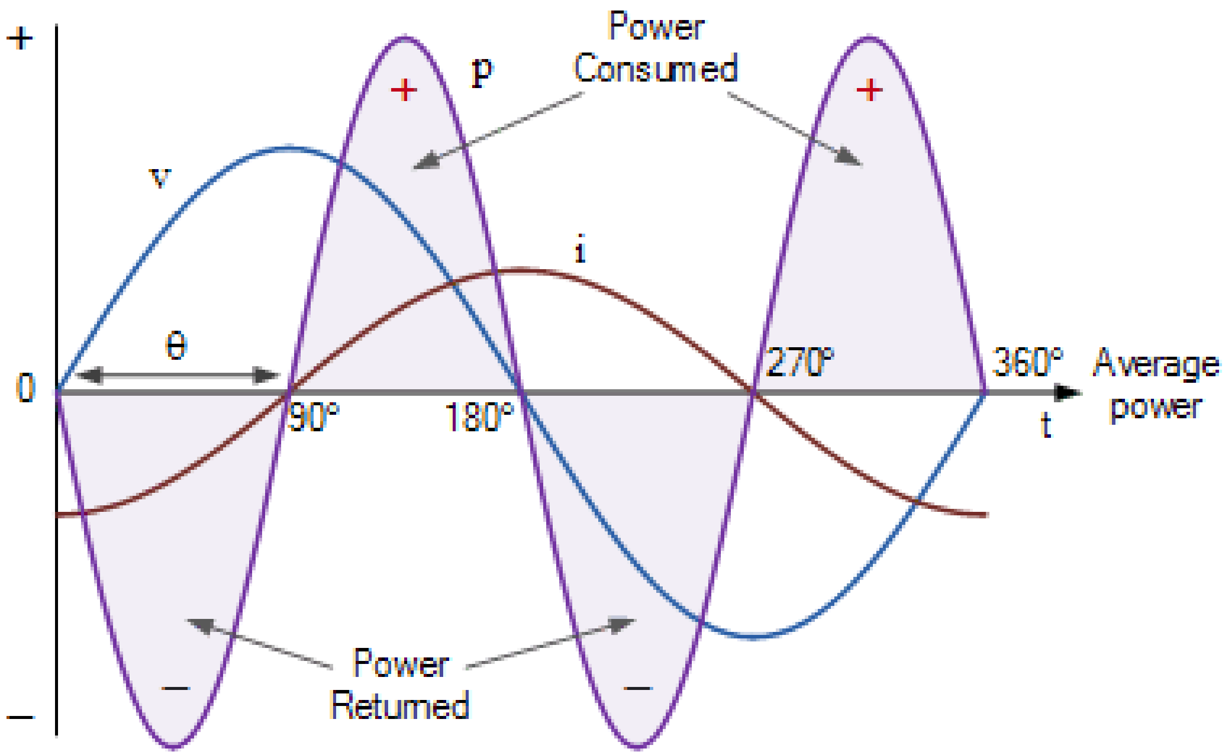
- For purely reactive circuit, current & voltage are out of phase (±90°). waveform on next page

$$P = \frac{1}{2}V_m I_m \cos 90^\circ = 0$$

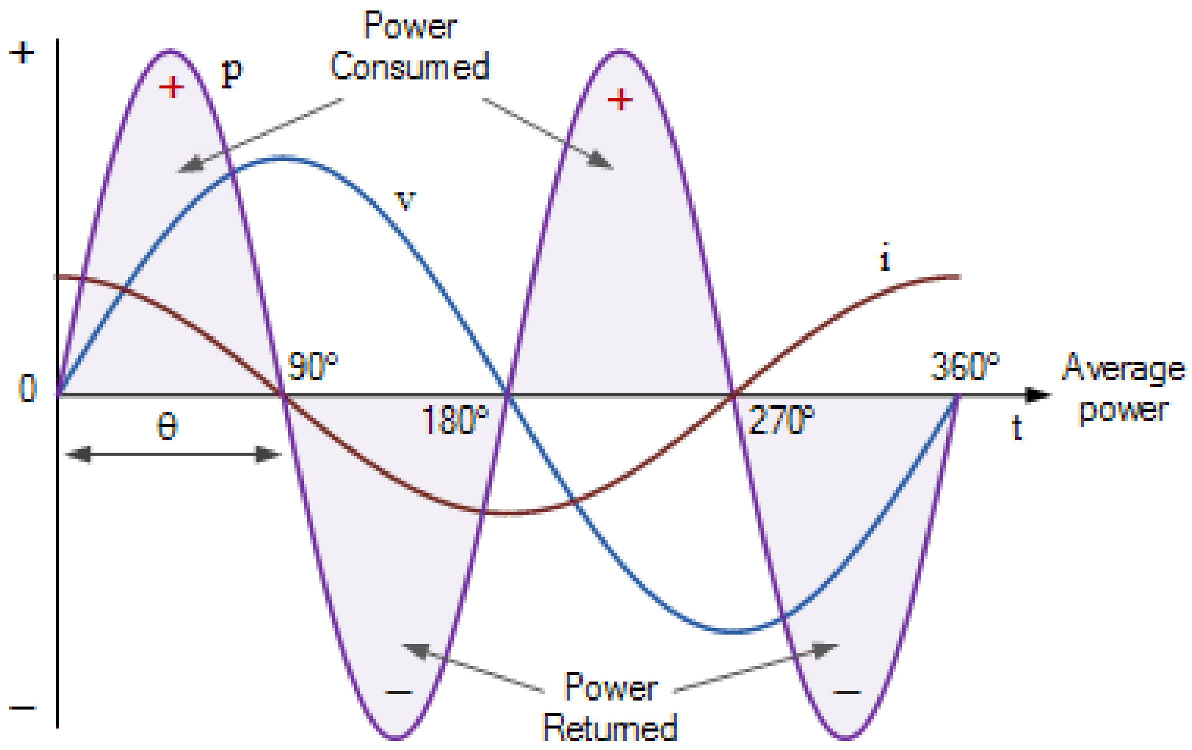
✓ A resistive load (R) absorbs power at all times, while a reactive load (L *or* C) absorbs zero average power.

continued…

Pure Inductor



Pure Capacitor



Example

Given that

$$v(t) = 120\cos(377t + 45°)\text{ V} \quad \text{and} \quad i(t) = 10\cos(377t - 10°)\text{ A}$$

find the instantaneous power and the average power absorbed by the passive linear network

$$p = vi = 1200\cos(377t + 45°)\cos(377t - 10°)$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$= 600[\cos(754t + 35°) + \cos 55°]$$

$$= 344.2 + 600\cos(754t + 35°)\text{ W}$$

average power is 344.2 W

You can use this too:

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}120(10)\cos[45° - (-10°)]$$

$$= 600\cos 55° = 344.2\text{ W}$$

continued…

Calculate the average power absorbed by an impedance $\mathbf{Z} = 30 - j70\ \Omega$ when a voltage $\mathbf{V} = 120\underline{/0^\circ}$ is applied across it.

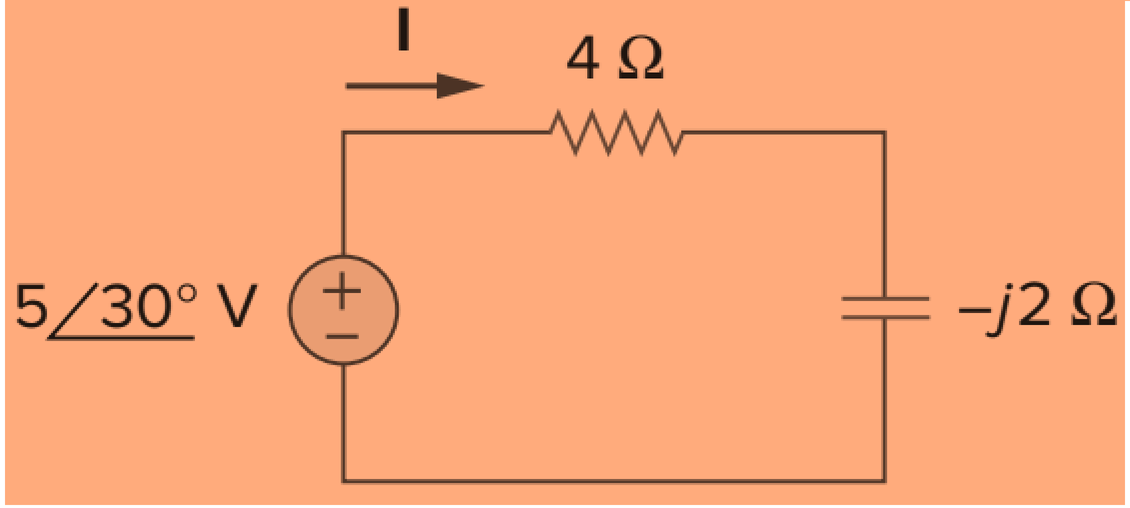
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{120\underline{/0^\circ}}{30 - j70} = 1.576\underline{/66.8^\circ}\ \text{A}$$

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(120)(1.576)\cos(0 - 66.8^\circ)$$

$$= 37.24\ \text{W}$$

continued...

For the circuit shown, find the average power supplied by the source and the average power absorbed by the resistor.



$$\mathbf{I} = \frac{5\angle 30^\circ}{4 - j2} = 1.118\angle 56.57^\circ \text{ A}$$

$$\mathbf{I}_R = \mathbf{I}$$

$$\mathbf{V}_R = 4\mathbf{I}_R = 4.472\angle 56.57^\circ \text{ V}$$

average power supplied by the voltage source

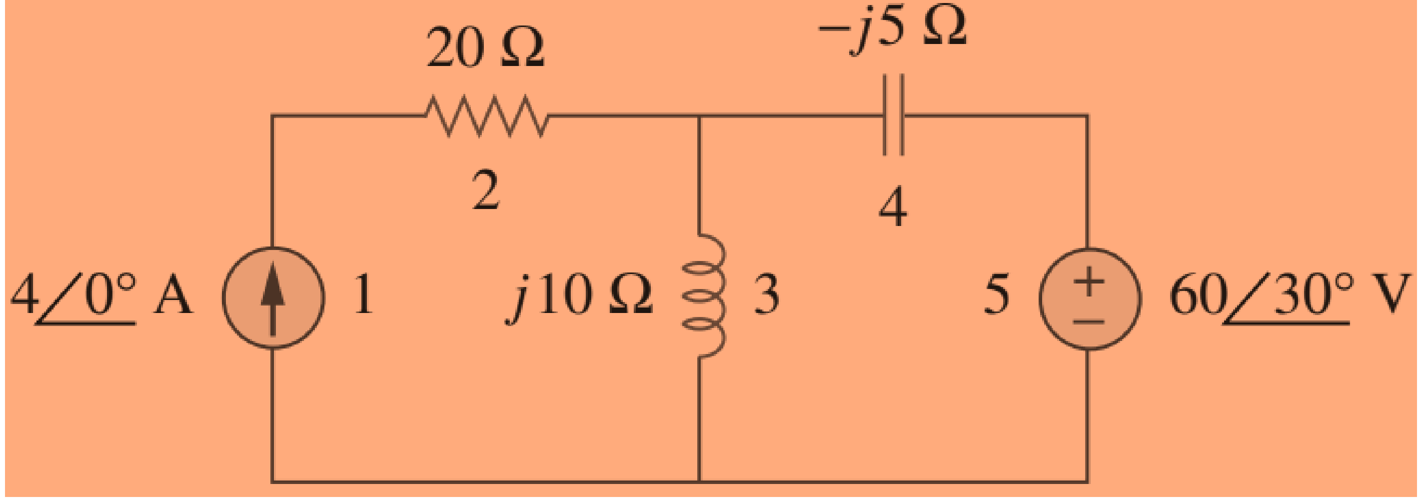
$$P = \frac{1}{2}(5)(1.118)\cos(30^\circ - 56.57^\circ) = 2.5 \text{ W}$$

average power absorbed by the resistor

$$P = \frac{1}{2}(4.472)(1.118) = 2.5 \text{ W}$$

continued…

Determine the average power generated by each source and the average power absorbed by each passive element in the circuit



For mesh 1,

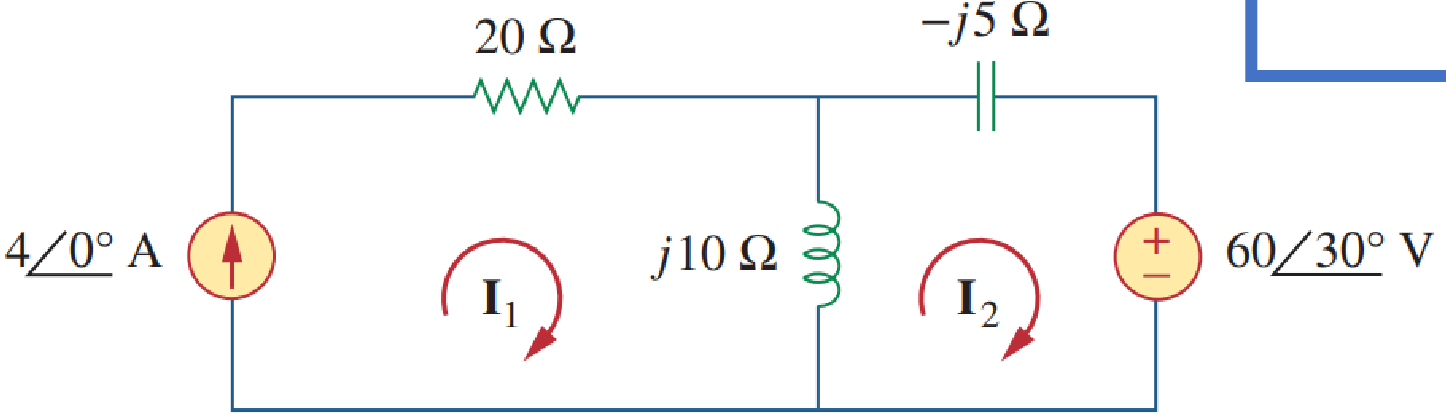
$$\mathbf{I}_1 = 4 \text{ A}$$

For mesh 2,

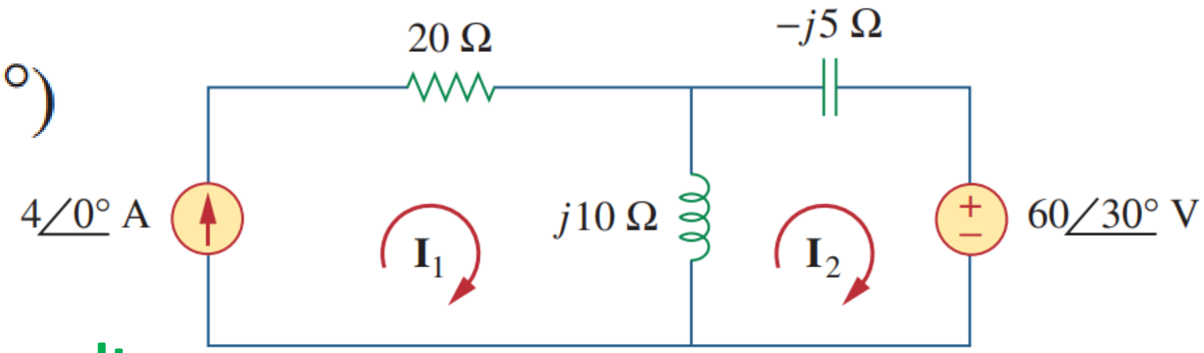
$$(j10 - j5)\mathbf{I}_2 - j10\mathbf{I}_1 + 60\angle 30^\circ = 0$$

$$\mathbf{I}_2 = -12\angle -60^\circ + 8$$

$$= 10.58\angle 79.1^\circ \text{ A}$$



continued...



$$P_5 = \frac{1}{2}(60)(10.58)\cos(30^\circ - 79.1^\circ)$$
$$= 207.8 \text{ W}$$

The circuit is delivering average power to the **voltage source**.

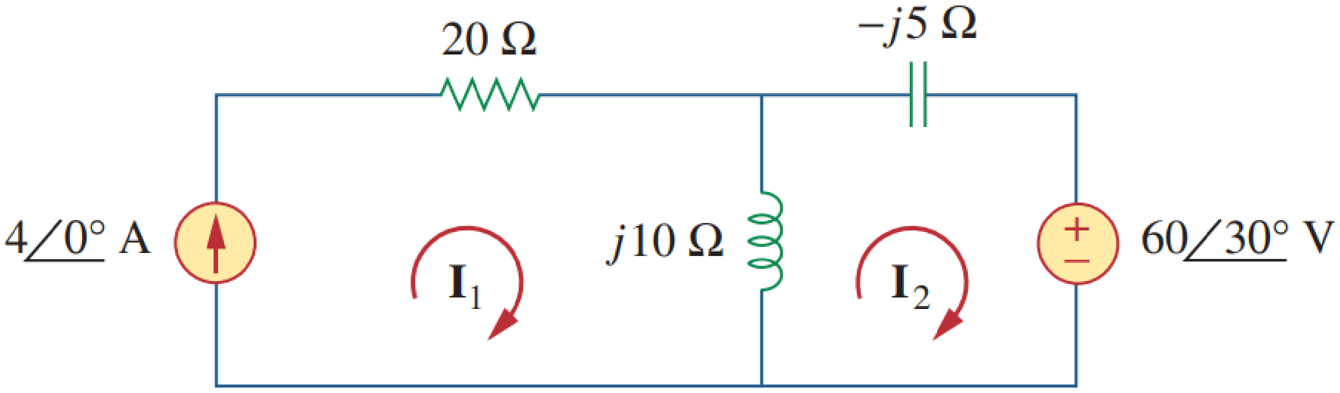
$$\mathbf{V}_1 = 20\mathbf{I}_1 + j10(\mathbf{I}_1 - \mathbf{I}_2) = 80 + j10(4 - 2 - j10.39)$$
$$= 184.984\angle 6.21^\circ \text{ V}$$

$$P_1 = -\frac{1}{2}(184.984)(4)\cos(6.21^\circ - 0) = -367.8 \text{ W}$$

The **current source** is delivering average power to the circuit.

$$P_2 = \frac{1}{2}(80)(4) = 160 \text{ W}$$ → This average power is absorbed by the **resistor**.

continued...



For the capacitor,

$$\mathbf{I}_2 = 10.58\underline{/79.1^\circ}$$

voltage across it is $-j5\mathbf{I}_2$

$$= (5\underline{/-90^\circ})(10.58\underline{/79.1^\circ}) = 52.9\underline{/79.1^\circ - 90^\circ}$$

$$P_4 = \frac{1}{2}(52.9)(10.58)\cos(-90^\circ) = 0$$

For the inductor,

current through it is $(\mathbf{I}_1 - \mathbf{I}_2) = 4 - 10.58\angle 79.1^\circ$

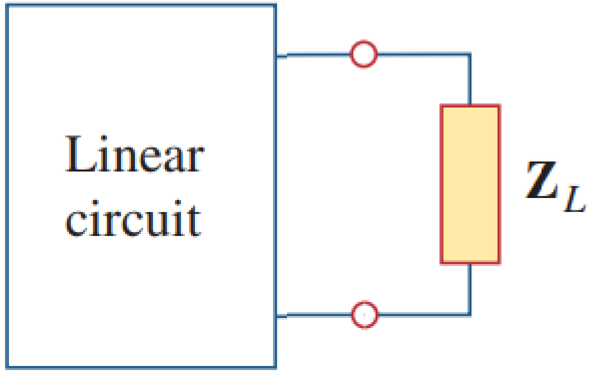
$$= 10.58\angle -79.1^\circ \text{ A}$$

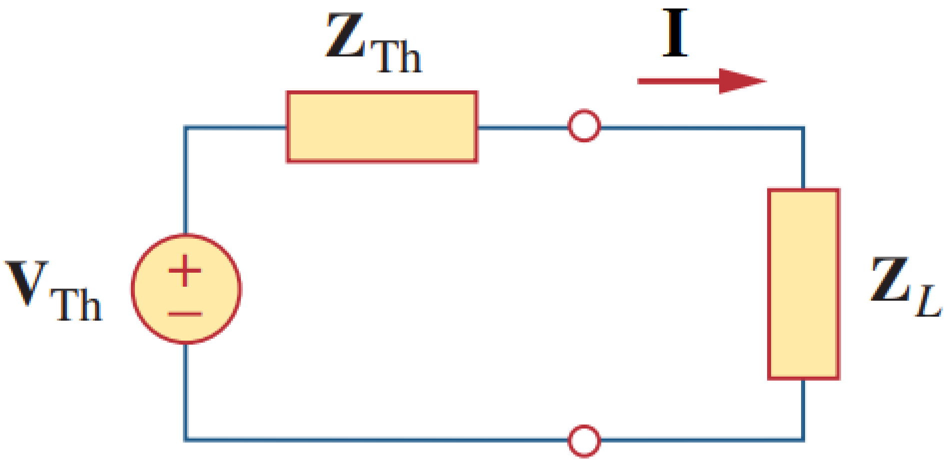
voltage across it is $j10(\mathbf{I}_1 - \mathbf{I}_2) = (10\angle 90^\circ)(10.58\angle -79.1^\circ)$

$$= 105.8\angle 10.9^\circ \text{ V}$$

$$P_3 = \frac{1}{2}(105.8)(10.58)\cos 90^\circ = 0$$

Maximum Average Power Transfer





$$\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$$

$$\mathbf{Z}_L = R_L + jX_L$$

For maximum average power transfer, the load impedance $\mathbf{Z}_L$ must be equal to the complex conjugate of the Thevenin impedance $\mathbf{Z}_{Th}$.

$$\mathbf{Z}_L = R_L + jX_L = R_{Th} - jX_{Th} = \mathbf{Z}_{Th}^*$$

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{8R_{Th}}$$

continued…

- In a special situation in which the load is purely real:
  The load R_L must be chosen such that

$$R_L = \sqrt{{R_{Th}}^2 + {X_{Th}}^2} = |\mathbf{Z}_{Th}|$$

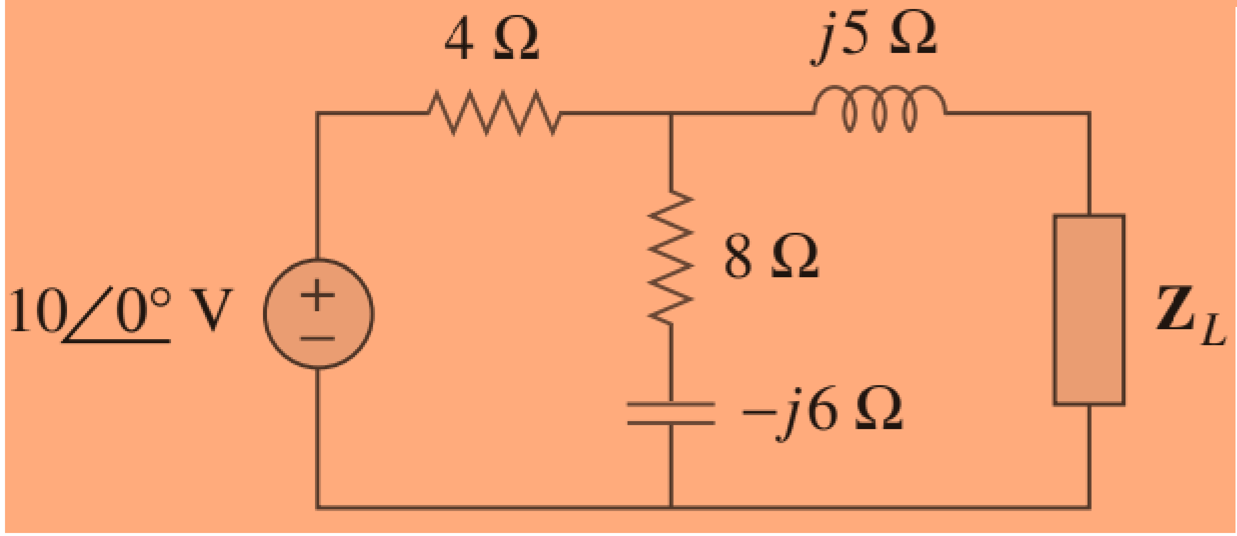
Then,

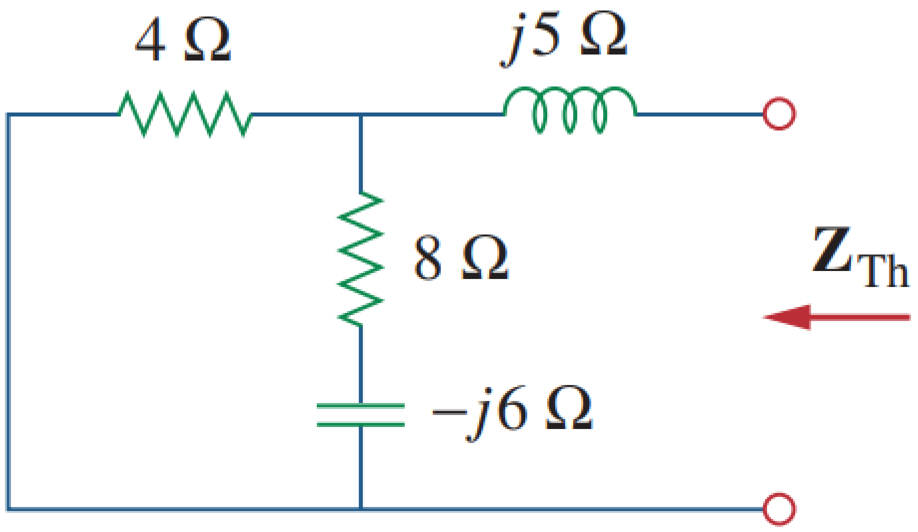
$$P_{max} = \frac{1}{2}|\mathbf{I}|^2 R_L$$

where $\mathbf{I}$ is the current phasor flowing through load R_L.

Example

Determine the load impedance $\mathbf{Z}_L$ that maximizes the average power drawn from the circuit What is the maximum average power?

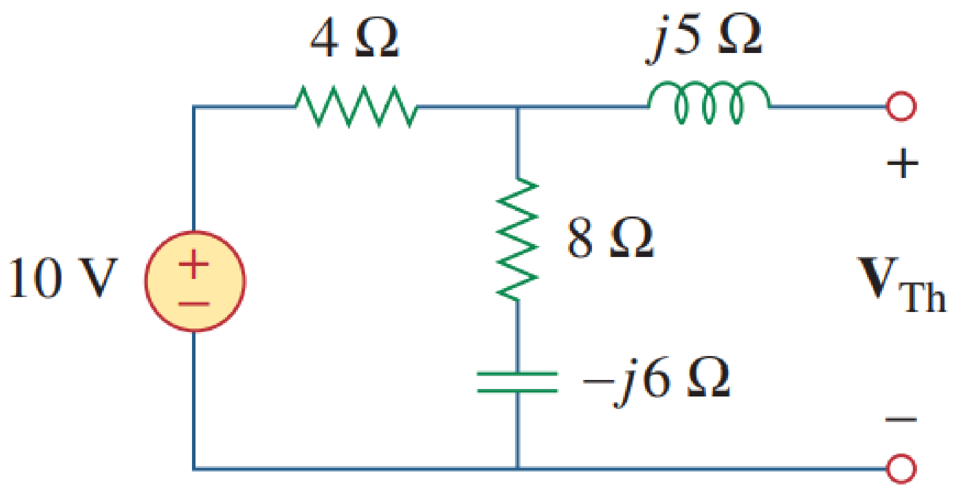




$$\mathbf{Z}_{\text{Th}} = j5 + 4 \parallel (8 - j6)$$

$$= j5 + \frac{4(8 - j6)}{4 + 8 - j6} = 2.933 + j4.467\ \Omega$$

continued…



$$\mathbf{V}_{Th} = \frac{8 - j6}{4 + 8 - j6}(10) = 7.454\angle{-10.3^\circ} \text{ V}$$

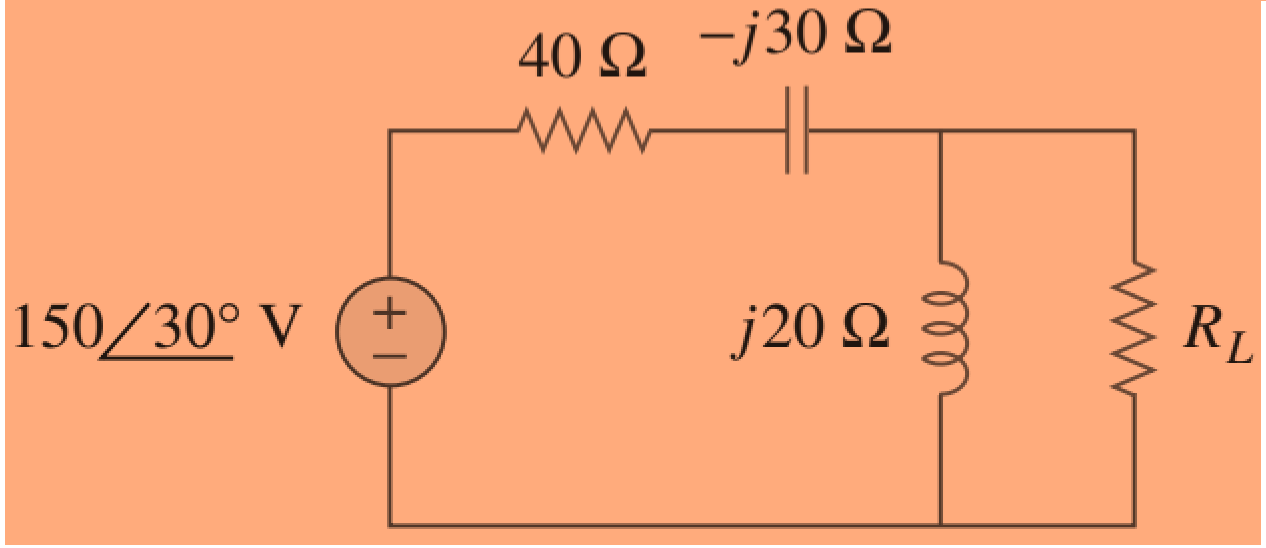
The load impedance draws the maximum power from the circuit when

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* = 2.933 - j4.467 \ \Omega$$

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{8R_{Th}} = \frac{(7.454)^2}{8(2.933)} = 2.368 \text{ W}$$

continued...

In the circuit , find the value of R_L that will absorb the maximum average power. Calculate that power.



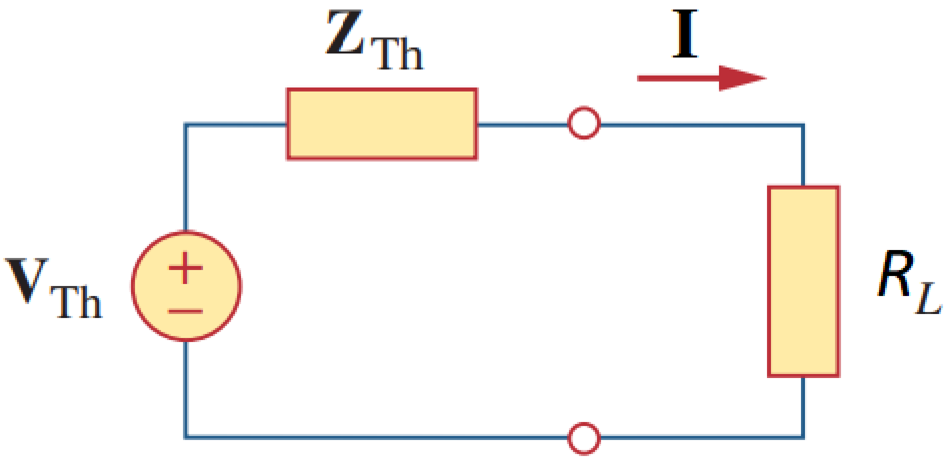
$$\mathbf{Z}_{Th} = (40 - j30) \parallel j20 = \frac{j20(40 - j30)}{j20 + 40 - j30} = 9.412 + j22.35\ \Omega$$

$$\mathbf{V}_{Th} = \frac{j20}{j20 + 40 - j30}(150\angle 30^\circ) = 72.76\angle 134^\circ\ \text{V}$$

The value of R_L that will absorb the maximum average power is

$$R_L = |\mathbf{Z}_{Th}| = \sqrt{9.412^2 + 22.35^2} = 24.25\ \Omega$$

continued...



The current through the load is

$$\mathbf{I} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{Z}_{\text{Th}} + R_L} = \frac{72.76\angle 134^\circ}{33.66 + j22.35} = 1.8\angle 100.42^\circ \text{ A}$$

The maximum average power absorbed by R_L is

$$P_{\max} = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{1}{2}(1.8)^2(24.25) = 39.29 \text{ W}$$

Effective *or* RMS Value

❖The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

$$I_{\text{eff}} = I_{\text{rms}}, \qquad V_{\text{eff}} = V_{\text{rms}}$$

For any periodic function $x(t)$ in general,

$$X_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T x^2\,dt}$$

For the sinusoid $i(t) = I_m \cos\omega t$

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}}$$

Similarly, for $v(t) = V_m \cos\omega t$

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$

continued...

The average power

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i)$$

$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

the average power absorbed by a resistor R

$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = \frac{1}{2} |\mathbf{I}|^2 R$$

$$P = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$$

Apparent Power and Power Factor

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$
$$= S \cos(\theta_v - \theta_i)$$

$$S = V_{\text{rms}} I_{\text{rms}}$$

- The apparent power, *S* (in VA) is the product of the rms values of voltage and current.

$$\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

- Power Factor, pf (*dimensionless*) is the ratio of the average power to the apparent power.

continued…

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V_m\underline{/\theta_v}}{I_m\underline{/\theta_i}} = \frac{V_m}{I_m}\underline{/\theta_v - \theta_i} = \frac{V_{\text{rms}}}{I_{\text{rms}}}\underline{/\theta_v - \theta_i}$$

- Power Factor (pf) is the cosine of phase difference between voltage and current. It is also the cosine of the angle of the load impedance.
- Value of pf ranges between zero and unity.
- For a purely resistive load, the voltage and current are in phase, that means $\theta_v = \theta_i$; thus, pf = 1.
  - apparent power (*S*) = average power (*P*)
- For a purely reactive load, $\theta_v - \theta_i = \pm 90°$. Thus, pf = 0.
  - average power (*P*) is zero.
- Leading pf means: current *leads* voltage → capacitive load (negative pf angle)
- Lagging pf means: current *lags* voltage → inductive load (positive pf angle)

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(Part 2)

Electrical Circuits II

Apparent Power and Power Factor

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$
$$= S \cos(\theta_v - \theta_i)$$

$$S = V_{\text{rms}} I_{\text{rms}}$$

- The apparent power, *S* (in VA) is the product of the rms values of voltage and current.

$$\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

- Power Factor, pf (*dimensionless*) is the ratio of the average power to the apparent power.

continued…

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V_m\angle\theta_v}{I_m\angle\theta_i} = \frac{V_m}{I_m}\angle\theta_v - \theta_i = \frac{V_{\text{rms}}}{I_{\text{rms}}}\angle\theta_v - \theta_i$$

- Power Factor (pf) is the cosine of phase difference between voltage and current. It is also the cosine of the angle of the load impedance.
- Value of pf ranges between zero and unity.
- For a purely resistive load, the voltage and current are in phase, that means $\theta_v = \theta_i$; thus, pf = 1.
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  - average power (*P*) is zero.
- Leading pf means: current *leads* voltage → capacitive load (negative pf angle)
- Lagging pf means: current *lags* voltage → inductive load (positive pf angle)

Example

A series-connected load draws a current $i(t) = 4\cos(100\pi t + 10°)$ A when the applied voltage is $v(t) = 120\cos(100\pi t - 20°)$ V. Find the apparent power and the power factor of the load. Determine the element values that form the series-connected load.

apparent power $$S = V_{\text{rms}} I_{\text{rms}} = \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} = 240 \text{ VA}$$

power factor $$\text{pf} = \cos(\theta_v - \theta_i) = \cos(-20° - 10°) = 0.866 \quad \text{(leading)}$$

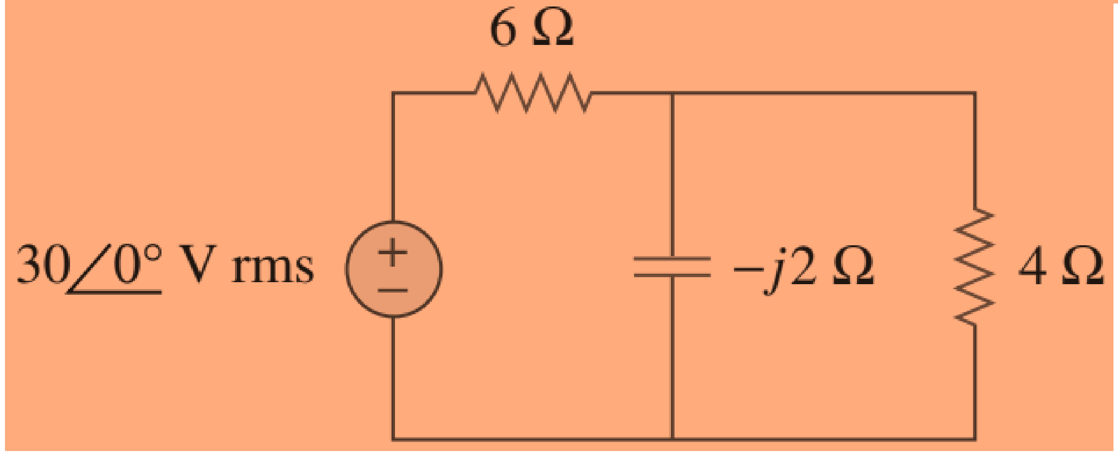
$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120\angle -20°}{4\angle 10°} = 30\angle -30° = 25.98 - j15\ \Omega$$

Load impedance **Z** can be modeled by a 25.98 Ω resistor in series with a capacitor.

$$-15 = -\frac{1}{\omega C} \rightarrow C = \frac{1}{15\omega} = 212.2\ \mu\text{F}$$

continued...

Determine the power factor of the entire circuit as seen by the source. Calculate the average power delivered by the source.



$$\mathbf{Z} = 6 + 4 \,\|\, (-j2)$$
$$= 6 + \frac{-j2 \times 4}{4 - j2}$$
$$= 6.8 - j1.6 = 7\angle -13.24^\circ \ \Omega$$

$$\text{pf} = \cos(-13.24) = 0.9734 \qquad \text{(leading)}$$

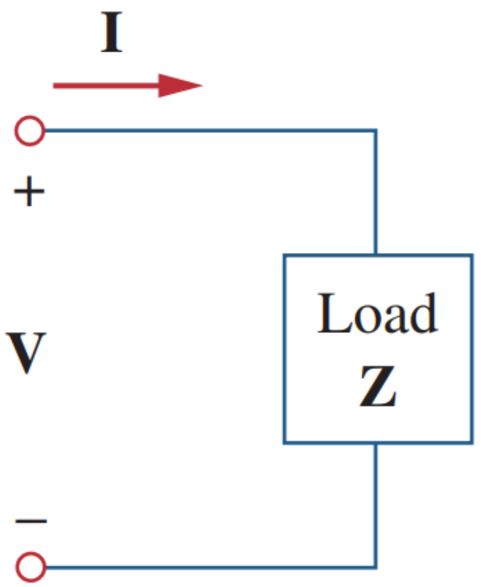
$$\mathbf{I}_{\text{rms}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{Z}} = \frac{30\angle 0^\circ}{7\angle -13.24^\circ} = 4.286\angle 13.24^\circ \text{ A}$$

average power supplied by the source

$$P = V_{\text{rms}} I_{\text{rms}}\, \text{pf} = (30)(4.286)0.9734 = 125 \text{ W}$$

Complex Power

Complex power **S** absorbed by the ac load is the product of the voltage and the complex conjugate of the current.



$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^*$$

$$\begin{aligned}\mathbf{S} &= \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} \\ &= V_{\text{rms}}I_{\text{rms}}\underline{/\theta_v - \theta_i} \\ &= V_{\text{rms}}I_{\text{rms}}\cos(\theta_v - \theta_i) + jV_{\text{rms}}I_{\text{rms}}\sin(\theta_v - \theta_i) \\ &= P + jQ\end{aligned}$$

$$\mathbf{S} = I^2_{\text{rms}}\mathbf{Z} = I^2_{\text{rms}}(R + jX) = P + jQ$$

$$P = \text{Re}(\mathbf{S}) = I^2_{\text{rms}}R$$

$$Q = \text{Im}(\mathbf{S}) = I^2_{\text{rms}}X$$

continued...

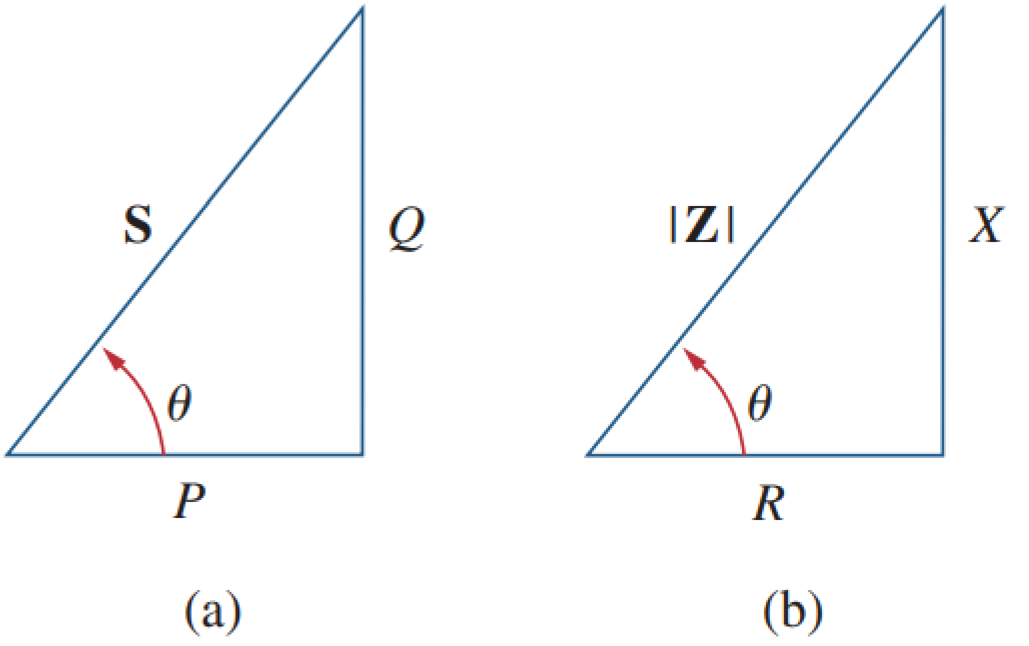
$$\text{Complex Power} = \mathbf{S} = P + jQ = \mathbf{V}_{\text{rms}}(\mathbf{I}_{\text{rms}})^*$$
$$= |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}|\underline{/\theta_v - \theta_i}$$
$$\text{Apparent Power} = S = |\mathbf{S}| = |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}| = \sqrt{P^2 + Q^2}$$
$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$
$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$
$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

$Q = 0$ for resistive loads (unity pf).
$Q < 0$ for capacitive loads (leading pf).
$Q > 0$ for inductive loads (lagging pf).

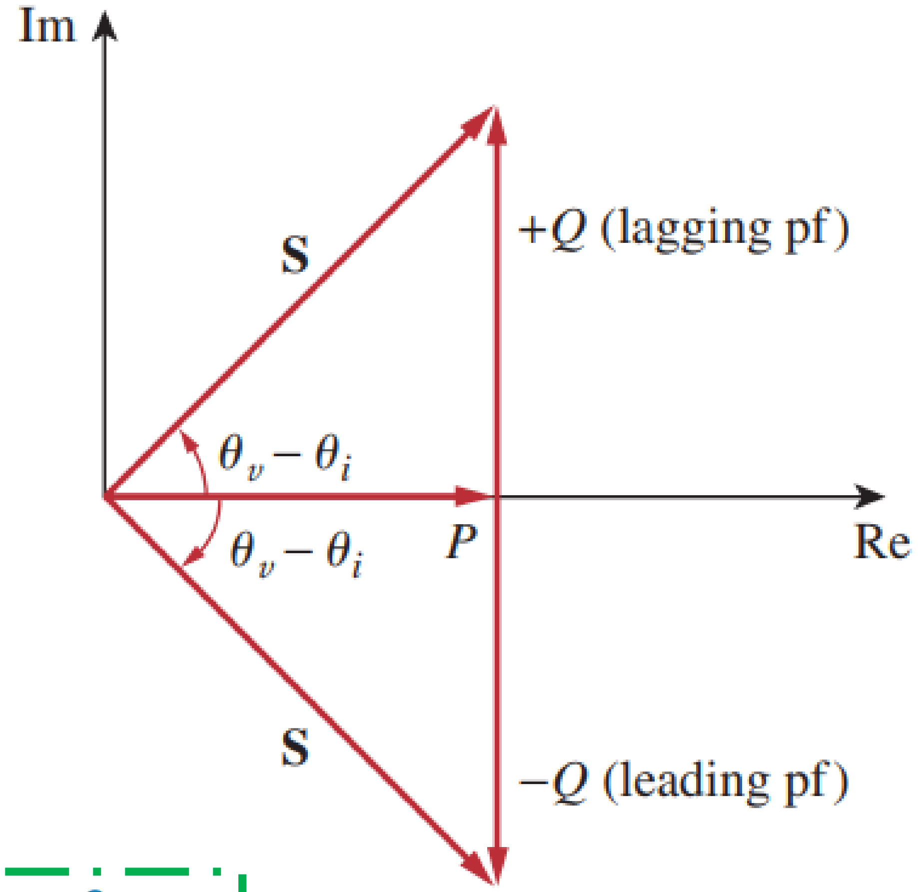
$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \cdots + \mathbf{S}_N$$

Total complex power in a network is the sum of complex powers of individual components.

Power Triangle



(a) Power triangle, (b) impedance triangle.



$$P = \mathrm{Re}(\mathbf{S}) = I_{\mathrm{rms}}^2 R$$

$$Q = \mathrm{Im}(\mathbf{S}) = I_{\mathrm{rms}}^2 X$$

Example

The voltage across a load is $v(t) = 60\cos(\omega t - 10°)$ V and the current through the element in the direction of the voltage drop is $i(t) = 1.5\cos(\omega t + 50°)$ A. Find: (a) the complex and apparent powers, (b) the real and reactive powers, and (c) the power factor and the load impedance.

(a) complex power $\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} = \left(\frac{60}{\sqrt{2}}\angle -10°\right)\left(\frac{1.5}{\sqrt{2}}\angle -50°\right)$

$$= 45\angle -60° \text{ VA}$$

apparent power $S = |\mathbf{S}| = 45$ VA

(b) $\mathbf{S} = 45\angle -60° = 22.5 - j38.97$

$P = 22.5$ W $\qquad Q = -38.97$ VAR

(c) pf $= \cos(-60°) = 0.5$ (leading)

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = {60\angle -10°}/_{1.5\angle 50°} = 20 - j34.64\ \Omega$$

continued...

A load $\mathbf{Z}$ draws 12 kVA at a power factor of 0.856 lagging from a 120-V rms sinusoidal source. Calculate: (a) the average and reactive powers delivered to the load, (b) the peak current, and (c) the load impedance.

(a) $\text{pf} = \cos\theta = 0.856 \longrightarrow \theta = \cos^{-1} 0.856 = 31.13^\circ$

apparent power is $S = 12{,}000$ VA

average or real power $P = S\cos\theta$

$$= 12{,}000 \times 0.856$$

$$= 10.272 \text{ kW}$$

Since pf is *lagging*, reactive power (*Q*) is positive.

reactive power $Q = S\sin\theta$

$$= 12{,}000 \times 0.517 = 6.204 \text{ kVAR}$$

(b) $\mathbf{S} = P + jQ = 10.272 + j6.204 \text{ kVA}$

continued…

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} \longrightarrow \mathbf{I}^*_{\text{rms}} = \frac{\mathbf{S}}{\mathbf{V}_{\text{rms}}} = \frac{10{,}272 + j6204}{120\angle 0^\circ}$$

$$= 100\angle 31.13^\circ \text{ A}$$

$$\mathbf{I}_{\text{rms}} = 100\angle -31.13^\circ$$

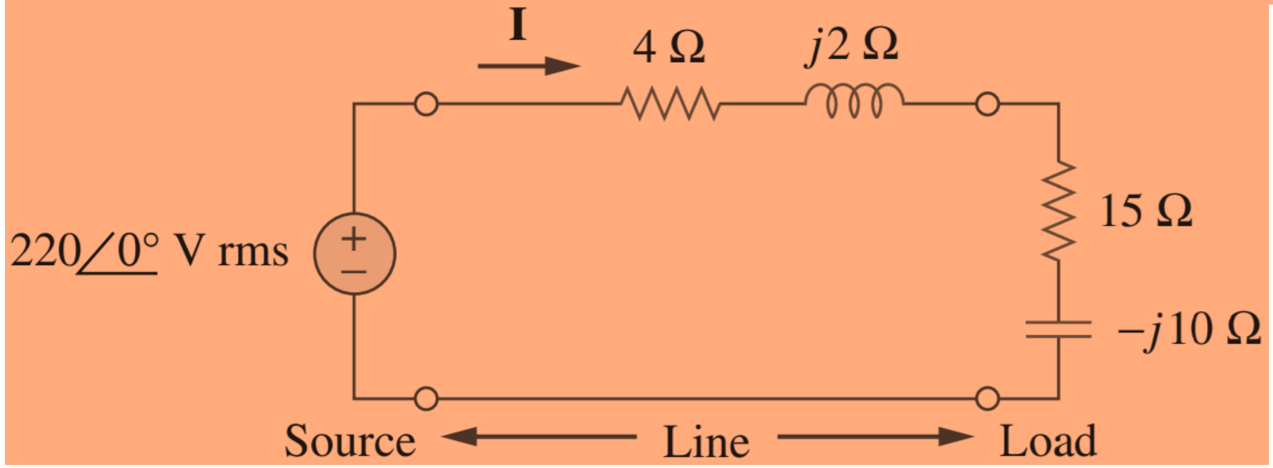
peak current $I_m = \sqrt{2}I_{\text{rms}} = \sqrt{2}(100) = 141.4 \text{ A}$

(c) $$\mathbf{Z} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{120\angle 0^\circ}{100\angle -31.13^\circ} = 1.027 + j0.62\ \Omega$$

which is an inductive impedance

continued…

a load being fed by a voltage source through a transmission line. The impedance of the line is represented by the $(4 + j2)\ \Omega$ impedance and a return path. Find the real power and reactive power absorbed by: (a) the source, (b) the line, and (c) the load.



total impedance

$$\mathbf{Z} = (4 + j2) + (15 - j10)$$
$$= 19 - j8$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = 10.67\underline{/22.83^\circ} \text{ A rms}$$

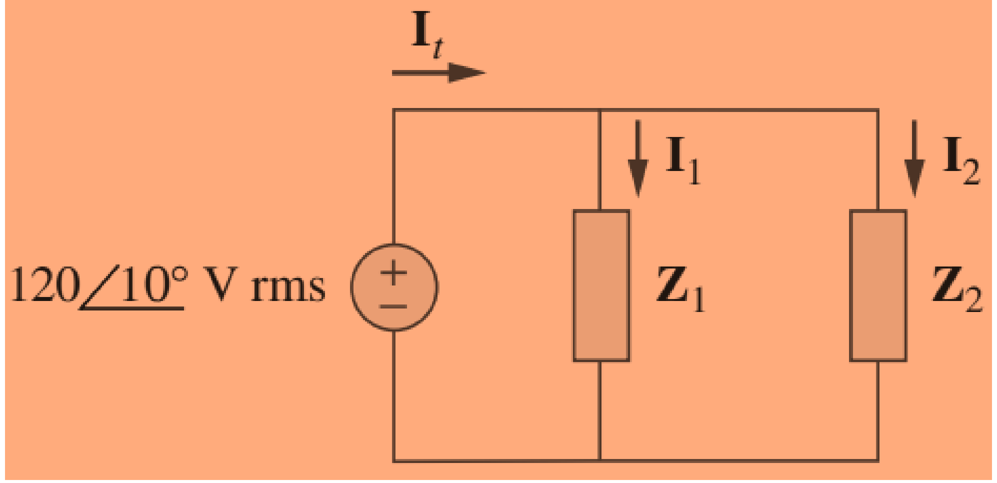
(a) $$\mathbf{S}_s = \mathbf{V}_s\mathbf{I}^* = (220\underline{/0^\circ})(10.67\underline{/-22.83^\circ})$$
$$= (2163.5 - j910.8) \text{ VA}$$ (leading)

continued…

$$\text{(b)}\quad \mathbf{V}_{\text{line}} = (4 + j2)\mathbf{I} = (4.472\angle 26.57^\circ)(10.67\angle 22.83^\circ)$$
$$= 47.72\angle 49.4^\circ \text{ V rms}$$
$$\mathbf{S}_{\text{line}} = \mathbf{V}_{\text{line}}\mathbf{I}^* = (47.72\angle 49.4^\circ)(10.67\angle -22.83^\circ)$$
$$= 455.4 + j227.7 \text{ VA} \qquad \text{(lagging)}$$
$$\text{(c)}\quad \mathbf{V}_L = (15 - j10)\mathbf{I} = (18.03\angle -33.7^\circ)(10.67\angle 22.83^\circ)$$
$$= 192.38\angle -10.87^\circ \text{ V rms}$$
$$\mathbf{S}_L = \mathbf{V}_L\mathbf{I}^* = (192.38\angle -10.87^\circ)(10.67\angle -22.83^\circ)$$
$$= (1708 - j1139) \text{ VA} \qquad \text{(leading)}$$

continued…

In the circuit $\mathbf{Z}_1 = 60\angle -30^\circ\ \Omega$ and $\mathbf{Z}_2 = 40\angle 45^\circ\ \Omega$. Calculate the total: (a) apparent power, (b) real power, (c) reactive power, and (d) pf, supplied by the source and seen by the source.



$$\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_1} = \frac{120\angle 10^\circ}{60\angle -30^\circ} = 2\angle 40^\circ \text{ A rms}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}}{\mathbf{Z}_2} = \frac{120\angle 10^\circ}{40\angle 45^\circ} = 3\angle -35^\circ \text{ A rms}$$

$$\mathbf{S}_1 = (\mathbf{V}_{\text{rms}})\left(\mathbf{I}_{1,\text{rms}}{}^*\right) = (120\angle 10^\circ)(2\angle -40^\circ) = 240\angle -30^\circ$$
$$= 207.85 - j120 \text{ VA}$$

$$\mathbf{S}_2 = (\mathbf{V}_{\text{rms}})\left(\mathbf{I}_{2,\text{rms}}{}^*\right) = (120\angle 10^\circ)(3\angle 35^\circ) = 360\angle 45^\circ$$
$$= 254.6 + j254.6 \text{ VA}$$

total complex power $\mathbf{S}_t = \mathbf{S}_1 + \mathbf{S}_2 = 462.4 + j134.6 \text{ VA}$

continued…

(a) The total apparent power

$$|\mathbf{S}_t| = \sqrt{462.4^2 + 134.6^2} = 481.6 \text{ VA}$$

(b) The total real power

$$P_t = \text{Re}(\mathbf{S}_t) = 462.4 \text{ W}$$

or $P_t = P_1 + P_2$

(c) The total reactive power

$$Q_t = \text{Im}(\mathbf{S}_t) = 134.6 \text{ VAR}$$

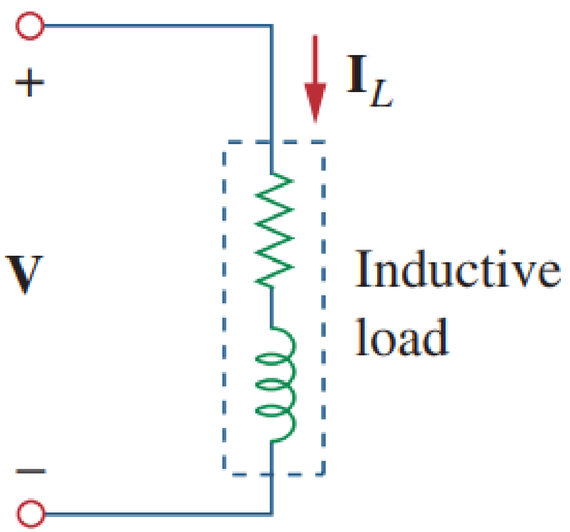
or $Q_t = Q_1 + Q_2$

(d) The pf

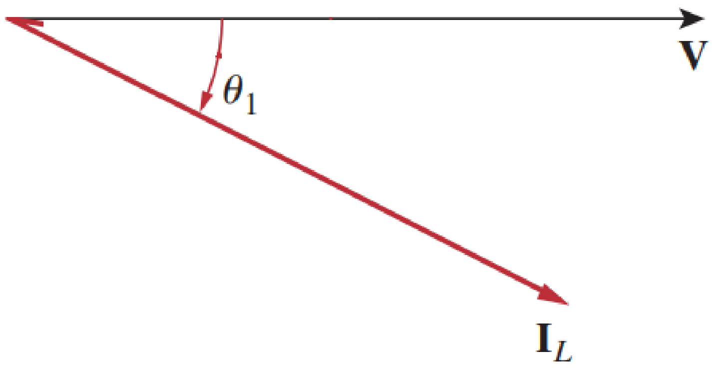
$$\text{pf} = P_t/|\mathbf{S}_t| = 462.4/481.6 = 0.96 \text{ (lagging)}$$

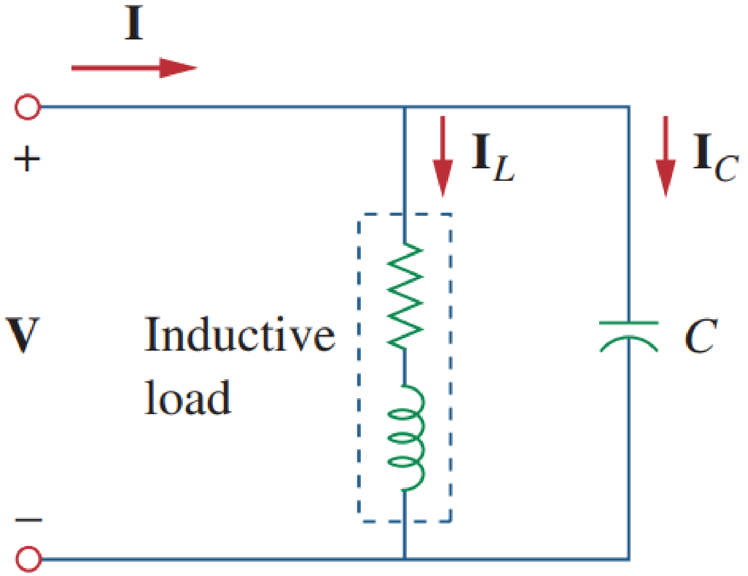
Power Factor Correction

The process of increasing the power factor without altering the voltage or current to the original load is known as power factor correction.

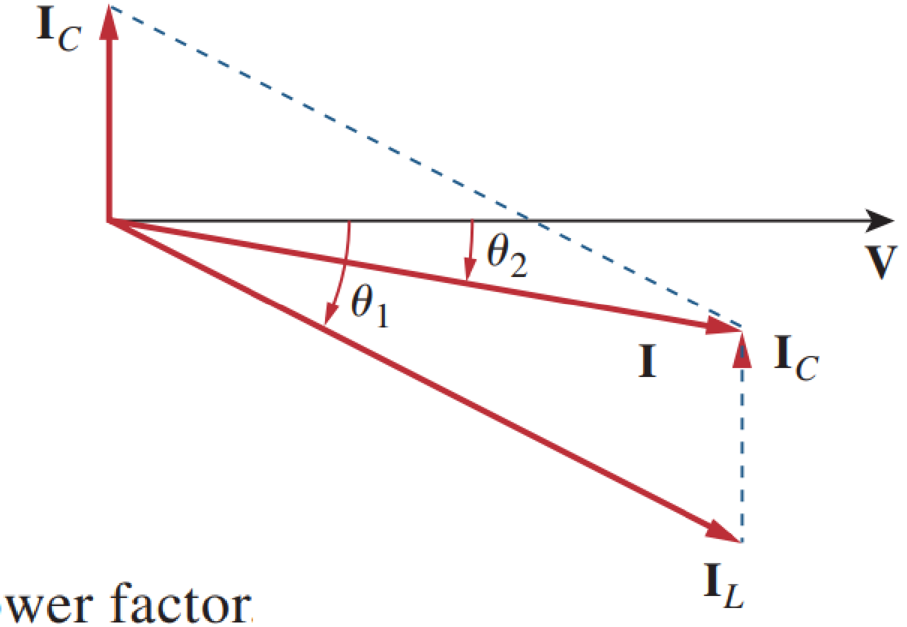


original inductive load



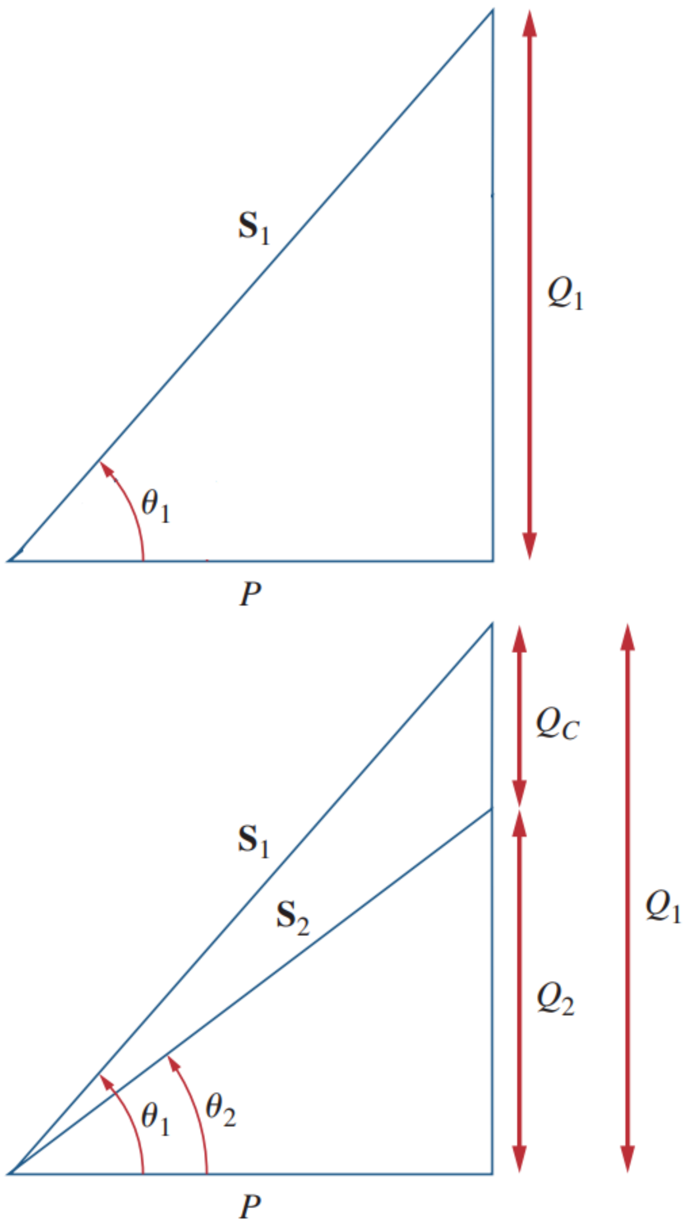


inductive load with improved power factor



- With the same supplied voltage, the original circuit (on left) draws larger current than the improved circuit (on right). $|\mathbf{I}_L| > |\mathbf{I}|$
- Power companies charge more for larger currents, because they result in increased power losses.
- Therefore, it is beneficial to both the power company and the consumer to keep the power factor as close to unity as possible.

continued…



If the original inductive load has apparent power S_1, then

$$P = S_1 \cos\theta_1$$
$$Q_1 = S_1 \sin\theta_1 = P \tan\theta_1$$

If we desire to increase the power factor from $\cos\theta_1$ to $\cos\theta_2$ without altering the real power, then the new reactive power is

$$Q_2 = P \tan\theta_2$$

The reduction in reactive power is caused by the shunt capacitor:

$$\begin{aligned} Q_C &= Q_1 - Q_2 \\ &= P(\tan\theta_1 - \tan\theta_2) \end{aligned}$$

$$Q_C = V_{\text{rms}}^2/X_C = \omega C V_{\text{rms}}^2$$

continued...

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\text{rms}}^2}$$

- <u>Rare case:</u> Load may be capacitive (pf is *leading*).

An inductor (*L*) should be connected across the load for pf correction.

$$Q_L = \frac{V_{\text{rms}}^2}{X_L} = \frac{V_{\text{rms}}^2}{\omega L} \longrightarrow L = \frac{V_{\text{rms}}^2}{\omega Q_L}$$

where $Q_L = Q_1 - Q_2$

(the difference between the new and old reactive powers)

Example

When connected to a 120-V (rms), 60-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.

$$\cos\theta_1 = 0.8 \rightarrow \theta_1 = 36.87^\circ$$

$$\cos\theta_2 = 0.95 \rightarrow \theta_2 = 18.19^\circ$$

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\text{rms}}^2}$$

The real power P has not changed

$$= \frac{4000(\tan 36.87^\circ - \tan 18.19^\circ)}{2\pi \times 60 \times 120^2} = \frac{1685.6}{2\pi \times 60 \times 120^2}$$

$$= 310.5\ \mu\text{F}$$

continued...

Find the value of parallel capacitance needed to correct a load of 140 kVAR at 0.85 lagging pf to unity pf. Assume that the load is supplied by a 110-V (rms), 60-Hz line.

$\cos\theta_1 = 0.85$

$\Rightarrow \theta_1 = 31.79°$

$\cos\theta_2 = 1$

$\Rightarrow \theta_2 = 0°$

Now,

$P = S\cos\theta_1$

$$\Rightarrow P = \frac{Q}{\sin\theta_1}\cos\theta_1$$

$$\Rightarrow P = \frac{140\times 10^3}{\sin 31.79°}\times 0.85 = 225.93\ \text{kW}$$

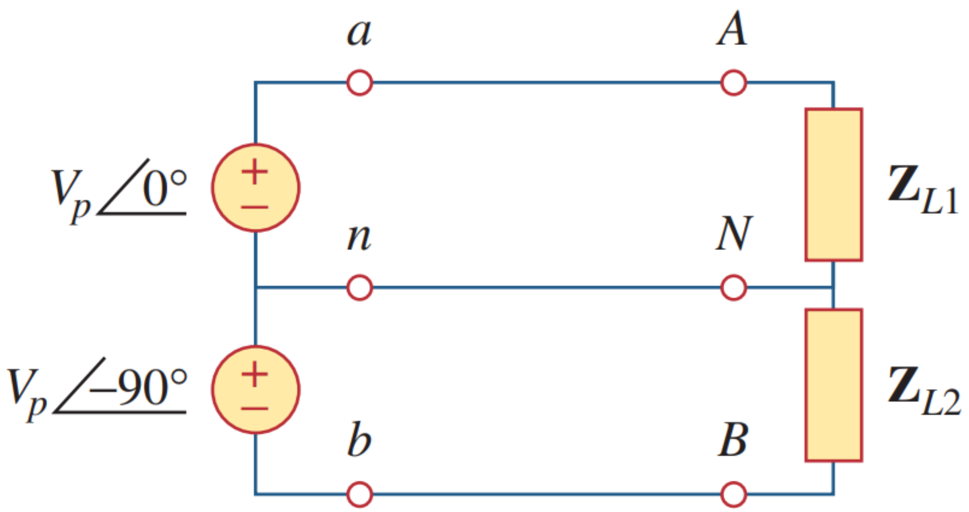
$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\text{rms}}^2} = \frac{225.93\times 10^3(\tan 31.79° - \tan 0°)}{2\pi\times 60\times 110^2}$$

$$= \mathbf{30.69\ mF}$$

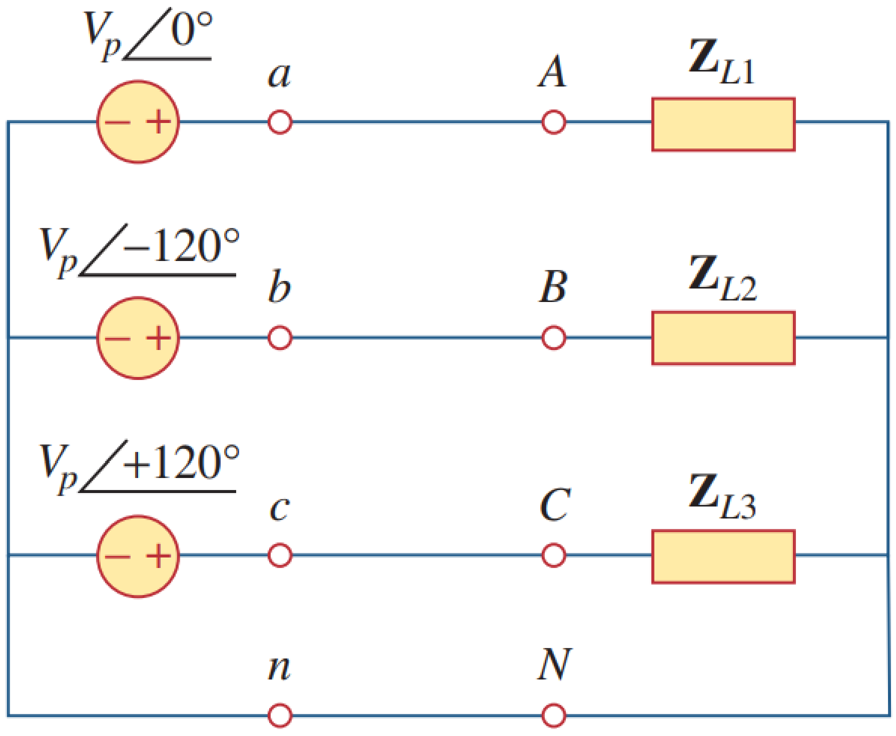
Three-Phase Circuits

Circuits or systems in which the ac sources operate at same frequency but different phases are known as polyphase.

Single-phase system
(*covered in previous chapter*)



Two-phase three-wire system



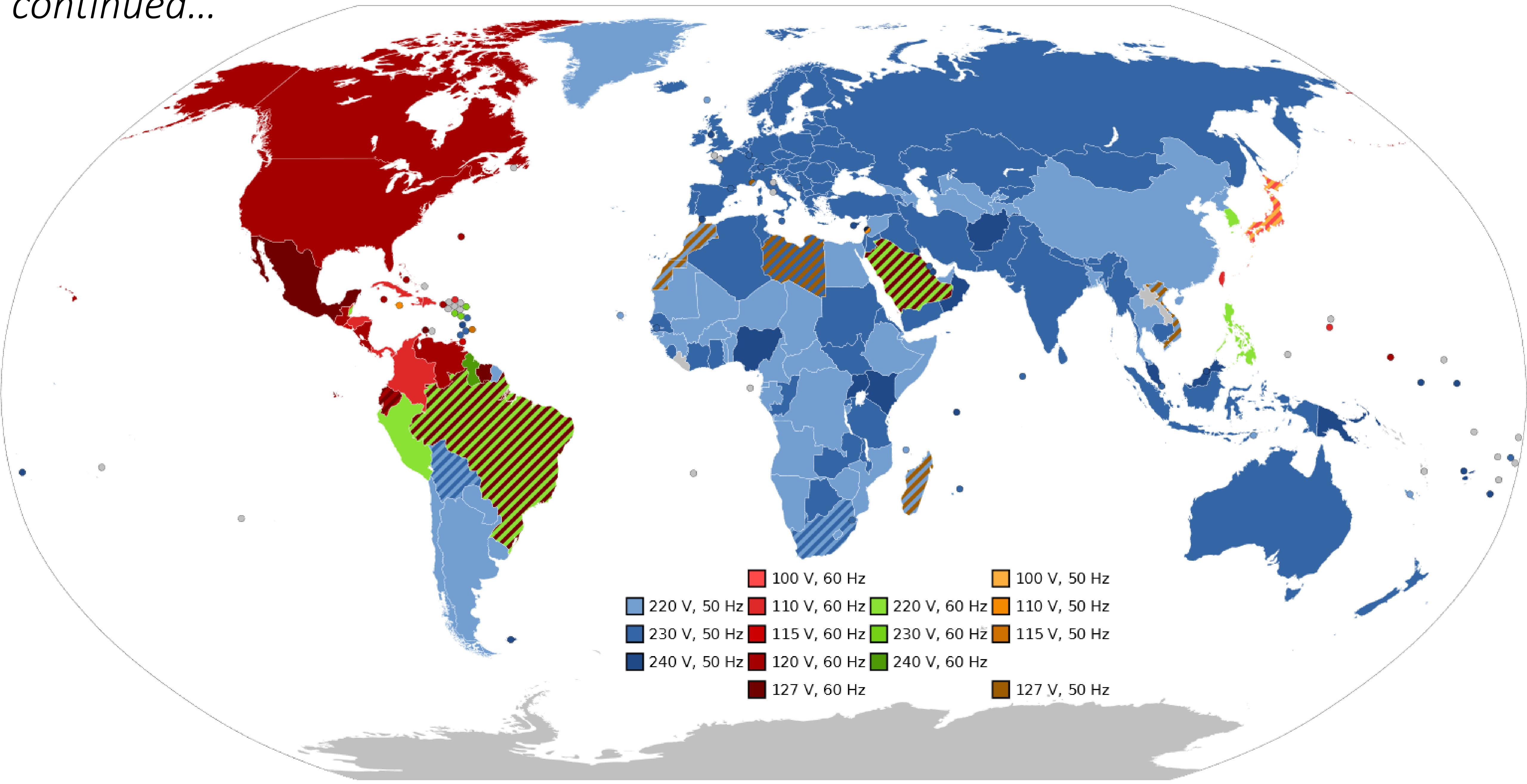
Three-phase four-wire system

continued…

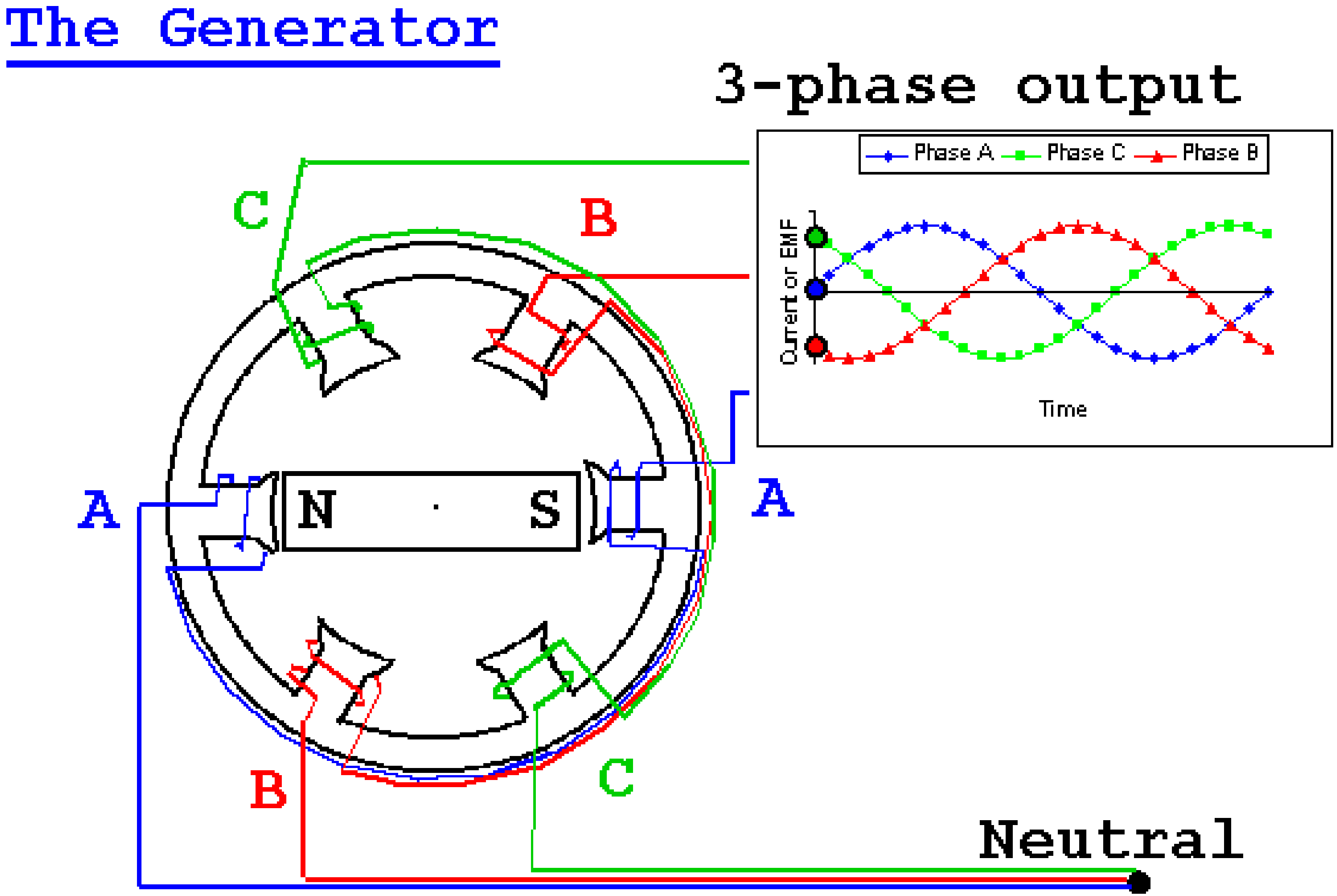
Why are 3-phase systems important?

- Nearly all electric power is generated and distributed in 3-phase, at the operating frequency of 60 Hz or 50 Hz.
  - ✓When 1-phase or 2-phase inputs are required, they are taken from the 3-phase system rather than generated independently.
- The instantaneous power in a 3-phase system can be constant.
  - ✓This results in uniform power transmission and less vibration.
- The amount of wire required for a 3-phase system is less than that required for an equivalent 1-phase system to deliver same power.
  - ✓The 3-phase system is more economical than 1-phase (*33% more material*).

continued…

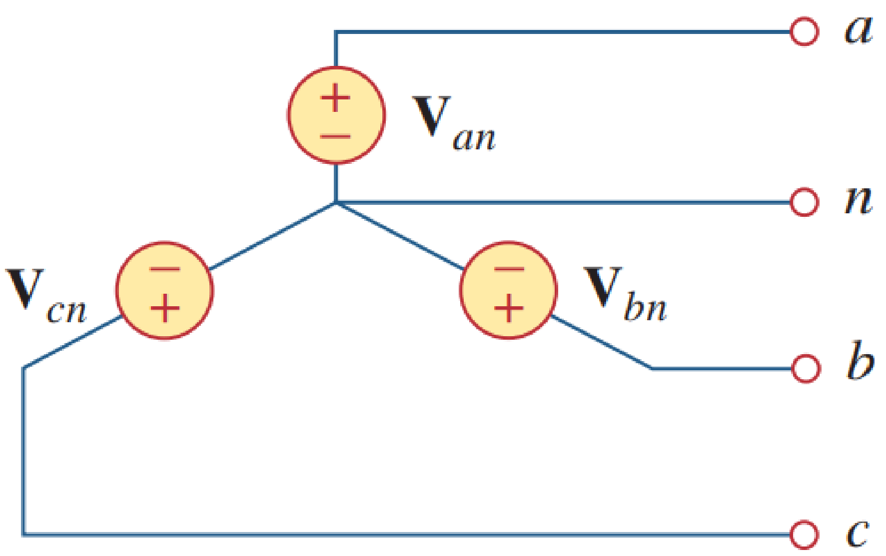

100 V, 60 Hz
100 V, 50 Hz
220 V, 50 Hz
110 V, 60 Hz
220 V, 60 Hz
110 V, 50 Hz
230 V, 50 Hz
115 V, 60 Hz
230 V, 60 Hz
115 V, 50 Hz
240 V, 50 Hz
120 V, 60 Hz
240 V, 60 Hz
127 V, 60 Hz
127 V, 50 Hz

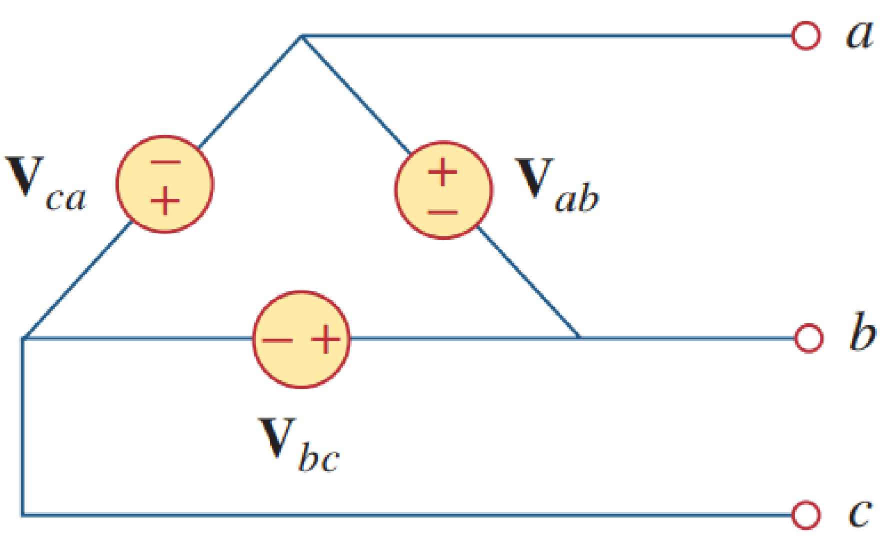
Balanced 3-Ø Voltage Source

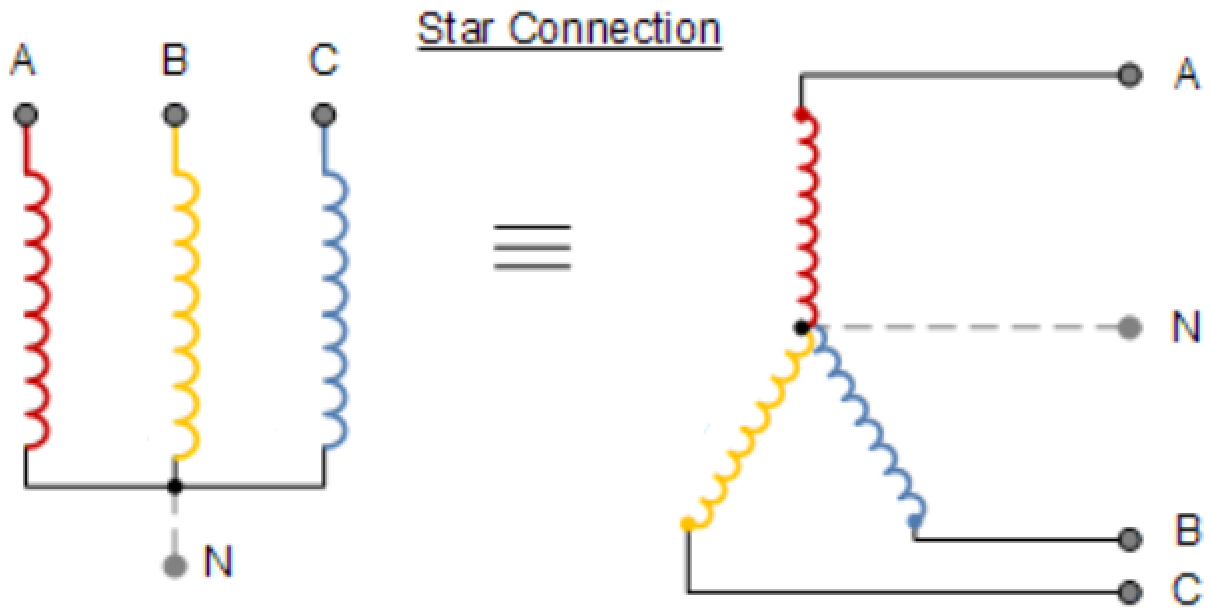


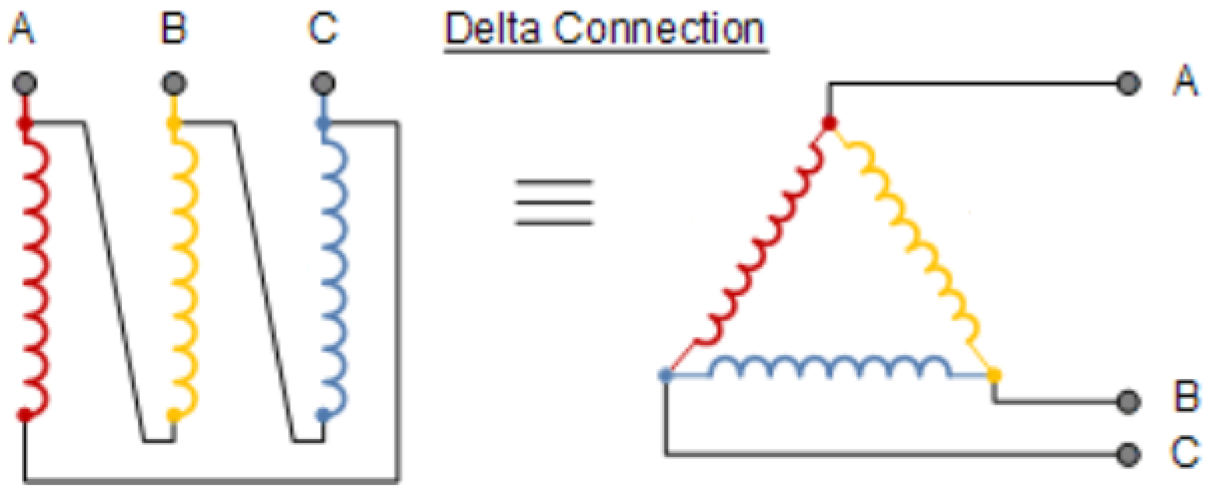
3-Ø current source is very scarce.

continued…


a
$\mathbf{V}_{an}$
n
$\mathbf{V}_{cn}$
$\mathbf{V}_{bn}$
b
c

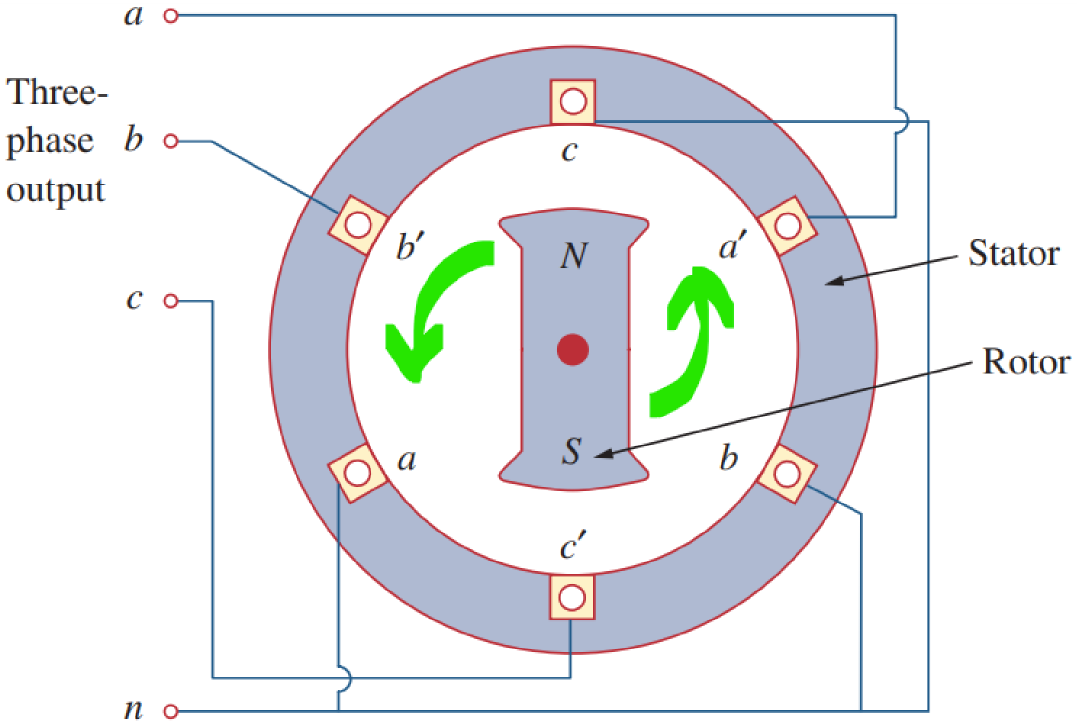

a
$\mathbf{V}_{ca}$
$\mathbf{V}_{ab}$
b
$\mathbf{V}_{bc}$
c


Star Connection
A
B
C
N
A
N
B
C


Delta Connection
A
B
C
A
B
C

continued (Star Connection)…

Positive (abc) Sequence

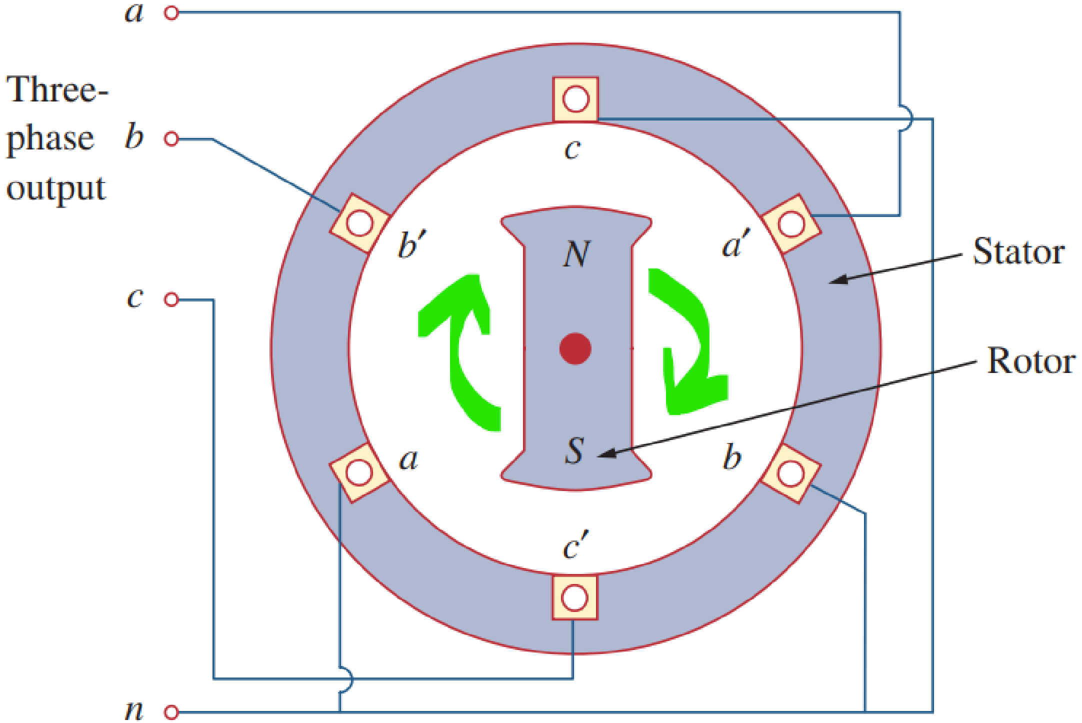


$$\mathbf{V}_{an} = V_p\angle 0°$$

$$\mathbf{V}_{bn} = V_p\angle -120°$$

$$\mathbf{V}_{cn} = V_p\angle -240° = V_p\angle +120°$$

Negative (acb) Sequence



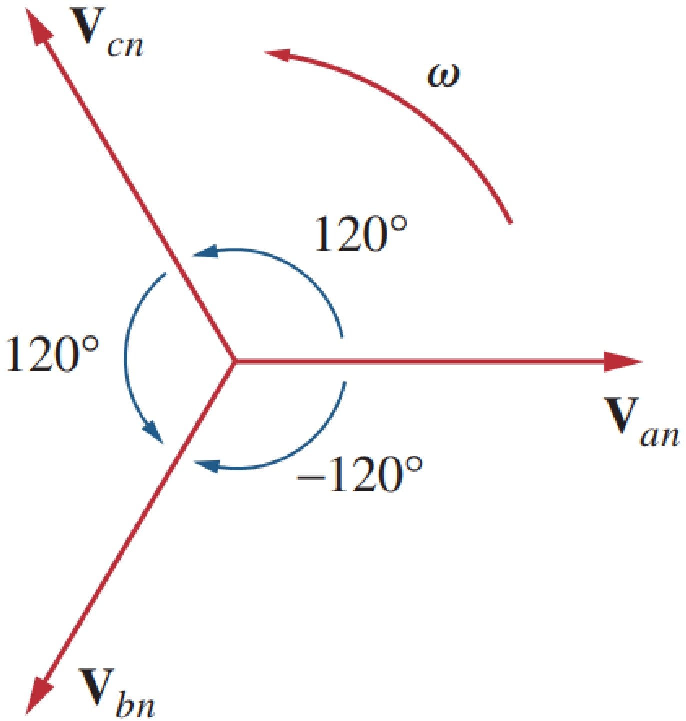
$$\mathbf{V}_{an} = V_p\angle 0°$$

$$\mathbf{V}_{cn} = V_p\angle -120°$$

$$\mathbf{V}_{bn} = V_p\angle -240° = V_p\angle +120°$$

continued (Star Connection)...

Positive (abc) Sequence

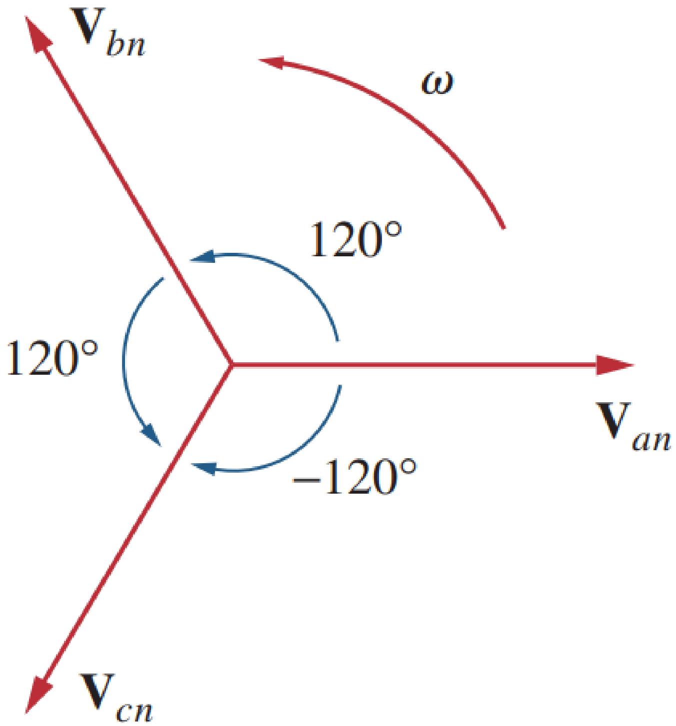


$$\mathbf{V}_{an} = V_p\angle 0^\circ$$

$$\mathbf{V}_{bn} = V_p\angle -120^\circ$$

$$\mathbf{V}_{cn} = V_p\angle -240^\circ = V_p\angle +120^\circ$$

Negative (acb) Sequence



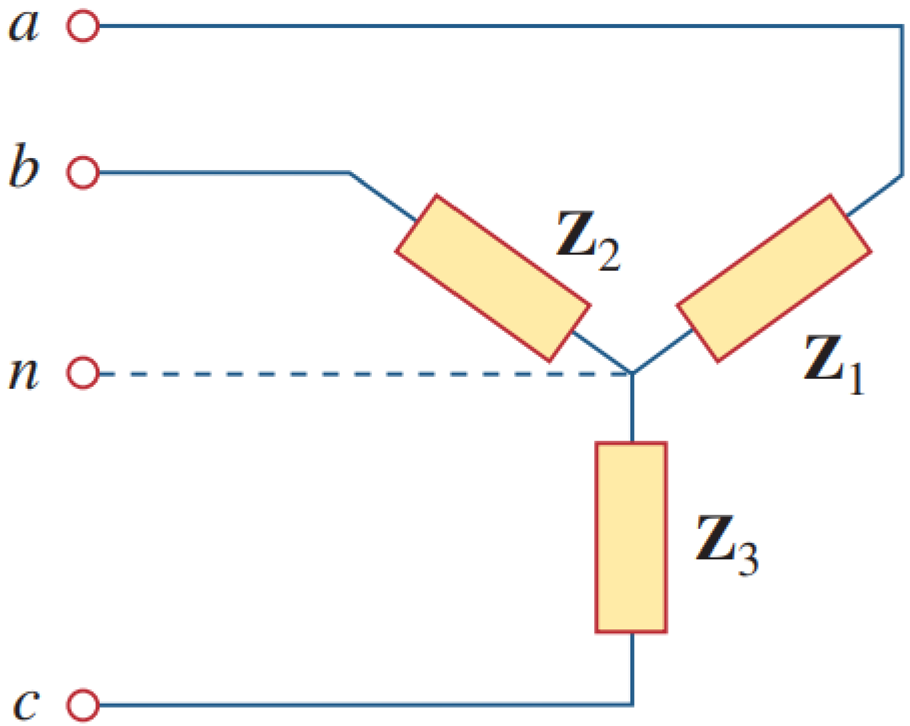
$$\mathbf{V}_{an} = V_p\angle 0^\circ$$

$$\mathbf{V}_{cn} = V_p\angle -120^\circ$$

$$\mathbf{V}_{bn} = V_p\angle -240^\circ = V_p\angle +120^\circ$$

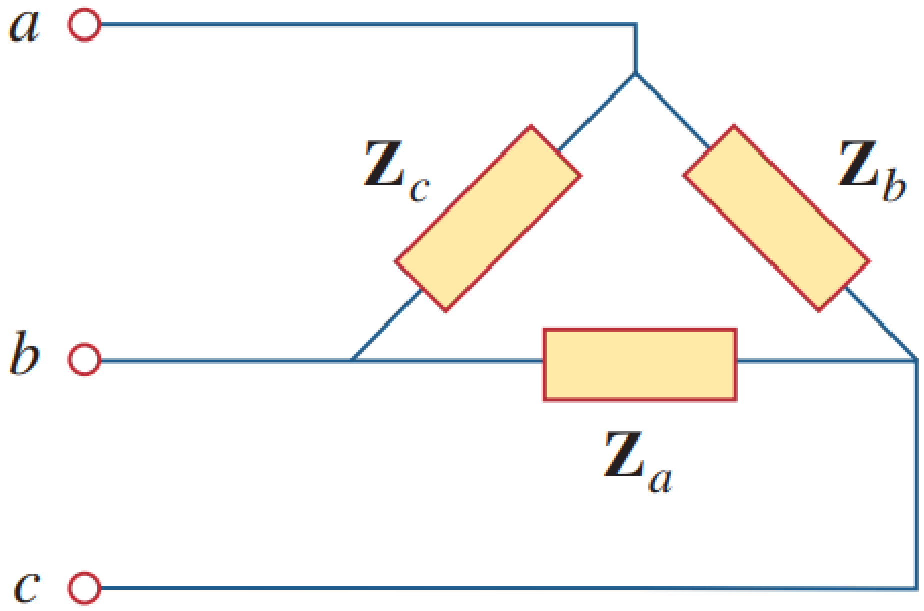
Balanced 3-Ø Load

Y-connected Load



$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y$$

Δ-connected Load



$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta$$

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y$$

$$\mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta$$

Example

Determine the phase sequence of the set of voltages

$$v_{an} = 200\cos(\omega t + 10°)$$

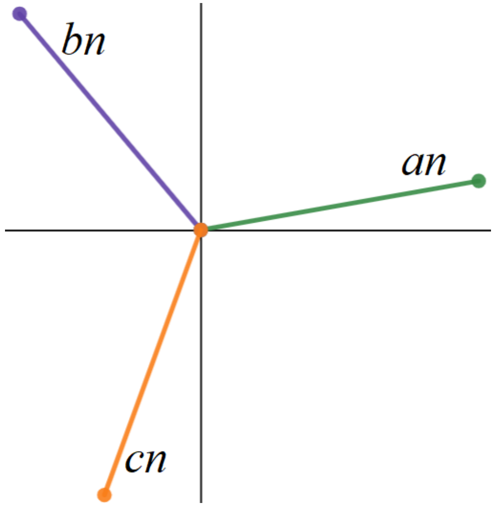
$$v_{bn} = 200\cos(\omega t - 230°), \qquad v_{cn} = 200\cos(\omega t - 110°)$$

$$\mathbf{V}_{an} = 200\angle 10°\ \text{V}$$

$$\mathbf{V}_{bn} = 200\angle -230°\ \text{V}$$

$$\mathbf{V}_{cn} = 200\angle -110°\ \text{V}$$

acb sequence

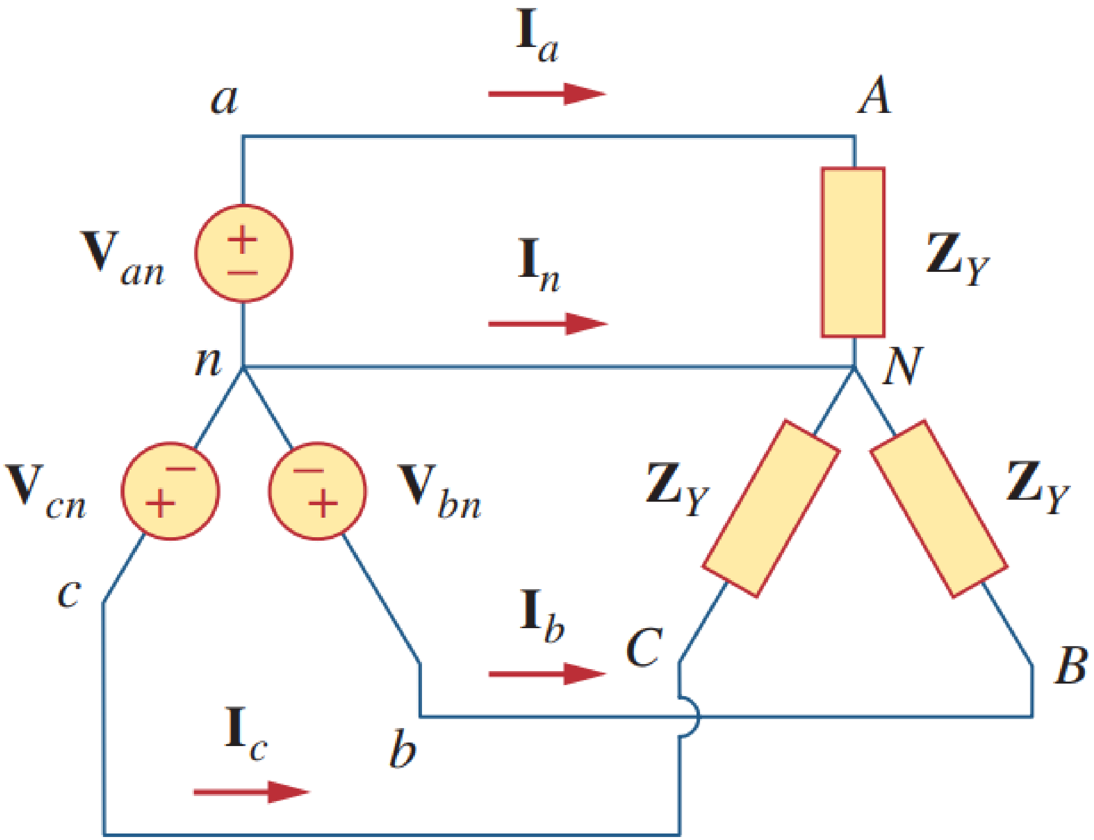


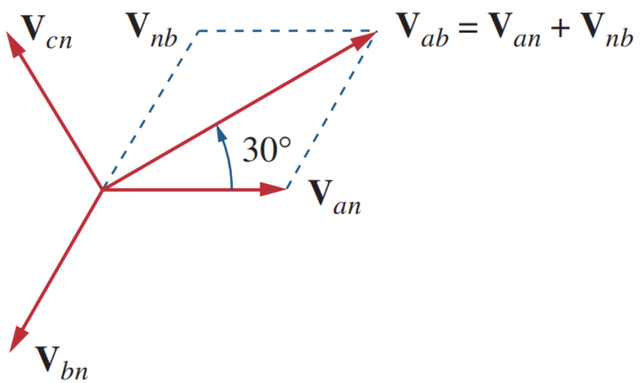
Given that $\mathbf{V}_{bn} = 110\angle 30°$ V, find $\mathbf{V}_{an}$ and $\mathbf{V}_{cn}$, assuming a positive (*abc*) sequence.

$110\angle 150°$ V

$110\angle -90°$ V

Balanced Y-Y Connection





Assuming the positive sequence,
the *phase* voltages (or *line-to-neutral* voltages):

$$\mathbf{V}_{an} = V_p\angle 0°$$

$$\mathbf{V}_{bn} = V_p\angle -120°, \qquad \mathbf{V}_{cn} = V_p\angle +120°$$

The *line-to-line* voltages or simply *line* voltages:

$$\begin{aligned}\mathbf{V}_{ab} &= \mathbf{V}_{an} + \mathbf{V}_{nb} \\ &= \mathbf{V}_{an} - \mathbf{V}_{bn} \\ &= V_p\angle 0° - V_p\angle -120° \\ &= \sqrt{3}V_p\angle 30°\end{aligned}$$

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}V_p\angle -90°$$

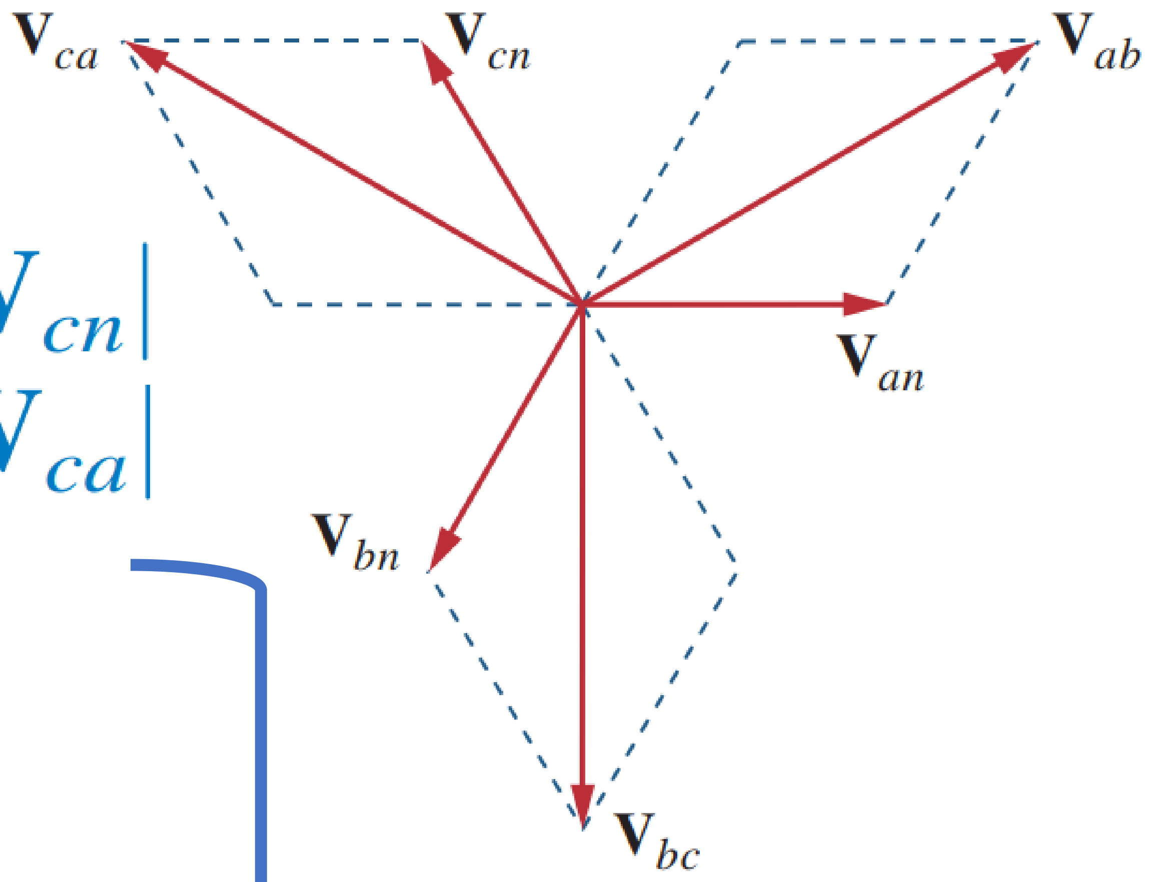
$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}V_p\angle -210°$$

continued…

$$V_L = \sqrt{3}V_p$$

$$V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$$



The *Line* Currents

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -120^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a\angle -120^\circ$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -240^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a\angle -240^\circ$$

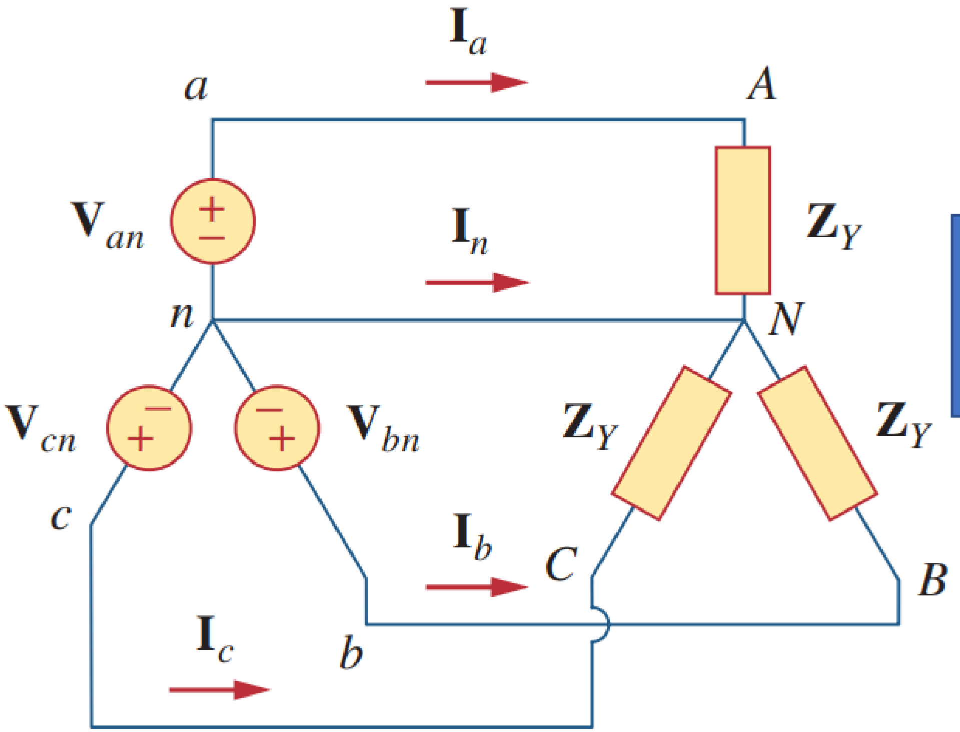
$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$$

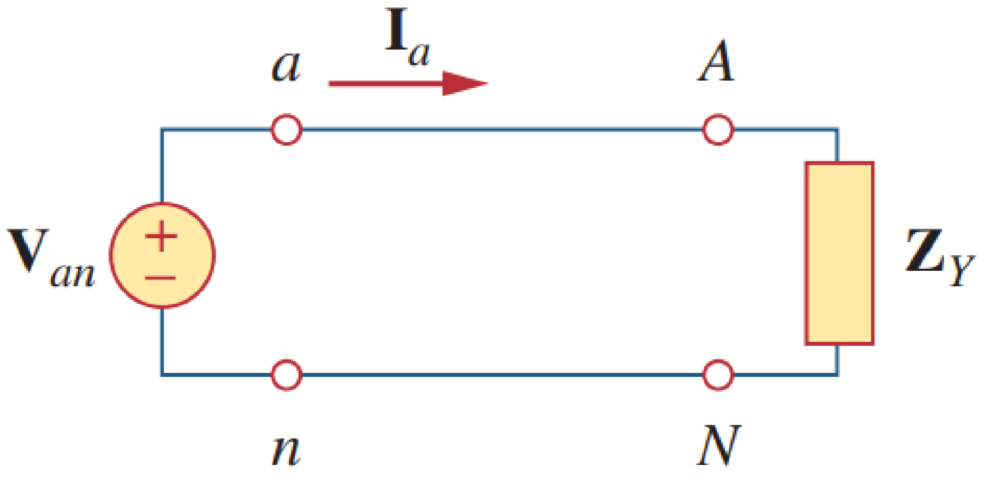
continued…

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 0$$

- Thus, voltage across the neutral wire is zero. The neutral line can thus be removed without affecting the system.

In Y-Y system, the *line* current is the same as the *phase* current.

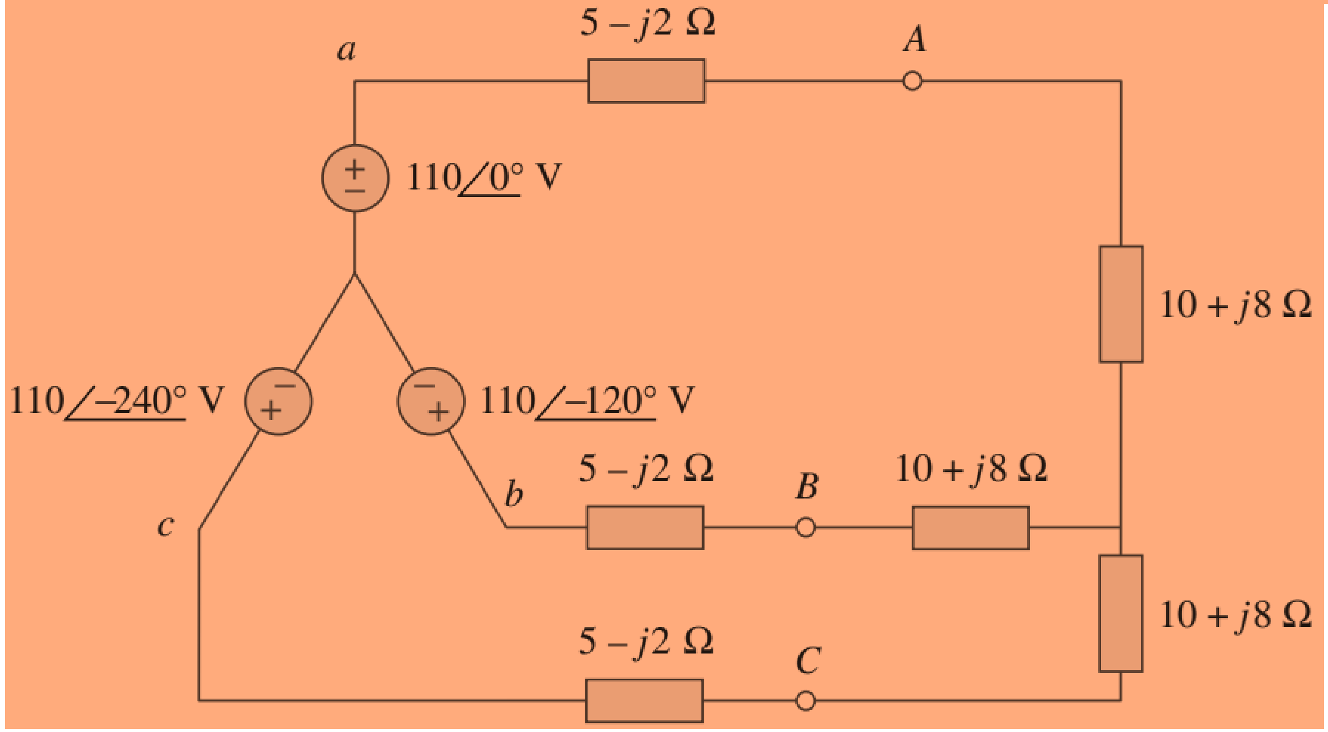




$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

Example

Calculate the line currents in the three-wire Y-Y system



$$\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 16.155\angle 21.8^\circ$$

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{110\angle 0^\circ}{16.155\angle 21.8^\circ} = 6.81\angle -21.8^\circ \text{ A}$$

Since the source voltages are in positive sequence, the *line* currents are also in positive sequence.

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 6.81\angle -141.8^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle -240^\circ = 6.81\angle -261.8^\circ \text{ A}$$

continued...

A Y-connected balanced three-phase generator with an impedance of $0.4 + j0.3\ \Omega$ per phase is connected to a Y-connected balanced load with an impedance of $24 + j19\ \Omega$ per phase. The line joining the generator and the load has an impedance of $0.6 + j0.7\ \Omega$ per phase. Assuming a positive sequence for the source voltages and that $\mathbf{V}_{an} = 120\angle 30^\circ$ V, find: (a) the line voltages, (b) the line currents.

(a) $$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ = \sqrt{3}(120)\angle(30^\circ + 30^\circ)$$

$$= 207.85\angle 60^\circ$$

$$\mathbf{V}_{bc} = 207.8\angle -60^\circ \text{ V}$$

$$\mathbf{V}_{ca} = 207.8\angle -180^\circ \text{ V}$$

continued…

(b) $\mathbf{Z}_Y = (0.4 + j0.3) + (24 + j19) + (0.6 + j0.7)$

$$= 32\angle 38.66^\circ$$

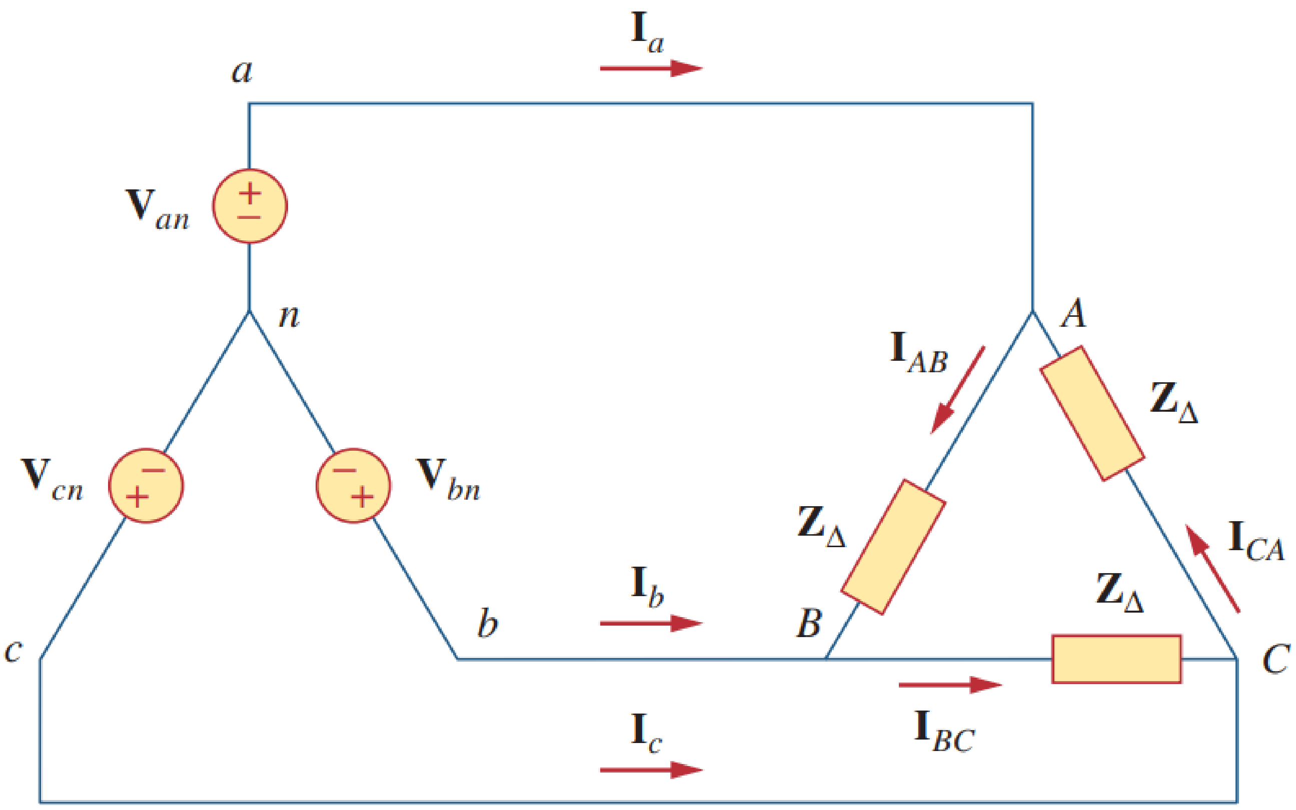
$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{120\angle 30^\circ}{32\angle 38.66^\circ}$$

$$= 3.75\angle -8.66^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 3.75\angle -128.66^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle -240^\circ = 3.75\angle 111.34^\circ \text{ A}$$

Balanced Y-Δ Connection



Assuming the positive sequence, the *phase* voltages are:

$$\mathbf{V}_{an} = V_p\underline{/0^\circ}$$

$$\mathbf{V}_{bn} = V_p\underline{/-120^\circ}, \qquad \mathbf{V}_{cn} = V_p\underline{/+120^\circ}$$

The *line* voltages are:

$$\mathbf{V}_{ab} = \sqrt{3}V_p\underline{/30^\circ} = \mathbf{V}_{AB}$$

$$\mathbf{V}_{bc} = \sqrt{3}V_p\underline{/-90^\circ} = \mathbf{V}_{BC}$$

$$\mathbf{V}_{ca} = \sqrt{3}V_p\underline{/+150^\circ} = \mathbf{V}_{CA}$$

The *phase* currents are:

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta}$$

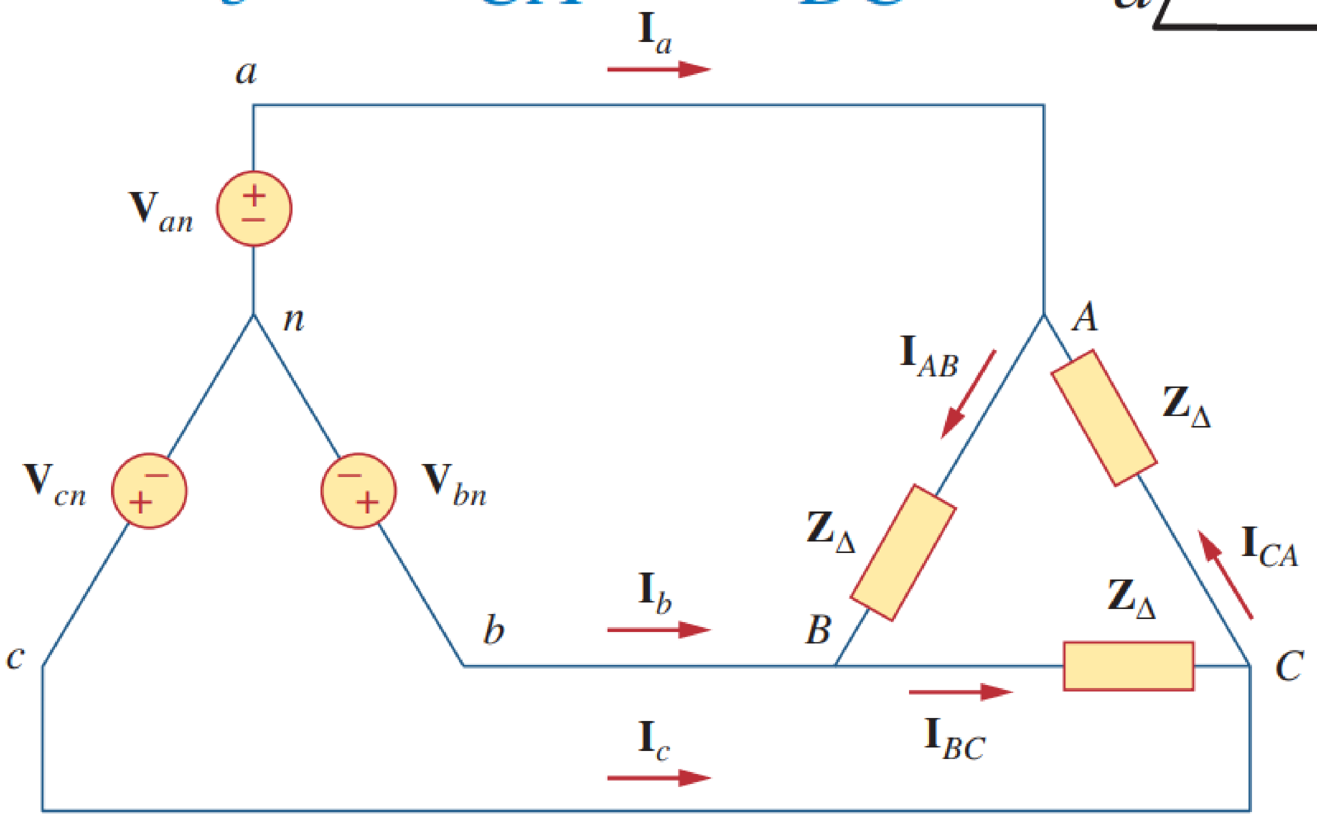
continued…

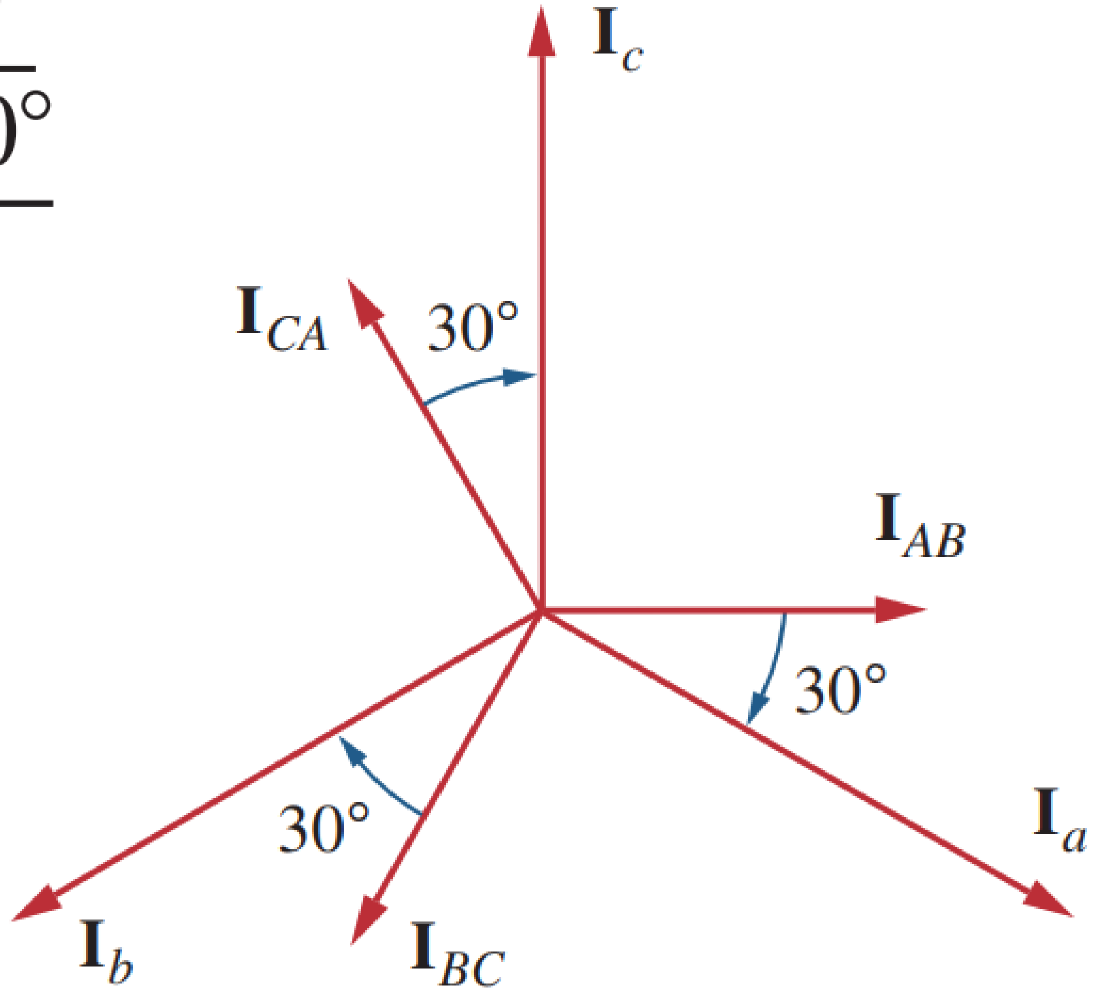
The *line* currents are:

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB} - \mathbf{I}_{AB}\angle -240° = \mathbf{I}_{AB}\sqrt{3}\angle -30°$$

$$\mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB} = \mathbf{I}_a\angle -120°$$

$$\mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} = \mathbf{I}_a\angle +120°$$



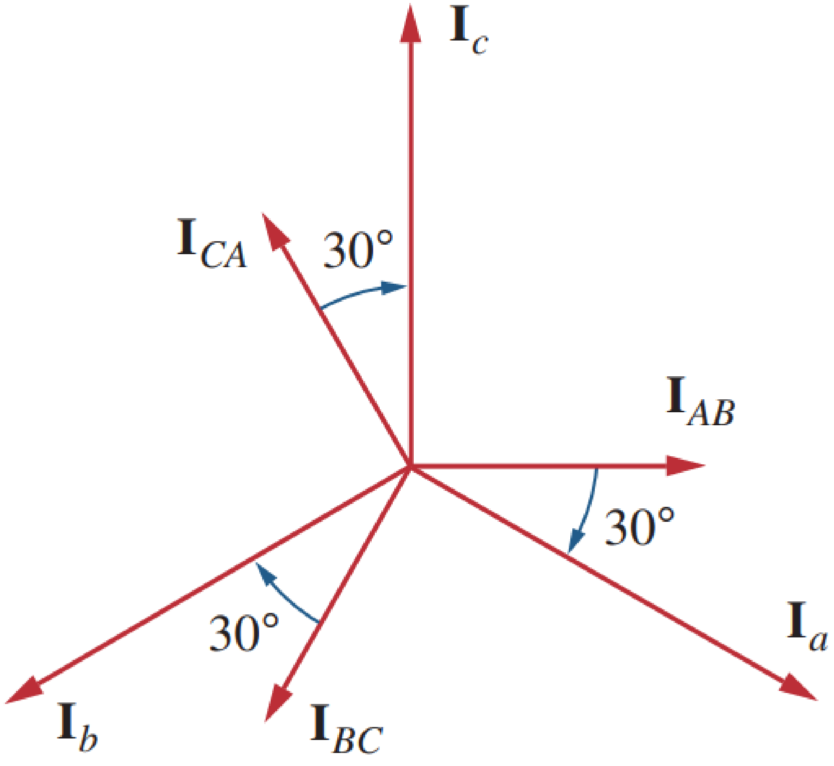


continued...

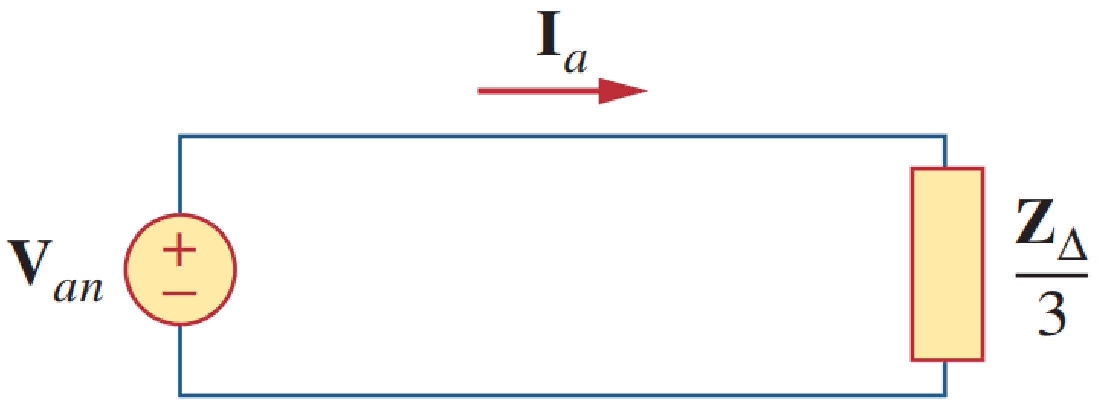
$$I_L = \sqrt{3} I_p$$

$$I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$$

$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$



Alternatively, we can transform Y-Δ system to a Y-Y system. Then solve the *line* current using 1-Ø equivalent circuit.



Then the *phase* currents are solved using:

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle{-30^\circ}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle{-120^\circ}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle{+120^\circ}$$

Example

A balanced *abc*-sequence Y-connected source with $\mathbf{V}_{an} = 100\angle 10^\circ$ V is connected to a Δ-connected balanced load $(8 + j4)\ \Omega$ per phase. Calculate the phase and line currents.

$$\mathbf{Z}_\Delta = 8 + j4 = 8.944\angle 26.57^\circ\ \Omega$$

using single-phase analysis, line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_\Delta/3} = \frac{100\angle 10^\circ}{2.981\angle 26.57^\circ} = 33.54\angle -16.57^\circ\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 33.53\angle -136.57^\circ\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ = 33.53\angle 103.43^\circ\ \text{A}$$

continued…

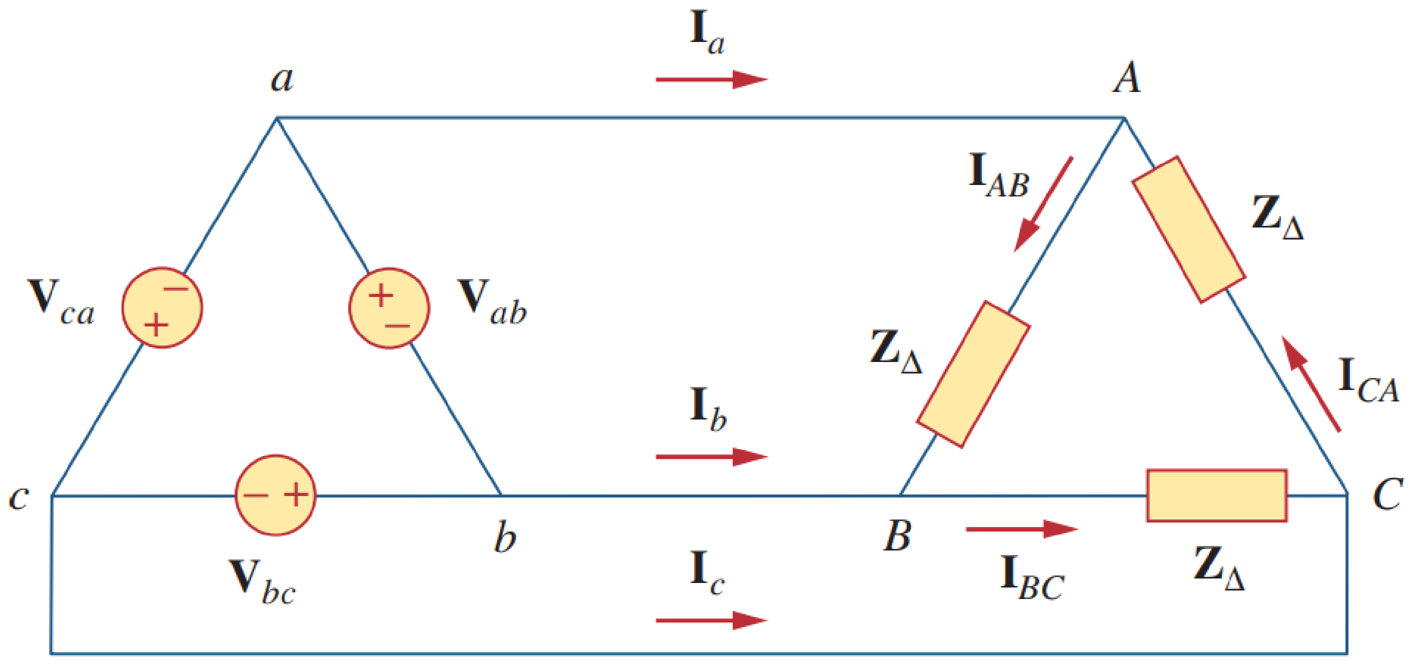
The *phase* currents are:

$$\mathbf{I}_{AB} = \frac{\mathbf{I}_a}{\sqrt{3}\angle -30°} = 19.36\angle 13.43° \text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = 19.36\angle -106.57° \text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle +120° = 19.36\angle 133.43° \text{ A}$$

Balanced Δ-Δ Connection



Assuming a positive sequence, the *phase* voltages are:

$$\mathbf{V}_{ab} = V_p\angle 0^\circ$$

$$\mathbf{V}_{bc} = V_p\angle -120^\circ, \qquad \mathbf{V}_{ca} = V_p\angle +120^\circ$$

Line voltages = *Phase* voltages

$$\mathbf{V}_{ab} = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \mathbf{V}_{BC}, \qquad \mathbf{V}_{ca} = \mathbf{V}_{CA}$$

The *phase* currents are:

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_\Delta}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_\Delta}$$

The *line* currents are:

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$

$$= \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$$

$$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$$

Example

A balanced Δ-connected load having an impedance $20 - j15\ \Omega$ is connected to a Δ-connected, positive-sequence generator having $\mathbf{V}_{ab} = 330\angle 0^\circ$ V. Calculate the phase currents of the load and the line currents.

$$\mathbf{Z}_\Delta = 20 - j15 = 25\angle -36.87^\circ\ \Omega$$

Since $\mathbf{V}_{AB} = \mathbf{V}_{ab}$, the phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{330\angle 0^\circ}{25\angle -36.87} = 13.2\angle 36.87^\circ \text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^\circ = 13.2\angle -83.13^\circ \text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle +120^\circ = 13.2\angle 156.87^\circ \text{ A}$$

continued…

the line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle{-30^\circ}$$

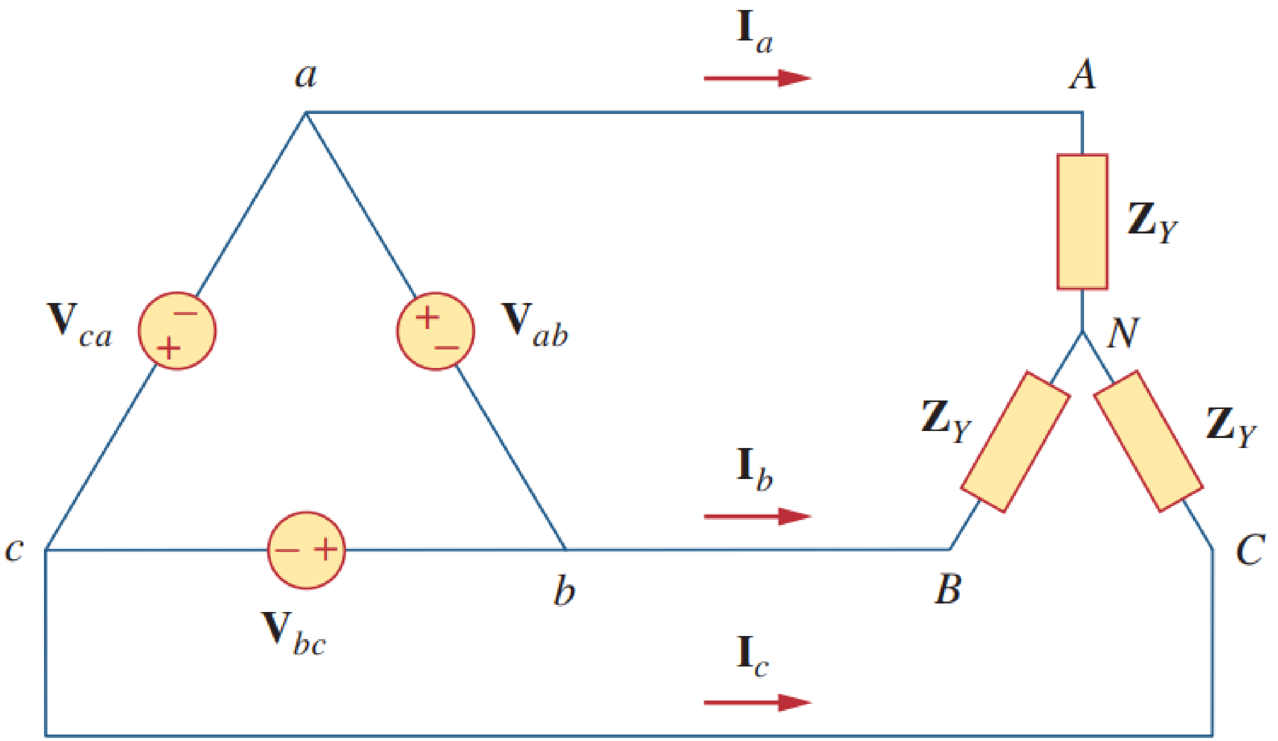
$$= (13.2\angle{36.87^\circ})(\sqrt{3}\angle{-30^\circ})$$

$$= 22.86\angle{6.87^\circ}\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle{-120^\circ} = 22.86\angle{-113.13^\circ}\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle{+120^\circ} = 22.86\angle{126.87^\circ}\ \text{A}$$

Balanced Δ-Y Connection



The *line* voltages as well as the *phase* voltages:

$$\mathbf{V}_{ab} = V_p\angle 0°, \qquad \mathbf{V}_{bc} = V_p\angle -120°$$

$$\mathbf{V}_{ca} = V_p\angle +120°$$

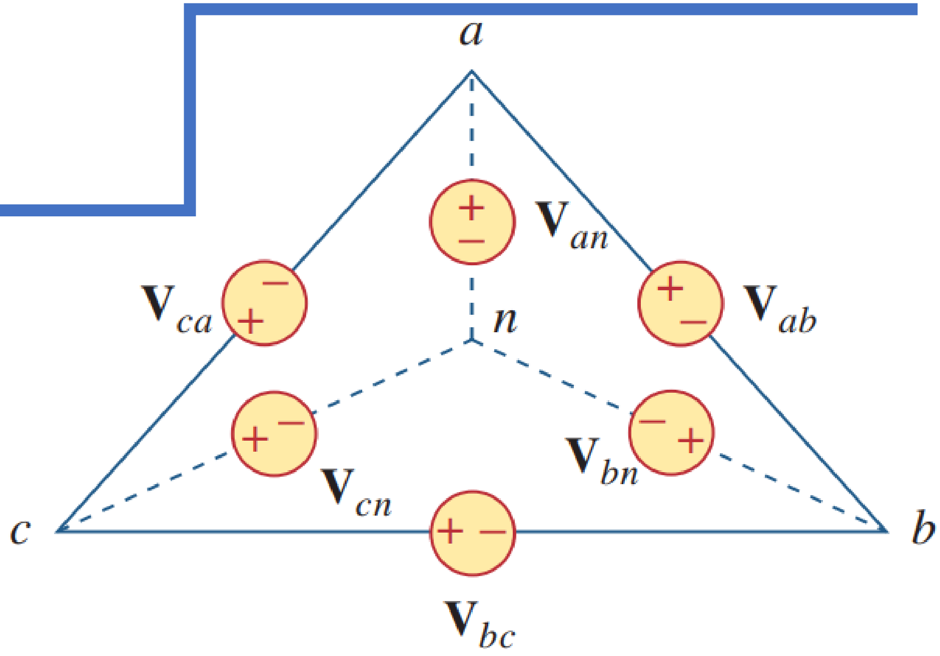
Replace the Δ-connected source with its equivalent Y-connected source.

- Now we find a Y-Y connection.
- Then solve using equivalent 1-Ø circuit.

Equivalent Y-connected source has the *phase* voltages:

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30°$$

$$\mathbf{V}_{bn} = \frac{V_p}{\sqrt{3}}\angle -150°, \qquad \mathbf{V}_{cn} = \frac{V_p}{\sqrt{3}}\angle +90°$$



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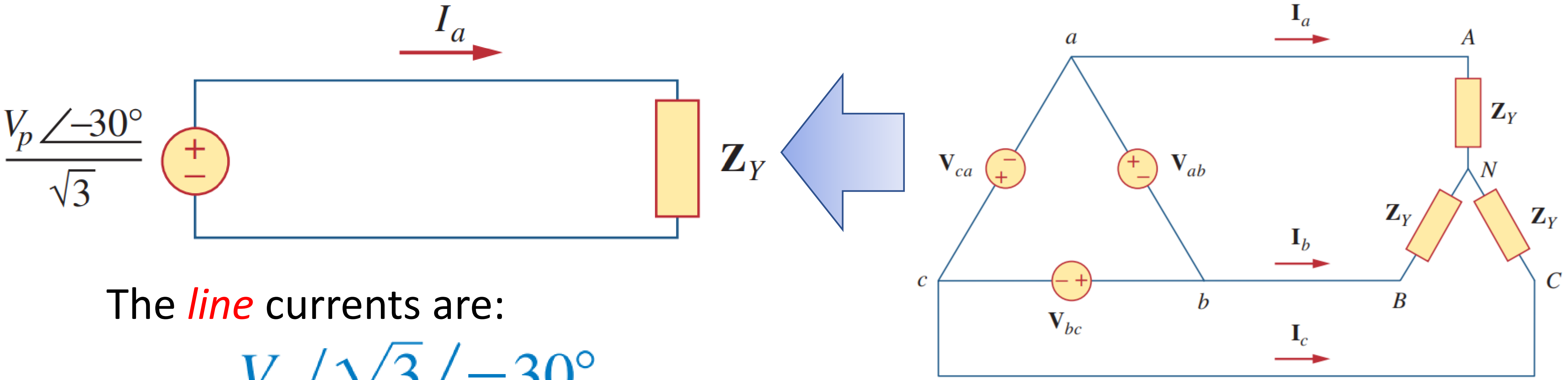


The *line* currents are:

$$\mathbf{I}_a = \frac{V_p/\sqrt{3}\angle -30°}{\mathbf{Z}_Y}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120°$$

$$\mathbf{I}_c = \mathbf{I}_a\angle +120°$$

Phase currents = *Line* currents

Example

A balanced Y-connected load with a phase impedance of $40 + j25\ \Omega$ is supplied by a balanced, positive sequence Δ-connected source with a line voltage of 210 V. Calculate the phase currents. Use $\mathbf{V}_{ab}$ as a reference.

$$\mathbf{Z}_Y = 40 + j25 = 47.17\angle 32^\circ\ \Omega \qquad \mathbf{V}_{ab} = 210\angle 0^\circ\ \text{V}$$

Δ-connected source is transformed to a Y-connected source

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\angle -30^\circ = 121.2\angle -30^\circ\ \text{V}$$

line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{121.2\angle -30^\circ}{47.12\angle 32^\circ} = 2.57\angle -62^\circ\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 2.57\angle -178^\circ\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120^\circ = 2.57\angle 58^\circ\ \text{A}$$

same as the phase currents

Summary

Δ-connected load is more desirable than the Y-connected load:

- It is easier to alter the loads in any one phase of the Δ-connected loads, as the individual loads are connected directly across the lines.

The Δ-connected source is hardly used in practice:

- Any slight imbalance in the phase voltages will result in unwanted circulating currents.

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0°$ $\mathbf{V}_{bn} = V_p\angle -120°$ $\mathbf{V}_{cn} = V_p\angle +120°$ Same as line currents	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30°$ $\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120°$ $\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120°$ $\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$ $\mathbf{I}_b = \mathbf{I}_a\angle -120°$ $\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0°$ $\mathbf{V}_{bn} = V_p\angle -120°$ $\mathbf{V}_{cn} = V_p\angle +120°$ $\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30°$ $\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120°$ $\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120°$ $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30°$ $\mathbf{I}_b = \mathbf{I}_a\angle -120°$ $\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0°$ $\mathbf{V}_{bc} = V_p\angle -120°$ $\mathbf{V}_{ca} = V_p\angle +120°$ $\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	Same as phase voltages $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30°$ $\mathbf{I}_b = \mathbf{I}_a\angle -120°$ $\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0°$ $\mathbf{V}_{bc} = V_p\angle -120°$ $\mathbf{V}_{ca} = V_p\angle +120°$ Same as line currents	Same as phase voltages $\mathbf{I}_a = \frac{V_p\angle -30°}{\sqrt{3}\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a\angle -120°$ $\mathbf{I}_c = \mathbf{I}_a\angle +120°$

Positive or *abc* sequence is assumed.

Power in a Balanced System

- Total instantaneous power, $p(t) = 3V_pI_p \cos\theta$
- Total average power, $P = 3V_pI_p \cos\theta = \sqrt{3}V_LI_L \cos\theta$
- Total reactive power, $Q = 3V_pI_p \sin\theta = \sqrt{3}V_LI_L \sin\theta$
- Total complex power, $\mathbf{S} = 3\mathbf{V}_p\left(\mathbf{I}_p\right)^* = 3I_p{}^2\mathbf{Z}_p = \sqrt{3}V_LI_L\angle\theta$
- Average power per phase, $P_p = V_pI_p \cos\theta$
- Reactive power per phase, $Q_p = V_pI_p \sin\theta$
- Apparent power per phase, $S_p = V_pI_p$
- Complex power per phase, $\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p\left(\mathbf{I}_p\right)^*$

Remember that V_p, I_p, V_L, I_L are rms values.

Example

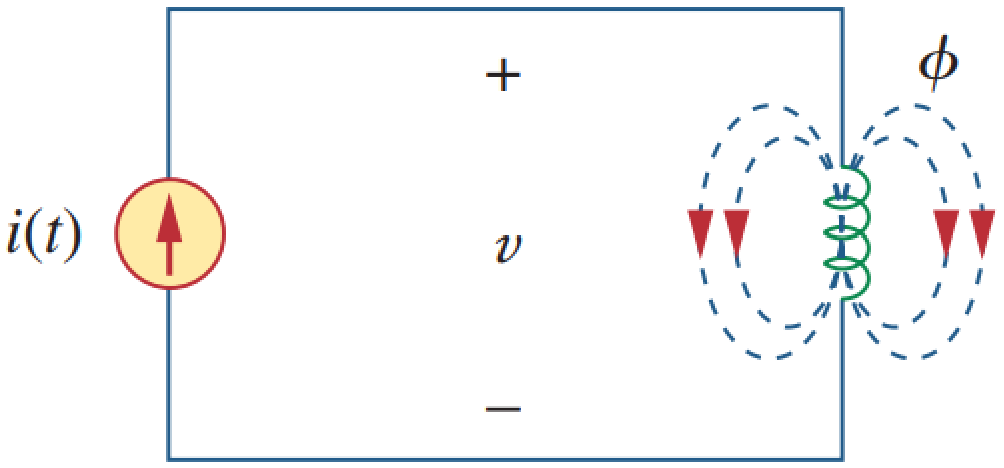
A three-phase motor can be regarded as a balanced Y-load. A three-phase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of the motor.

The apparent power

$$S = \sqrt{3}V_L I_L = \sqrt{3}(220)(18.2) = 6935.13 \text{ VA}$$

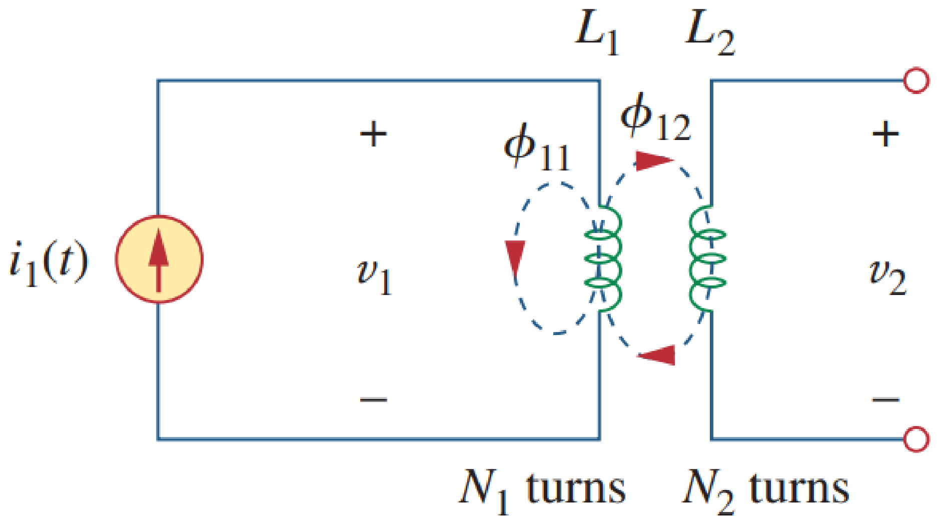
$$\text{pf} = \cos\theta = \frac{P}{S} = \frac{5600}{6935.13} = 0.8075$$

Mutual Inductance



$$v = N\frac{d\phi}{dt} = N\frac{d\phi}{di}\,\frac{di}{dt} = L\frac{di}{dt}$$

Self-inductance (L)



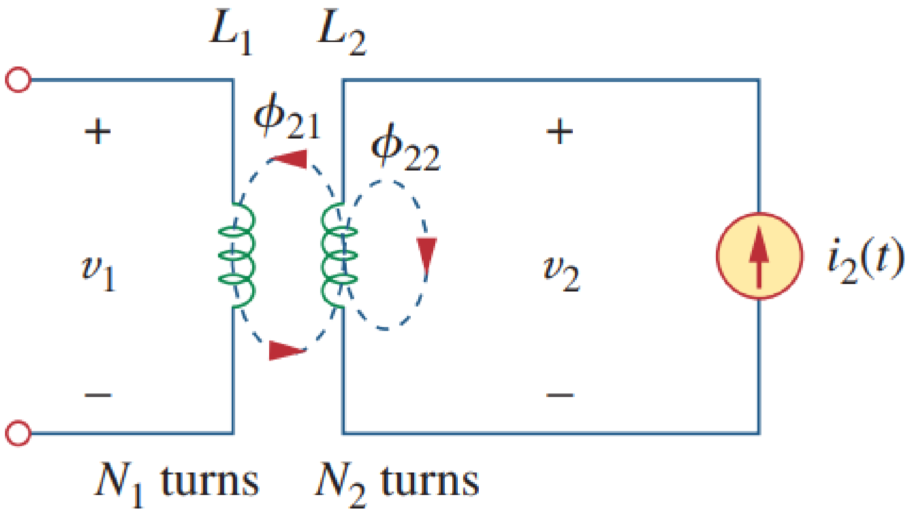
$$\phi_1 = \phi_{11} + \phi_{12}$$

$$v_1 = N_1\frac{d\phi_1}{dt} = N_1\frac{d\phi_1}{di_1}\,\frac{di_1}{dt} = L_1\frac{di_1}{dt}$$

$$v_2 = N_2\frac{d\phi_{12}}{dt} = N_2\frac{d\phi_{12}}{di_1}\,\frac{di_1}{dt} = M_{21}\frac{di_1}{dt}$$

continued...

M_{21} : mutual inductance of coil 2 with respect to coil 1.



$$v_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi_2}{di_2}\frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

$$v_1 = N_1 \frac{d\phi_{21}}{dt} = N_1 \frac{d\phi_{21}}{di_2}\frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

$$\phi_2 = \phi_{21} + \phi_{22}$$

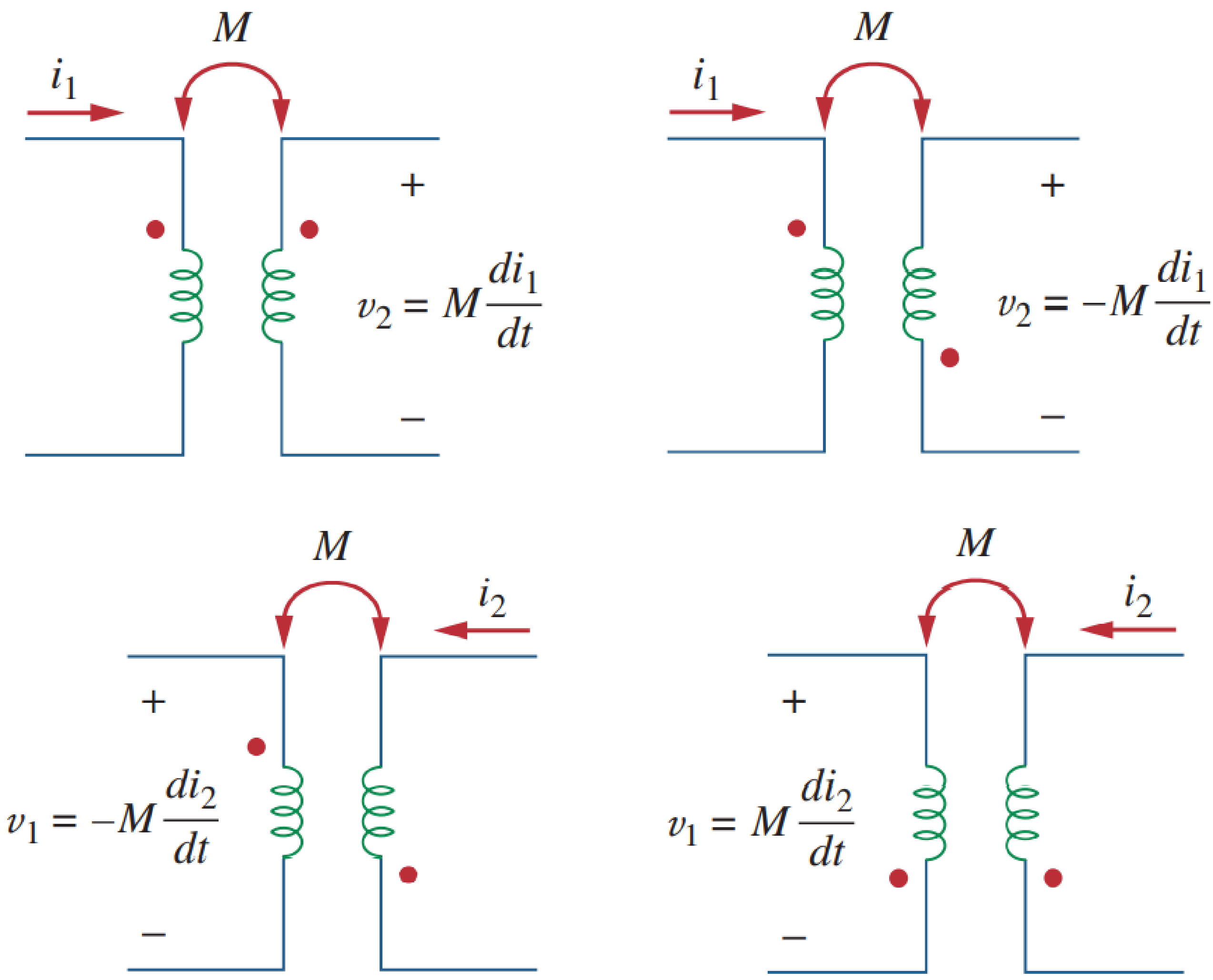
M_{12} : mutual inductance of coil 1 with respect to coil 2.

$$M_{12} = M_{21} = M$$

Mutual inductance is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H).

Dot Convention: Magnetically Coupled Circuit

- If a current enters the *dotted terminal* of one coil, the reference polarity of the mutual voltage in the second coil is **positive** at the *dotted terminal* of the second coil.
- If a current leaves the *dotted terminal* of one coil, the reference polarity of the mutual voltage in the second coil is **negative** at the *dotted terminal* of the second coil.



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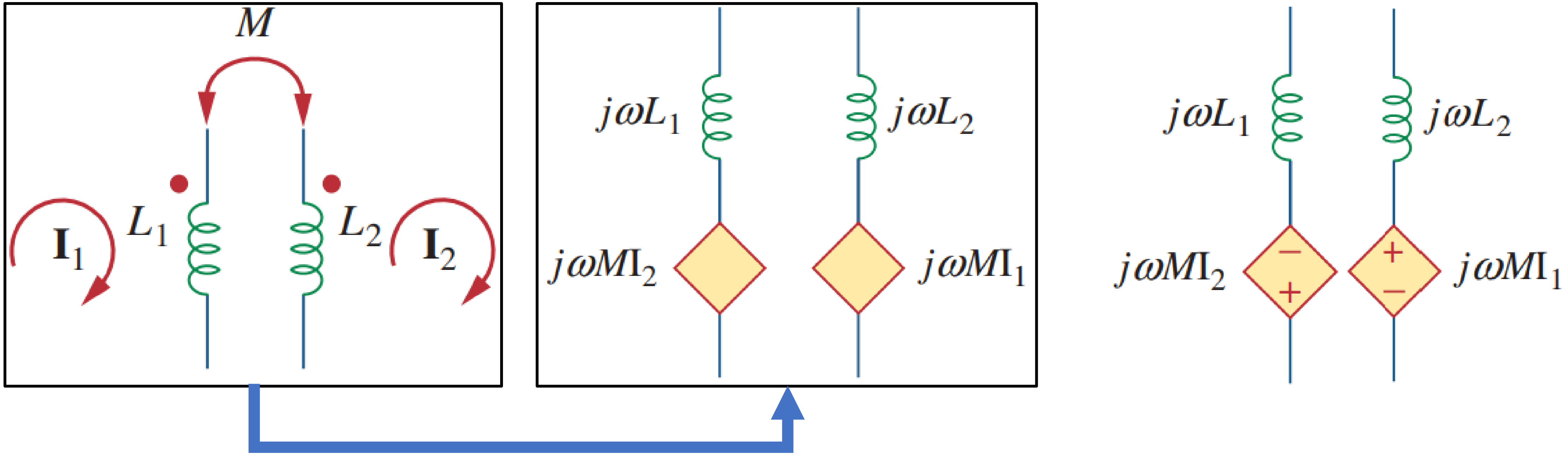
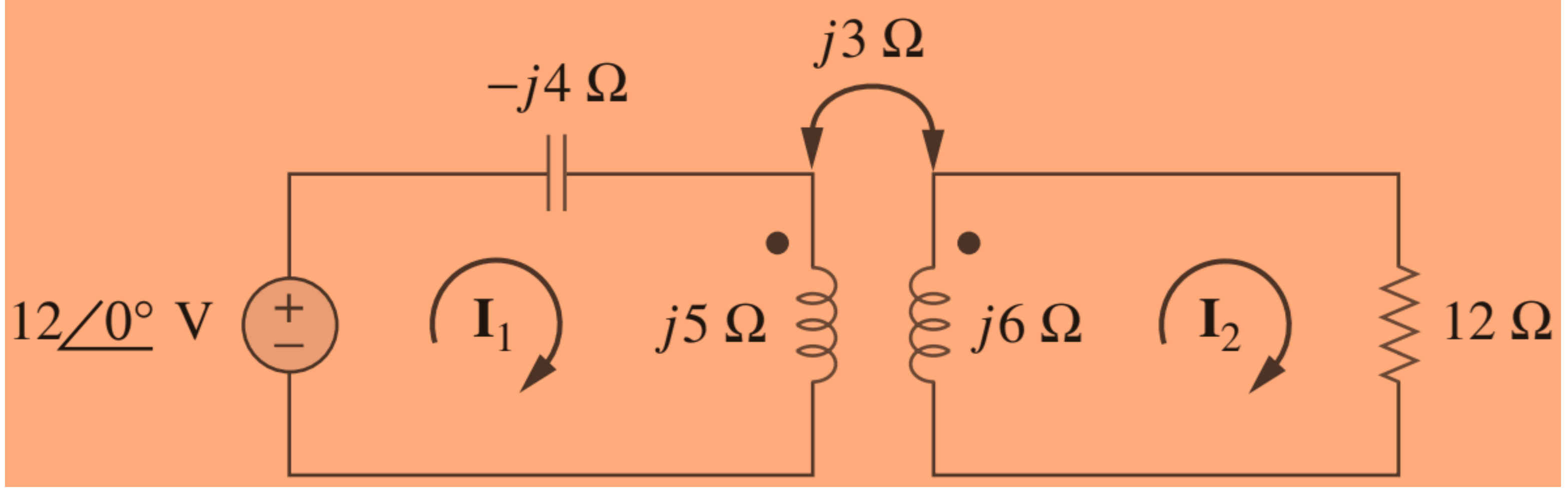
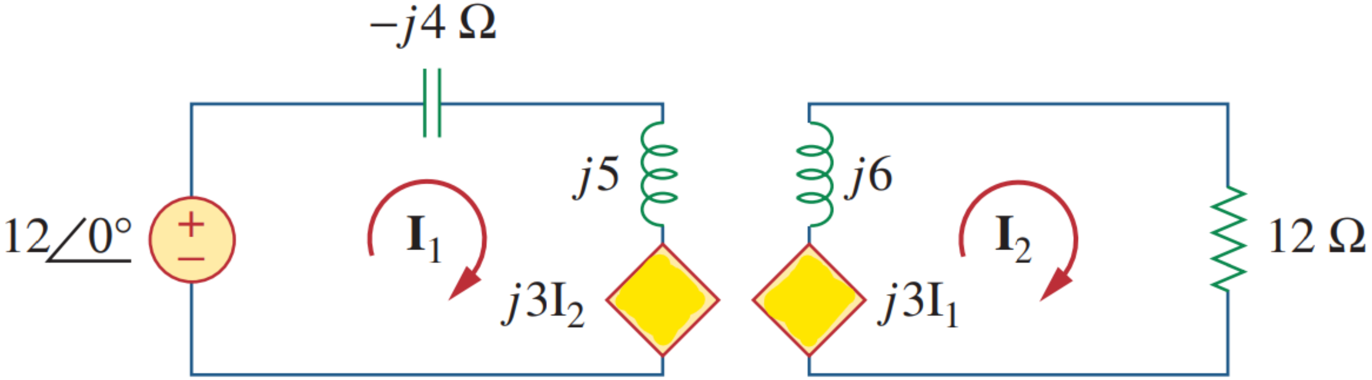


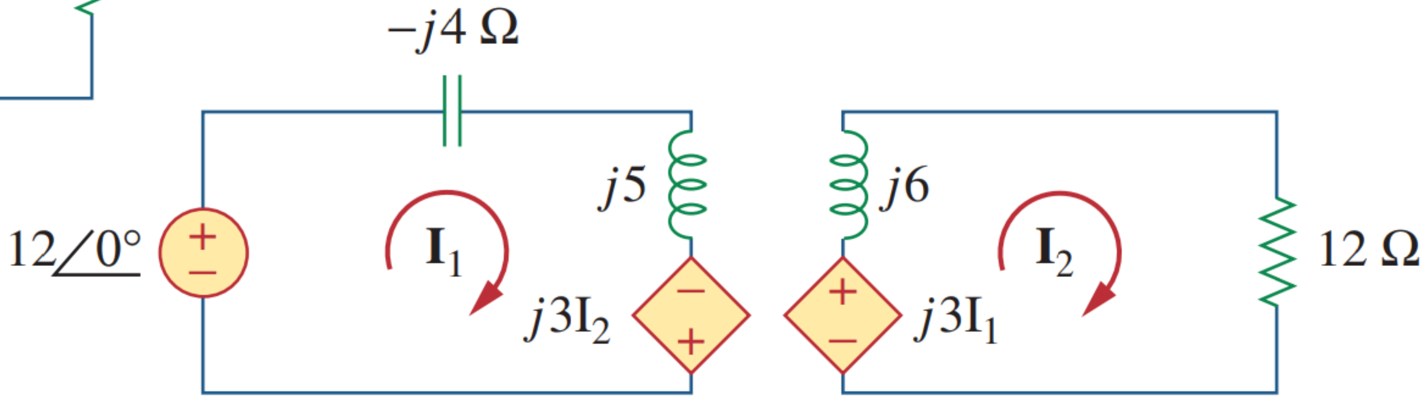
- $\mathbf{I}_1$ enters L_1 at the dotted end, it induces a voltage in L_2 that tries to force a current out of the dotted end of L_2.
- $\mathbf{I}_2$ leaves L_2 at the dotted end, it induces a voltage in L_1 that tries to force a current into the dotted end of L_1.

Example

Calculate the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit







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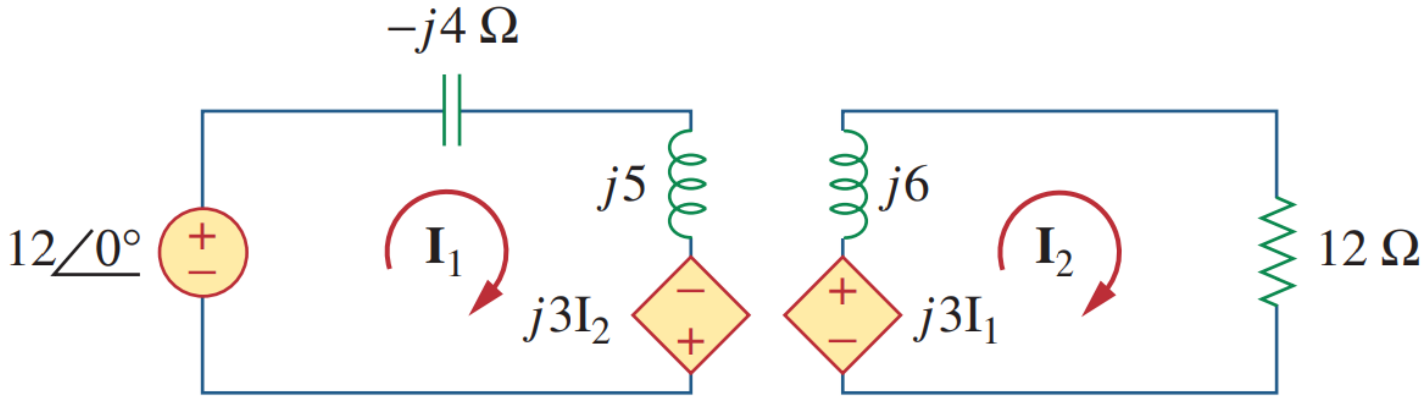


For loop 1,

$$-12 + (-j4 + j5)\mathbf{I}_1 - j3\mathbf{I}_2 = 0 \longrightarrow j\mathbf{I}_1 - j3\mathbf{I}_2 = 12$$

For loop 2,

$$-j3\mathbf{I}_1 + (12 + j6)\mathbf{I}_2 = 0$$

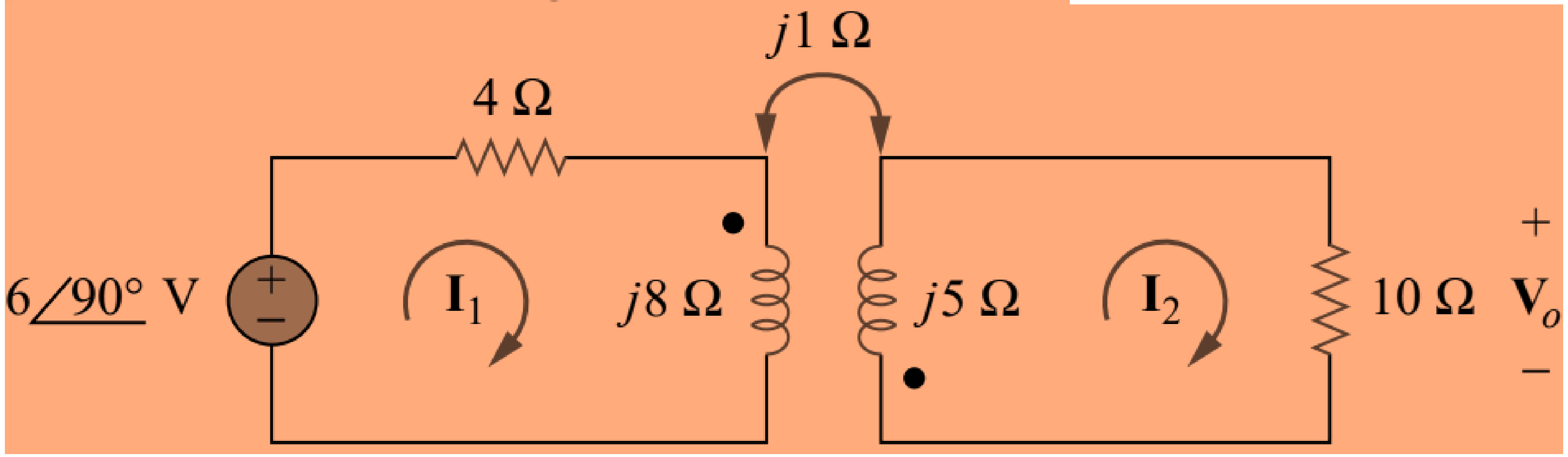
After solving,

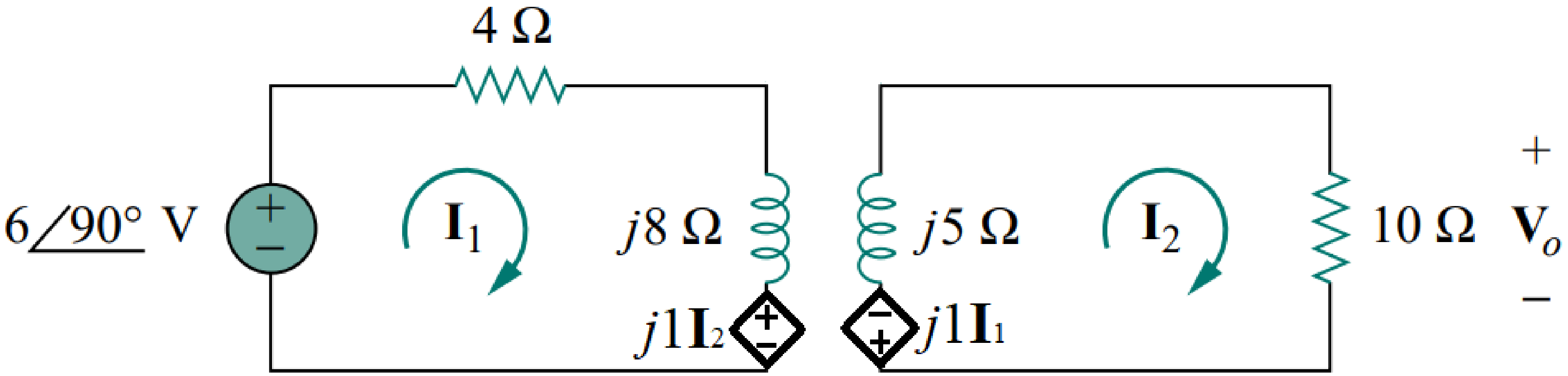
$$\mathbf{I}_1 = 13.01\angle{-49.39^\circ}\ \text{A}$$

$$\mathbf{I}_2 = 2.91\angle{14.04^\circ}\ \text{A}$$

continued...

Determine the voltage $\mathbf{V}_o$ in the circuit





For mesh 1, $-6j + (4 + j8)\mathbf{I}_1 + j1\mathbf{I}_2 = 0$

For mesh 2, $j1\mathbf{I}_1 + (j5 + 10)\mathbf{I}_2 = 0$

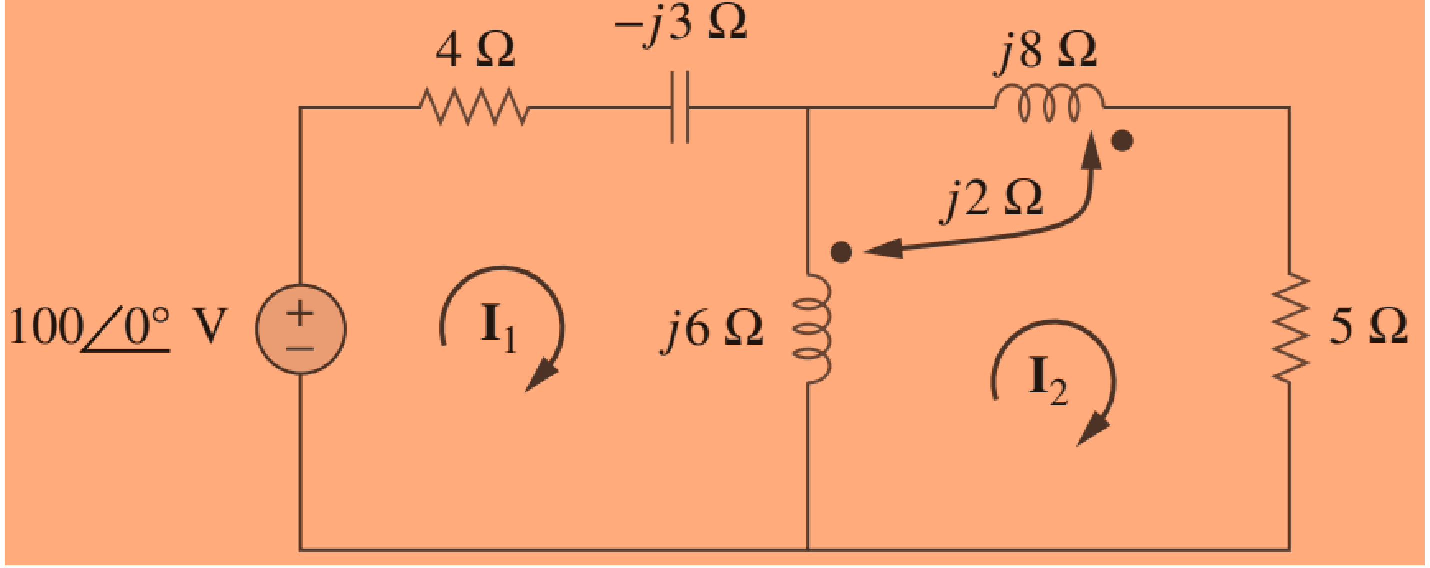
After solving, $\mathbf{I}_2 = -j0.06$

$\therefore \mathbf{V}_o = 10I_2$

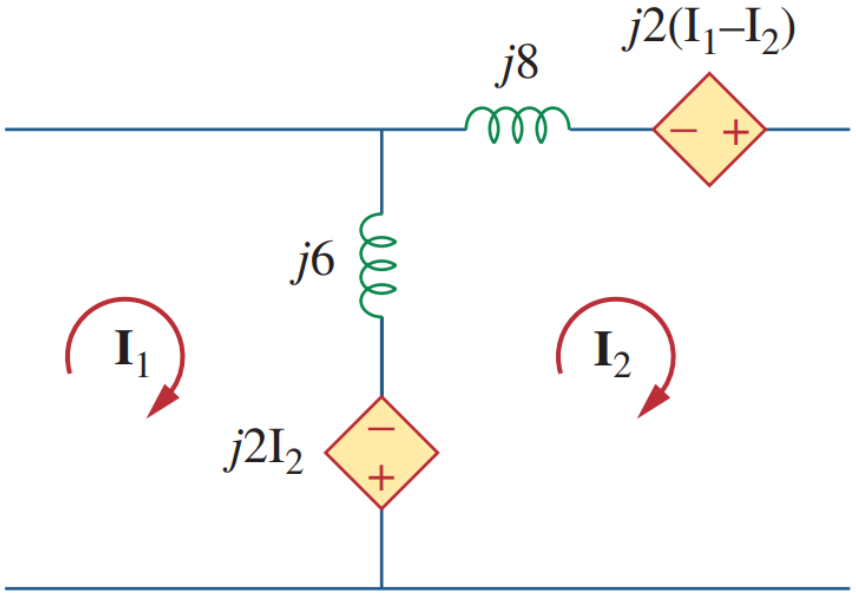
$\Longrightarrow \mathbf{V}_o = -j0.6 = 0.6\angle -90°\ V$

continued…

Calculate the mesh currents in the circuit



$$\mathbf{I}_1 = 20.3\angle 3.5^\circ \text{ A}$$
$$\mathbf{I}_2 = 8.693\angle 19^\circ \text{ A}$$



- For mesh 1,

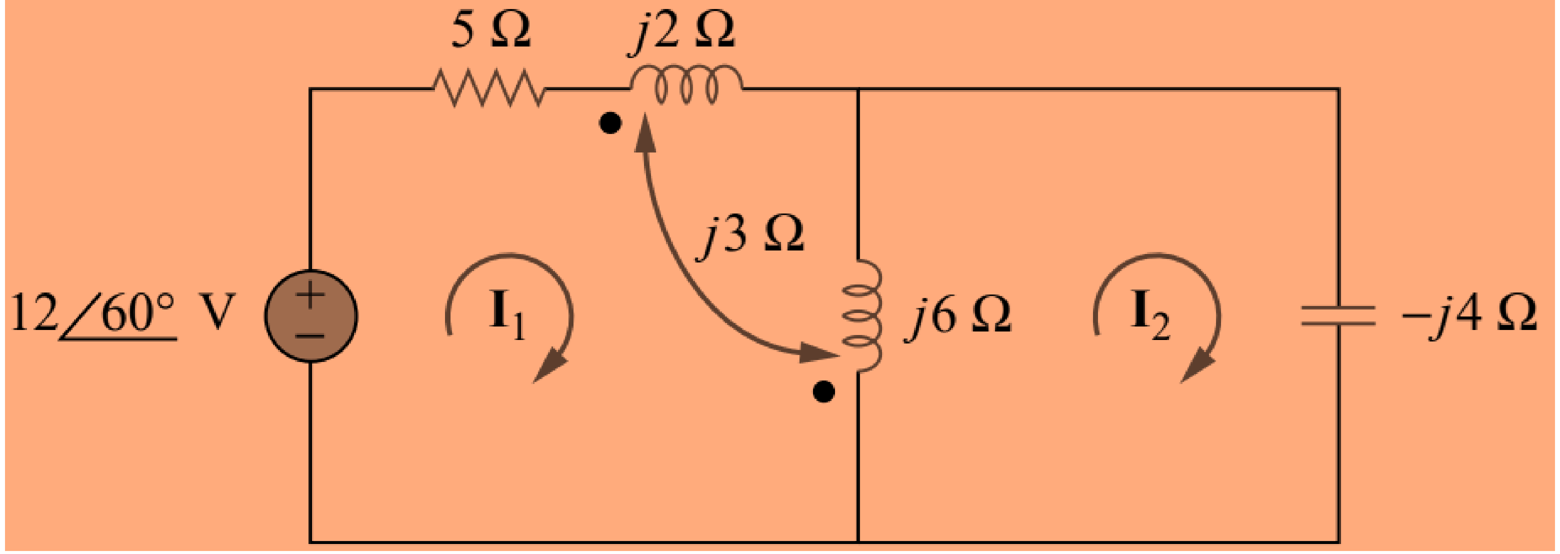
$$-100 + (4 - j3 + j6)\mathbf{I}_1 - j6\mathbf{I}_2 - j2\mathbf{I}_2 = 0$$
$$\Rightarrow (4 + j3)\mathbf{I}_1 - j8\mathbf{I}_2 = 100$$

- For mesh 2,

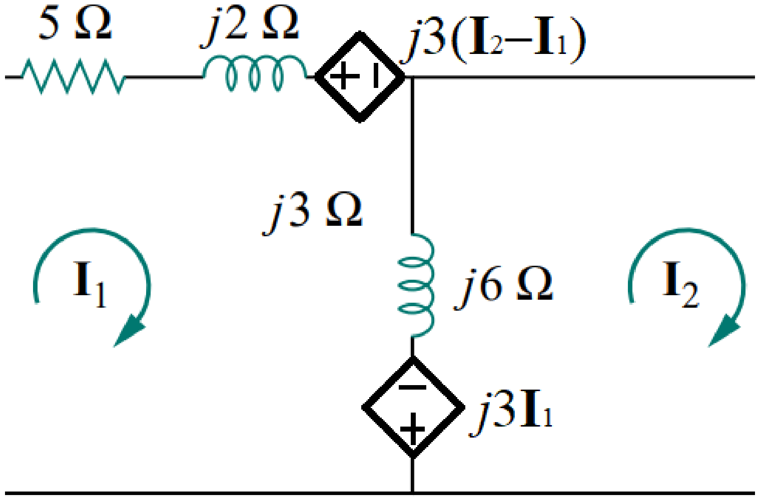
$$j2\mathbf{I}_2 + (j6 + j8 + 5)\mathbf{I}_2 - j6\mathbf{I}_1 - j2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$
$$\Rightarrow -j8\mathbf{I}_1 + (5 + j18)\mathbf{I}_2 = 0$$

continued…

Determine the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit



$$\mathbf{I}_1 = 2.15\angle 86.56^\circ$$
$$\mathbf{I}_2 = 3.23\angle 86.56^\circ \text{ A}$$



- For mesh 1,

$$-(12\angle 60^\circ) + (5 + j2 + j3)\mathbf{I}_1 - j6\mathbf{I}_2 + j3(\mathbf{I}_2 - \mathbf{I}_1) = 0$$
$$\Longrightarrow (5 + j2)\mathbf{I}_1 - j3\mathbf{I}_2 = 6 + j6\sqrt{3}$$

- For mesh 2,

$$j3\mathbf{I}_1 + (j6 - j4)\mathbf{I}_2 - j6\mathbf{I}_1 = 0$$
$$\Longrightarrow -1.5\mathbf{I}_1 + \mathbf{I}_2 = 0$$